A COMPARISON OF THE COMMONSENSE AND
FIXED POINT THEORIES OF NONMONOTONICITY

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ABSTRACT

The mathematical fixed point theories of nonmonotonic reasoning are examined and compared to a commonsense theory of nonmonotonic reasoning which models our intuitive ability to reason about defaults. It is shown that all of the known problems of the fixed point theories are solved by the commonsense theory. The concepts of this commonsense theory do not involve mathematical fixed points, but instead are explicitly defined in a monotonic modal quantificational logic which captures the modal notion of logical truth.

I INTRODUCTION

A number of recent papers [McDermott & Doyle, McDermott, Moore, and Reiter] have attempted to formalize the commonsense notion of something being possible with respect to what is assumed. All these papers have been based on the mathematical theory of fixed points. For example, [McDermott & Doyle] describes a rather baroque theory of nonmonotonicity in which sentences such as 'A are discovered to be theorems of a system by determining if 'A is in the intersection of possibly infinite numbers of sets which are the fixed points of the theorems generated by applying inference rules to axioms and possibility statements in all possible ways. Explicitly if K is the set of axioms it must be determined whether:

('A is in the (intersection of all S such that
                    (S is a fixed point of K)))
where:
(S is a fixed point of K) iff S =
    (Theorems of (union K
        {'(,P is possible with respect
                            to what is assumed):
        P is not in S}))).

The main problem with such "mathematical fixed point" theories of nonmonotonicity is that even if the theorems of these theories were in accord with our primitive intuitions (which they are not as we shall see in section 3) and even if deductions could be carried out in such theories (and this is not likely since they inherently involve proofs by mathematical induction over both the classical theorem generation process and the process of generating sentences) by no stretch of the imagination would those deductions reflect our commonsense understanding of the concept of something being possible with respect to what is assumed. For what after all have intersections of infinite sets, mathematical fixed points, infinite sets of theorems generated by formalized deduction procedures, mathematical induction over formalized deduction procedures, or even formalized deduction procedures themselves to do with commonsense arguments about nonmonotonicity (such as for example the argument presented in section 2 below)? In our opinion, commonsense nonmonotonic arguments do not involve such concepts, at any conscious level of human reasoning, and therefore to try to explain such concepts in that terminology is an extraordinary perversion of language that is likely to lead only to unintuitive theories. The unintuitiveness of these fixed point theories is in fact recognized by some of the very proponents of these theories although they tend to view said unintuitiveness as an intrinsic property of nonmonotonic reasoning rather than as a mere artifact of their particular theories. For example, [McDermott] states "As must be clear to everyone by now, using defaults in reasoning is not a simple matter of 'commonsense', but is computationally impossible to perform without error" and "we must attempt another wrenching of existing intuitions." Generally, we suggest that the problems with these fixed point theories is a consequence of trying to model commonsense reasoning by semantic analysis rather than by developing a calculus which directly models that commonsense reasoning.

We briefly describe our commonsense theory on nonmonotonicity in section 2 and then compare it to the fixed point theories in section 3.

II THE COMMONSENSE THEORY

The basic idea of our commonsense theory of nonmonotonicity is that nonmonotonicity is already encompassed in the normal intensional logic of everyday commonsense reasoning and can be explained precisely in that terminology.

For example, a knowledge base consisting of a simple default axiom expressing that a particular bird flies whenever that bird flies is possible with respect to what is assumed is stated as:

(that which is assumed is
    (if (A is possible with respect
                    to what is assumed) then A))

where A stands for the proposition that that particular bird flies.

Reflection on the meaning of this knowledgebase leads immediately to the conclusion that either A is logically possible and the knowledgebase is synonymous to A, or A is not logically possible and the knowledgebase is synonymous to logical truth. This conclusion is obtained by simple case analysis: for either A is possible with respect to what is assumed or it is not. If A is possible with respect to what is assumed then, since (if truth then A) is just A, that which is assumed is indeed A. Since that which is assumed is A, A is possible with respect to what is assumed only if A is logically possible. On the other hand, if A is not possible with respect to what is assumed then since falsity implies A is just truth, that which is assumed is truth. Since that which is assumed is truth, A is not possible with respect to what is assumed only if A is not logically possible. Thus if it is further assumed that A is logically possible, then it follows that the knowledgebase is synonymous to A itself. The nonmonotonic nature of these expressions becomes apparent if an additional proposition that that particular bird does not fly is added to the knowledgebase:

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(that which is assumed is
   (and (not A)
        (if (A is possible with respect
                 to what is assumed) then A)))
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Reflection on this new knowledgebase leads immediately to the conclusion that it is synonymous to not A. This conclusion is again obtained by simple case analysis: for if A is possible with respect to what is assumed then, since (if truth then A) is just A, that which is assumed is indeed ((not A)and A) which is falsity. Since that which is assumed is falsity A is possible with respect to what is assumed only if A and falsity is logically possible which it is not. Thus A is not possible with respect to what is assumed. On the other hand, if A is not possible with respect to what is assumed then, since falsity implies A is just truth, that which is assumed is just not A. Since that which is assumed is (not A), A is not possible with respect to what is assumed only if A and (not A) is not logically possible which is the case. Thus it follows that the knowledgebase is synonymous to (NOT A).

Therefore, whereas the original knowledgebase was synonymous to A the new knowledgebase, obtained by adding (not A), is synonymous, not to falsity, but to (not A) itself.

These simple intuitive nonmonotonic arguments involve logical concepts such as not, implies, truth, falsity, logical possibility, possibility with respect to some assumed knowledgebase, and synonymity to a knowledgebase. The concepts: not, implies, truth (i.e. T), and falsity (i.e. NIL) are all concepts of (extensional) quantificational logic and are well known. The remaining concepts: logical possibility, possibility with respect to something, and synonymity of two things can be defined in a very simple modal logic extension of quantificational logic,

which we call Z[Brown 1,2,3,4]. The axiomatization of the modal logic Z is described in detail below. But briefly, it consists of (extensional quantificational logic) plus the intensional concept of something being logically true written as the unary predicate: (LT P). The concept of a proposition P being logicaly possible and the concept of two propositions being synonymous are then defined as:

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(POS P) = (NOT(LT(NOT P))) P is logically possible
(SYN P Q) = (LT(IFF P Q))  P is synonymous to Q
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The above knowledgebases and arguments can be formalized in the modal logic Z quite simply by letting some letter such as K stand for the knowledgebase under discussion. The idiom "that which is assumed is X" can then be rendered to say that K is synonymous to X, and the idiom "X is possible with respect to what is assumed" can be rendered to say that K and X is possible:

```
(that which is assumed is X) = (SYN K X)
(X is possible with respect
        to what is assumed) = (POS(AND K X))
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These two idioms are indexial symbols referring implicitly to some particular knowledgebase K under discussion. This knowledgebase referenced by the (X is possible with respect to what is assumed) idiom is always the meaning of the symbol generated by the enclosing (that which is assumed is X) idiom. Each occurrence of the (that which is assumed is X) idiom always generates a symbol (unique to the theory being discussed) to stand for the database under discussion. These knowledgebases have been expressed solely in terms of the modal quantificational logic Z. In particular, the nonmonotonic concepts were explicitly defined in this logic. The intuitive arguments about the meaning of these nonmonotonic knowledgebases can be carried out solely in the modal quantificational logic Z. Most importantly, our commonsense understanding and reasoning about nonmonotonicity is directly represented by the inference steps of this formal theory. Therefore, it is clear that nonmonotonic reasoning needs no special axioms or rules of inference because it is already inherent in the normal intensional logic of everyday commonsense reasoning as modeled by the modal quantificational logic Z. It remains only to axiomatize the logic Z.

Our theory Z of commonsense intensional reasoning is a simple modal logic [Lewis] that captures the notion of logical truth. The symbols of this modal logic consist of the symbols of (extensional) quantificational logic plus the primitive modal symbolism: (LT p) which is truth whenever the proposition p is logically true.

The axioms and inference rules of this modal logic include the axioms and inference rules of (extensional) quantificational logic similar to that used by Frege in Begriffsschrift [Frege], plus the following inference rule and axioms about the concept of logical truth.

The Modal Logic Z

R0: from p infer (LT p)

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A1: (IMPLY(LT P) P)
A2: (IMPLY(LT(IMPLY P Q)) (IMPLY(LT P)(LT Q)))
A3: (OR(LT P) (LT(NOT(LT P))))
A4: (IMPLY(ALL Q(IMPLY(WORLD Q)
                                    (LT(IMPLY Q P))))
        (LT P))
A5: (ALL S(POS(meaning of the generator
                                subset S)))
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The inference rule R0 means that p is logically true may be inferred from the assertion of p to implicitly be logically true. The consequence of this rule is that a proposition P may be asserted to be logically true by writing just:

P

and that a proposition P is asserted to be true in a particular world or state of affairs W by writing:

(LT(IMPLY W P))

The axiom A1 means that if P is logically true then P. Axiom A2 means that if it is logically true that P implies Q then if P is logically true then Q is logically true. Axiom A3 means that P is logically true or it is logically true that P is not logically true. The inference rule R0 and the axioms A1, A2 and A3 constitute an S5 modal logic. A good introduction to modal logic in general and in particular to the properties of the S5 modal logic is given in [Hughes and Cresswell]. Minor variations of the axioms A1, A2, and A3 were shown in [Carnap] to hold for the modal concept of logical truth. We believe that the additional axioms, namely A4 and A5, are needed in order to precisely capture the notion of logical truth.

The axiom A4 states that a proposition is logically true if it is true in all worlds. We say that a proposition P is a world iff P is possible and P is complete, that P is complete iff for all Q, P determines Q, that P determines Q iff P entails Q or P entails not Q, that P entails Q iff it is logically true that P implies Q, and that P is possible iff it is not the case that not P is logically true. These definitions are given below:

```
(WORLD P)=df (AND(POS P)(COMPLETE P))  P is a world
(COMPLETE P) = df (ALL Q(DET P Q))     P is complete
(DET P Q)= df (OR(ENTAIL P Q)
                            (ENTAIL P(NOT Q)))  P determines Q
(ENTAIL P Q) = df (LT(IMPLY P Q))     P entails Q
(POS P) = df (NOT(LT(NOT P)))         P is possible
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Thus a world is a possible proposition which for every proposition entails it or its negation. The axiom A5 states that the meaning of every conjunction of the generated contingent propositions or their negations is possible. We call this axiom "The Axiom of the Possibility of Contingent facts" or simply the "Possibility Axiom". The need for this axiom follows from the fact that the other axioms of the modal logic do not imply certain elementary facts about the possibility of conjunctions of distinct possibly negated atomic expressions consisting of nonlogical symbols. For example, if we have a theory formulated in our modal logic which contains the nonlogical atomic expression (ON A B) then since (ON A B) is not logically true, it

follows that (NOT(ON A B)) must be possible. Yet (POS(NOT(ON A B))) does not follow from these other axioms. Likewise, since (NOT(ON A B)) is not logically true (ON A B) must be possible. Yet (POS(ON A B)) does not follow from the other axioms. Thus these contingent propositions (ON A B) and (NOT(ON A B)) need to be asserted to be possible. There are a number of ways in which this may be done and these ways essentially correspond to different ways the idiom: (P is a meaning combination of the generators) may be rendered. In this paper we have chosen a general method which is applicable to just about any contingent theory one wishes. This rendering is given below:

```
(meaning of the generator subset S) = df
        (ALL G(IMPLY(GENERATORS G)
                            (IFF(S G)(GMEANING G)) ))
(GMEANING '(p ,X1...,XN)) df
        (p(GMEANING X1)...(GMEANING XN))
    for every contingent symbol p of arity n.
(GENERATORS) - df (LAMBDA(A)(A is a contingent
                        variable free simple sentence))
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We say that the meaning of the generator subset S is the conjunction of the GMEANINGs of every generator in S and the negation of the GMEANINGS of all the generators not in S. The generator meaning of any expression beginning with a contingent symbol 'p is p of the GMEANING of its arguments. The generators are simply any contingent variable free atomic sentences we wish to use.

One of the most striking features of nonmonotonic knowledgebases is that they are sometimes described in terms of themselves. Such knowledgebases are said to be reflexive[Hayes]. For example, the knowledgebase K purportedly defined by the axiom:

(SYN K (IMPLY(POS(AND K A))A))

is defined as being synonymous to the default: (IMPLY(POS(AND K A))A) which in turn is defined in terms of K. Thus this purported definition of K is not actually a definition at all but is merely an axiom describing the properties possessed by any knowledgebase K satisfying this axiom. In general, a purported definition of a knowledgebase:

(SYN K(f K))

will be implied by zero or more explicit definitions of the form:

(SYN K g)

where K does not occur in g. The explicit definitions which imply a purported definition of a knowledgebase are called the solutions of that purported definition. In general a purported definition may have zero or more solutions. For example, (SYN K(NOT K)) is (LT(IFF K(NOT K))) which is (LT NIL) which is NIL and therefore has no solutions, and (SYN K K) is (LT(IFF K K)) which is (LT T) which is T and therefore has all solutions. Finally, (SYN K G) where K does not occur in G is an explicit definition of K and therefore has only one solution namely itself.

Because K is the knowledgebase under discussion, it is not itself a contingent proposition of that knowledgebase. Thus 'K is not a GENERATOR and the possibility axiom A5 will not

apply to it.

As an example of how the modal logic Z is used, we carry out in Z a slightly more general argument similar to the unformalized nonmonotonic arguments described above. This argument is about a knowledgebase K consisting of (a conjunction of) axioms G not containing K plus one additional standard default axiom. A standard default axiom is an axiom of the form:

(IMPLY(POS(AND K A))(IMPLY B A))

This structure contains as instances default axioms such as:

(IMPLY(POS(AND K(CAN-FLY ENTERPRISE))))
    (IMPLY(IS-SPACE-SCHUTTLE ENTERPRISE)
        (CAN-FLY ENTERPRISE)))

T1: A knowledgebase containing exactly one variable free standard default has precisely one solution.
(IFF(SYN K(AND G(IMPLY(POS(AND K A))(IMPLY B A))))
    (SYN K(AND G(IMPLY(POS(AND G A))(IMPLY B A)))))
                        proof
(SYN K(AND G(IMPLY(POS(AND K A))(IMPLY B A))))
(IF(POS(AND K A))
    (SYN K(AND G(IMPLY(AND B T)A)))
    (SYN K(AND G(IMPLY(AND B NIL)A))))
(IF(POS(AND K A))
    (SYN K(AND G(IMPLY B A)))
    (SYN K G))
(OR(AND(POS(AND K A))(SYN K(AND G(IMPLY B A))))
    (AND (NOT(POS(AND K A)))(SYN K G)))
(OR(AND(POS(AND G(IMPLY B A)A))
        (SYN K(AND G(IMPLY B A))))
    (AND(NOT(POS(AND G A)))(SYN K G)))
(OR(AND(POS(AND G A))(SYN K(AND G(IMPLY B A))))
    (AND(NOT(POS(AND G A)))(SYN K G)))
(IF(POS(AND G A))
    (SYN K(AND G(IMPLY B A)))
    (SYN K G))
(SYN K(IF(POS(AND G A))(AND G(IMPLY B A))G))
(SYN K(AND G(IF(POS(AND G A))(IMPLY B A)T)))
(SYN K(AND G(IMPLY(POS(AND G A))(IMPLY B A))))

The solutions to the two purported definitions of the informal arguments given at the start of this section are obtained from theorem T1 as corollaries for if G is T, B is T, and A is possible it follows that:
(IFF(SYN K(IMPLY(POS(AND K A))A))
    (SYN K A))
and if G is (NOT A) and B is T it follows that:
(IFF(SYN K(AND(NOT A)(IMPLY(POS(AND K A))A)))
    (SYN K(NOT A)))
We now compare our commonsense theory of nonmonotonicity to the fixed point theories.

III THE FIXED POINT THEORIES

In this section we examine four fixed point theories: [McDermott & Doyle, McDermott, Moore, and Reiter] and comment on their modelling of our commonsense intuitions and on their computational tractability.

[Reiter] presents a theory of nonmonotonicity called "A Logic for Default Reasoning" which is essentially a first order logic supplemented with additional inference rules of the form:
    from (A X),(m(B1 X)),...,(m(Bn X)) infer (C X)

where "m" is not a symbol of the theory, but like "infer" is merely part of the structural syntax of the inference rule itself. This rule is intended to mean that if A holds and all Bs are possible then C may be inferred. The problem with this default theory is that even though it uses the concept of being possible with respect to what is assumed, it does not allow the inference of any laws at all about the concept of being possible with respect to what is assumed because the possibility symbol "m" is not part of the formal language. Thus, although there is a certain pragmatic utility to this theory, it does not actually axiomatize the concept M of being possible with respect to what is assumed.

[McDermott & Doyle] describes a nonmonotonic logic which was intended to capture the notion of a sentence being consistent with the sentences in a given knowledgebase: "We first define a standard language of discourse including the nonmonotonic modality M ('consistent')." Since the intended meaning of their symbol M is essentially our idiom (that which is possible with respect to what is assumed) if the knowledgebase is K the intended meaning of the notion M could be defined in our logic as:
    (M X) = df (POS(AND K X))
There are two problems with this theory. First, as pointed out in [McDermott & Doyle] it is computationally intractible: "there seems to be no procedure which will tell you when something is a theorem" and in fact no proof procedure is given for even a first order quantificational nonmonotonic logic. Second, again as is pointed out in [McDermott & Doyle] this theory is too weak to actually capture the notion of consistency with a knowledgebase: "Unfortunately, the weakness of the logic manifests itself in some disconcerting exceptional cases which indicate that the logic fails to capture a coherent notion of consistency". All these disconcerting cases are solved in our theory.

The first such problem is that the knowledgebase K consisting of the expression:
    (AND(M A)(NOT A))
is not synonymous to falsity in their logic even though intuitively it should be since (NOT A) is in K and therefore (AND K A) is contradictory. This problem is solved in our theory of nonmonotonicity since:
    (IFF(SYN K(AND G(POS(AND K A))(NOT A)))
        (SYN K NIL))
A second problem with their logic, as they point out, is that (M A) does not follow from (M(AND A B)), even though intuitively it should. This problem is solved in our theory since:
    (IMPLY(POS(AND A B))(POS A))
    (IMPLY(NOT(LT(NOT(AND A B))))(NOT(LT(NOT A))))
    (IMPLY(LT(NOT A))(LT(NOT(AND A B))))
which by A2 of the modal logic Z is implied by:
    (LT(IMPLY(NOT A)(NOT(AND A B))))
    T
McDermott and Doyle consider their logic to have a third problem, namely that a theory consisting of '(AND(IMPLY(M A)B)(NOT B)) where 'A and 'B are simple sentences (ie. GENERATORS in our terminology) is incoherent because it has no fixed point. However, intuitively, whether the knowledgebase consisting of this axiom has a

solution or not depends precisely on whether (AND A(NOT B)) is logically possible or not; for if (AND A(NOT B)) is not logically possible, then it is not possible with respect to any K, and therefore K is synonymous to (NOT B) and if it is logically possible then B is in K and therefore the false proposition (AND A(NOT B)B) would have to be logically possible (which it cannot be) for there to be a solution. Since 'A and 'B are assumed to be generators, it follows that (AND A(NOT B)) is possible. Therefore intuitively such a knowledgebase K should not have any solutions. We therefore do not consider this example to be a defect of their theory. This same point is made in [Moore2] where this example was analyzed from the perspective of Stalnaker's [Moore2] theory. This example does, however, illustrate that the theory in [McDermott&Doyle] only applies to generators, for if A were falsity or were synonymous to B then there would be a solution, namely that K is synonymous to (NOT B). Therefore:

(IFF(SYN K(AND(IMPLY(POS(AND K A))B)(NOT B)))
    (AND(NOT(POS(AND(NOT B)A)))(SYN K(NOT B))))

Thus if 'A and 'B are assumed to be generators, it follows that

(IFF(SYN K(AND(IMPLY(POS(AND K A))B)(NOT B)))
    NIL)

[McDermott] makes a second attempt to find a coherent theory of nonmonotonicity. This attempt is based essentially on the idea of supplementing the theorem generation process with the rules of inference and axioms of a modal logic. Because it is based on the same general set theoretic fixed point constructions as in [McDermott & Doyle] this new theory is just as computationally intractible. The "necessity operator": L of these nonmonotonic modal logics intuitively mean that something is entailed by what is assumed (i.e. that the negation of that thing is not possible with respect to what is assumed). Thus the intuitive meaning of L could be captured in our modal logic Z by the definition:

(L A) = df (ENTAIL K A) (i.e (NOT(M(NOT A))))

Three modal logics: T, S4, and S5 are investigated because McDermott does not believe any one is superior to the others: "The reason why I study a variety of modal systems is that they are all closely related, and no one is obviously better than the others." This statement is entirely correct because none of these three modal logic extensions of the nonmonotonic theory captures the intuitive notion of being possible with respect to what is assumed. The problem with the first two logics: T and S4 is that they are too weak. For example, one problem with [McDermott]'s nonmonotonic S4, as is therein pointed out, is that a knowledgebase K consisting of the expression:

'(IMPLY(L(M A))(NOT A))

where 'A is a simple sentence (i.e. a GENERATOR in our terminology) is not contradictory although intuitively it should be. For if (L(M A)) is the case then the knowledgebase is synonymous to (NOT A) and (M A) is contradictory making (L(M A)) contradictory. And if (L(M A)) is not the case then the knowledgebase is synonymous to T and since (L(MT)) is the case a contradiction results. This problem is solved in our theory of

nonmonotonicity since:

(IFF(SYN K(IMPLY(LT(IMPLY K(POS(AND K A))))
                (NOT A)))
    (OR(AND(SYN A T)(SYN K NIL))
       (AND(SYN A NIL)(SYN K T))))

Thus, when 'A is a generator there are no solutions:

(IFF(SYN K(IMPLY(LT(IMPLY K(POS(AND K A))))
                (NOT A)))
    NIL)

Thus, even Nonmonotonic S4 (and since T is weaker than S4 it too) is too weak to capture the notion of being possible with respect to what is assumed. There remains only the question whether [McDermott]'s nonmonotonic S5 captures the notion of being possible with respect to what is assumed. One problem with this nonmonotonic S5 logic, as is therein pointed out, is that a knowledgebase consisting of the simple default:

(IMPLY(M A)A)

has a fixed point containing (NOT A). This bizarre result follows from the fact that in McDermott's theory the additional default:

'(IMPLY(M(NOT A))(NOT A))

which is logically derivable in the knowledgebase from the first default is (in our terminology) incorrectly assumed to be part of what entails the knowledgebase. Thus, in McDermott's S5 logic a knowledgebase containing a default always (in our terminology) includes in its purported definition the opposite default thus giving the situation:

(IFF(SYN K(AND(IMPLY(POS(AND K A))A)
              (IMPLY(POS(AND K(NOT A)))(NOT A))))
    (OR(SYN K A)(SYN K(NOT A))))

which states that a knowledgebase with two opposite defaults has two solutions A and (NOT A). The unintuitiveness of having a default actually default to the opposite of what is specified is recognized by McDermott: "Surely the logic should draw some distinction between a default and its negation if it is to be a logic of defaults at all." (In fact [McDermott]'s nonmonotonic S5 logic is so bizarre that as is pointed out therein it is not nonmonotonic after all as its theorems are just those of monotonic S5 modal logic.)

This problem of defaults does not appear in our theory of nonmonotonicity because we do not make the erronious assumption that the derived default is part of what entails the knowledgebase K:

(SYN K(IMPLY(POS(AND K A))A))

Thus, even though either default is equivalent in the knowledgebase K:

(IFF(ENTAIL K(IMPLY(POS(AND K A))A))
    (ENTAIL K(IMPLY(POS(AND K(NOT A)))(NOT A))))

and therefore that the first default is equivalent to the conjunction of two:

(IFF(ENTAIL K(IMPLY(POS(AND K A))A))
    (ENTAIL K(AND(IMPLY(POS(AND K A))A)
                 (IMPLY(POS(AND K(NOT A)))
                       (NOT A)))))

and that K entails the two defaults it does not follow that K is synonymous to the two defaults:

(SYN K(AND(IMPLY(POS(AND K A))A)
          (IMPLY(POS(AND K(NOT A)))(NOT A))))

is false because the two defaults do not entail K:

(ENTAIL(AND(IMPLY(POS(AND K A))A)
           (IMPLY(POS(AND K(NOT A)))(NOT A)))
       K) is false.

These facts are verified by theorem T1 which proves that a knowledgebase
(SYN K(IMPLY(POS(AND K A))A))
consisting of one default(even though the opposite default is entailed by it) has only one solution, namely A.

Another problem with [McDermotts]'s nonmonotonic S5, as [Moore2] points out is that for every A, the S5 axiom (IMPLY(L A)A) causes every knowledgebase to have (in the absence of information to the contrary) a fixed point which contains A. This is not a problem in our system because again we do not make the erronious assumption that this modal axiom is (in our terminology) part of what entails the knowledgebase. Thus, even though any knowledgebase K containing:
    (IMPLY(LT(IMPLY K A))A)
has in the absence of additional information, a solution containing A:
(IFF(SYN K(AND G(IMPLY(LT(IMPLY K A))A)))
    (OR(SYN K(AND G A))
        (AND(POS(AND G(NOT A)))(SYN K G))))
and even though: (ENTAIL K(LT(IMPLY K A))) it does not follow that: (SYN K(LT(IMPLY   K   A))). Thus, all the suggested deficiencies oF [McDermott]'s modal nonmonotonic logics are solved in our theory of nonmonotonicity.

[Moore] describes a theory of nonmonotonicity based on some ideas of Stalnaker[Moore]. He calls this theory autoepistemic logic because it "is intended to model the beliefs of an agent reflecting upon his own beliefs". The main problem with [Moore]'s theory is that it is too weak to capture the notion of being possible with respect to what is assumed. For example none of the following axioms of the S5 modal logic are theorems of autoepistemic logic:
    '(IMPLY(L P)P)
    '(IMPLY(L(IMPLY P Q))(IMPLY(L P)(L Q)))
    '(OR(L P)(NOT(L(NOT P))))
where 'P and 'Q are variables and L is concept of something being entailed by a knowledgebase, even though every variable free instance of these sentences are theorems. Thus, for example simple quantified laws such as:
    '(ALL X(IMPLY(L(P X))(P X)))
    '(ALL X(IMPLY(L(IMPLY(P X)(Q X)))
                (IMPLY(L(P X))(L(Q X)))))
    '(ALL X(OR(L(P X))(NOT(L(NOT(P X)))))
are not theorems of autoepistemic logic. One might try to repair this problem of autoepistemic logic by adding the axioms of S5. However, this does not solve the problem, because when the axiom:
    '(IMPLY(L P)P)
is added to autoepistemic logic, just as in [McDermott]'s S5 nonmonotonic logic, the result is that there is a fixed point of every knowledgebase containing P. For this reason [Moore] suggests that only the axioms of a weaker modal logic than S5 which does not include '(IMPLY(L P)P) be added. The problem with this is that the excluded axiom '(IMPLY(L P)P) where 'P is a variable is intuitively true of the concept of being possible with respect to what is assumed, and therefore should be deducible as a theorem. Moore tries to justify his system's failure to include this axiom by saying that his system tries to capture the notion M of something being possible with respect

to what is "believed" by an ideally rational agent and the concept L of something being entailed by what is believed: "The problem is that all of these logics also contain the schema LP->P, which means that, if the agent believes P then P is true, but this is not generally true". Moore then essentially argues that since, as it is well known, this law fails for the notion of belief when this sentence is asserted as being true in the real world it must be incorrect to assert it generally. (The other S5 modal laws hold for the concept of belief as can readily be proven in our modal logic Z when "believes" is defined to mean that which is entailed by one's explicit beliefs.) The problem with Moore's analysis is that it confuses the real world and the agent's belief world when it states that the second P in "LP->P" means P is true; for in autoepistemic logic the assertion of a sentence is a statement that that sentence is believed, not that it is true. Therefore, the correct rendering of this belief interpretation is:
    (That which is believed is:
        (if (P is believed) then P))
which intuitively is true.

These problems are solved in our theory of nonmonotonicity, because all the axioms and inference rules of the concept of being possible with respect to what is assumed, are theorems of the modal logic Z. An interesting number of these theorems are listed below. (LTK p) is interpreted to mean that p is entailed by what is assumed. The ... in the purported definition represents the conjunction of axioms asserted into the knowledgebase.

        Interpretation in Z of the Modal Logic KZ
TKR0: (IMPLY (KTRUE P) (KTRUE (LTK P)))
TKA1: (KTRUE (IMPLY(LTK P)P)))
TKA2: (KTRUE (IMPLY(LTK(IMPLY P Q))
                (IMPLY(LTK P) (LTK Q))))
TKA3: (KTRUE (OR(LTK P) (LTK(NOT(LTK P)))))
TKA4: (KTRUE (IMPLY (ALL Q(IMPLY(WORLDK Q)
                        (LTK(IMPLY Q P))))
                (LTK P)))
TKA5: (ALL S(IMPLY(ENTAIL(meaning of the generator
                    subset S)K)
                (KTRUE(POSK(meaning of the
                    generator subset S)))))
PURPORTED-DEFINITION: (SYNK ...)

DEF
(WORLDK W)   df (AND(POSK W)(COMPLETEK W))
(COMPLETEK W) df (ALL Q(DETK W Q))
(DETK P Q)   df (OR(ENTAILK P Q)(ENTAILK P(NOTQ)))
(ENTAILK P Q) df (LTK(IMPLY P Q))
(POSK P)   df (NOT(LTK(NOT P)))
(LTK P)   df (LT(IMPLY K P))
(SYNK P)   df (SYN K P)
(KTRUE P)   df (LT(IMPLY K P))

We now answer the general question which [McDermott&Doyle,McDermott,and Moore] attempted to answer, namely, from the viewpoint of asserting things into a knowledgebase, what precisely are the laws which capture the notion of something being possible with respect to a knowledgebase. Here they are:

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                The Modal Logic KZ
KR0: from p infer (LTK p)
KA1: (IMPLY(LTK P)P)
KA2: (IMPLY(LTK(IMPLY P Q))
            (IMPLY(LTK P)(LTK Q)))
KA3: (OR(LTK P) (LTK(NOT(LTK P))))
KA4: (IMPLY(ALL Q(IMPLY(WORLDK Q)
                       (LTK(IMPLY Q P))))
           (LTK P))
KA5: for the meaning of all the generator
     subsets s which entail K:
     (POSK(meaning of the generator subset s))
PURPORTED-DEFINITION: ...
     Reflection: (entail ... K)
```

where ... is the conjunction of axioms actually being asserted into the knowledgebase. It should be noted that the notion of entailment is precisely defined in the modal logic Z and therefore KA5 does not involve a circular definition as do the fixed point theories. An examination of these laws, ironically, shows that the problem with [McDermott&Doyle, McDermott,and Moore] is not with choice of modal laws such as KA1,KA2, KA3, and KA4, since all these laws are true, but rather with the basic fixed point construction itself which is (incorrectly) far stronger than KA5 and the reflection portion of the purported definition.

IV CONCLUSION

Any scientific theory must be judged by its correctness (Does it predict all the phenomena so far examined or are there counterexamples?), by its experimental feasibility (Is it possible to make predictions from the theory, or are the deductions so computationally intractable that it is practically impossible to determine the consequences of the theory?), and by its generality (Does it apply to just the current problem at hand or does it also provide solutions to other radically different problems). By these criteria, unlike the fixed point theories, our theory of nonmonotonicity based on the modal logic Z fairs extremely well. For, indeed, first, we have not found any phenomena predicted by our theory which clashes with our primitive intuitions and in fact even after examining the example problems of four other theories of nonmonotonicity, we have not found any example therein described for which our theory does not give the intuitively correct result. Secondly, unlike the fixed point theories, our theory of nonmonotonicity is computationally tractable in that deductions can be made from it merely by deducing theorems in the modal quantificational logic Z (which is monotonic) in the traditional manner by applying inference rules to axioms and previously deduced theorems. Finally, unlike the fixed point theories, our theory of nonmonotonicity which is essentially nothing more than the axioms and inference rules of the modal quantificational logic Z is a quite general theory applicable to many problem areas. For example it has been used to define a wide range of intensional concepts [Brown 4,5] such as those found in doxastic logic, epistemic logic, and deontic logic.

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