RULE REFINEMENT USING THE PROBABILISTIC RULE GENERATOR

Won D. Lee and Sylvian R. Ray

Department of Computer Science, University of Illinois, Urbana, Illinois

ABSTRACT

This work treats the case of expert-originated hypotheses which are to be modified or refined by training event data. The method accepts the hypotheses in the form of weighted VL expressions and uses the probabilistic rule generator, PRG. The theory of operation, verified by experimental results, provides for any degree of hypothesis modification, ranging from minor perturbation to complete replacement according to supplied confidence weightings.

I INTRODUCTION

There are many situations where we would like to construct a knowledge base initially as a set of hypotheses, introduced by a human expert, which are later systematically modified by experimental training data events. Indeed, one might say that this ordering of the learning process is analogous to theoretical study of a problem's solution methods followed by practical experience with the problem, wherein modification or adaptation of the initial rules or hypotheses occur. Required modifications may range from small perturbations of the hypotheses through major or minor deletions and addition of new rules.

This problem has been called,"rule refinement", in which cases it was viewed as an incremental learning of machine generated rules. Here, we extend the idea to modifying hypotheses originated either by human agent or machine. Therefore, communication between human expert and machine becomes possible in the sense of human introduction of a bias or preliminary problem treatment. Our approach hinges upon a probabilistic formulation of the rule generation problem and deviates significantly from previous approaches because of this and its embodiment in the Probabilistic Rule Generator (PRG) (Lee and Ray, 1986a & b). The form of expression of the rules must also be equally convenient both for a human and for the rule generator.

First, some of the difficulties arising from the rule refinement problem are discussed. Next, related works are examined, and then, a language is presented describing the initial hypotheses appropriately to communicate to a machine. Finally, a scheme to modify initial hypotheses with the training data set is described with some practical application results.

II DIFFICULT NATURE OF THE PROBLEM

To examine the difficult nature of the problem, let us consider a simple case first. Assume that we have only one initial hypothesis $V_1$ for a class $C_1$, and one hypothesis, $V_2$, for a class $C_2$. Let $F_1$ and $F_2$ be new training event sets for $C_1$ and $C_2$, respectively, to be used in rule refinement.

If $V_1$ and $V_2$ perfectly describe classes $C_1$ and $C_2$, respectively, then all the events in $F_1$ will be covered by the hypothesis $V_1$, and likewise, there will be no events in $F_2$ which are not covered by $V_2$. But, in general, hypotheses are not perfect, hence usually not consistent with the new event sets. Therefore, hypotheses need to be modified to accommodate newly acquired facts. But, the modification process is complicated by various interactions among hypotheses and events as described below.

A hypothesis $V_1$ is incomplete if it does not cover all the events in $F_1$ (we are still assuming that there is only one hypothesis $V_1$ for the class $C_1$).

A hypothesis is inconsistent if it covers events which belong to other classes.

A hypothesis $V_1$ collides with another hypothesis $V_2$ if their intersection is non-null. If the two hypotheses belong to different classes, then a contradiction between the hypotheses results. On the other hand, if the two hypotheses belong to the same class, then they might have to be merged together to form a new hypothesis.

Now, let us consider all of the three problems mentioned above at the same time (see Figure 1). More complications arise since when we try to resolve incompleteness by expanding a hypothesis, then inconsistency might occur during the process by covering exception events belonging to other classes.

Likewise, if we shrink a hypothesis to resolve inconsistency by not covering exception events, then incompleteness might occur during the process since some of the events included in the former hypothesis might not belong

\[ V_1 + \]
\[ + \]
\[ - \]
\[ V_2 \]
\[ + \]
\[ + \]
\[ - \]
\[ \]

Figure 1. Simple example of a general rule refinement problem: here, "+" are events in $F_1$, and "-" are those in $F_2$. Notice the probabilistic nature of the problem.

Of course, we can always adjust hypothesis \( V' \) so that it includes some of the events in \( F' \), and does not cover any event in \( F'' \). But, in doing so, the newly generated hypothesis might become malformed in shape as it might have to be shrunk too much to exclude all exception events. Therefore, the classical viewpoint of strict completeness and consistency of a concept is no longer adequate. A scheme should be flexible enough to generate an appropriate form of hypotheses according to user specification, and it is desirable to have a process with a probabilistic nature, allowing partially complete and partially consistent concepts. The point here is that a general rule refinement scheme must be able to deal with all the possible situations that the usually noisy data set might present.

As we can see, the problem of dealing with collisions between hypotheses will become increasingly complex, as the situation worsens by the addition of problems of incompleteness and inconsistency.

We have considered only a simple case so far: each class having only one initial hypothesis. But, what about more complex cases when there is more than one hypothesis for each class? A bewildering variety of interactions among the hypotheses and the event sets can occur then.

Implicit in the above discussion is that we modify the hypotheses to fit with the example events. But, if the hypotheses represent only a few cases of the facts, and if the new training data set contains a large amount of events not described by the initial hypotheses, the idea of starting with the hypotheses and adjusting them to fit the data might not be the right one. Rather, we might want to produce complexes from the data events first, and adjust those complexes according to hypotheses. An extreme case arises when the initial hypotheses do not cover any of the events in the new training data set. The idea is that we should not give any special privilege to the initial hypotheses, but treat the hypotheses equally with the event sets according to their importance of how many data events they represent.

III PREVIOUS WORK*

A. AQ Rule Refinement

The AQ rule refinement scheme was derived in (Michalski and Larson, 1978). There are some shortcomings in this scheme. First, the objective of AQ incremental rule generation is to produce a new decision rule consistent with the input hypotheses and the observed events. Initial hypotheses are not modified to make them consistent with observed events; they are rather used to find events that cause inconsistency and incompleteness to the hypotheses, and to generate a cover of an event set against those hypotheses.

Since complexes are generated around the example events only, and no attempt is made to modify the initial hypotheses to make them consistent with the example events, it is argued that the expansion strategy used in incremental rule generation in AQ should be dropped (O'Rorke, 1982).

Secondly, it has been observed that new hypotheses generated are usually overly complex compared to former hypotheses. This is because there is a lack of facility to capture complexes with some exception events in them, and therefore all the new hypotheses are formed to include strictly positive events only.

There have been attempts to remedy the problems mentioned above (Reinke and Michalski, 1985). But since those methods are based on the modified AQ star synthesis, they do not address the probabilistic nature of rule refinement fully. Some problems associated with AQ still remain in them (Lee and Ray, 1986a). The capability to capture complexes probabilistically becomes important since there will be a large number of interactions among complexes and events in the modification process.

B. ID3 iterative rule generation — methodology and discussion

A rule can be generated by ID3 iteratively by selecting a subset of training events, called a "window", at each iteration (Quinlan, 1983).

There are two methods of forming a new window in ID3. One way is to add the exceptions to the current window up to some specified number to form an enlarged window. The other one is to select "key" events in the current window and replace the rest with the exceptions, thus keeping the window size constant.

Notice that ID3 itself does not have the capability to accept initial hypotheses, but uses already existing data events iteratively to generate a decision tree. Also, ID3 itself is intended to make a decision tree, rather than to synthesize variable-valued logic expressions. Therefore, the ID3 rule refinement scheme is not intended for initial hypotheses modification.

Yet, when we relax some of the constraints in the scheme, we can extract some useful ideas from it.

First, let us consider the case of making a new window by adding some specified number of exceptions to the current window. Let us relax the requirement of "some specified number", so that we can add a large number of exceptions to the current window, if we desire. Then we observe that the ID3 rule refinement scheme will not differentiate between already available rules and the training events. In other words, the current rule can be grown in any direction. Therefore, a new rule can have any shape from almost identical to the current rule to almost entirely different from the current one.

Secondly, let us consider the idea of selecting the key events in the current window to represent the current rule. It is important to select these key events to represent the current rule faithfully as they will be mixed with "foreign" exception events.

If there is no further information available about the general distribution tendency of the events in each variable domain, we might want to choose events which are distributed equally over the subspace of the complex to which these events belong.

Let us further imagine the situation that, after we generate a rule, we lost all the actual data sets of events.
Then one way to recover the rule for the next iteration is to generate events artificially, thus simulating the hypotheses. Because the event space is generally huge, and the number of available training events is usually small, and more importantly, human conceptual knowledge may be very condensed, a concept by a human expert can play an important role. Thus, a scheme to modify an expert-driven concept systematically may be especially effective.

### IV CASE STUDIES

$V_{l_j}$ expressions (Michalski, 1975) have been used to describe rules generated by inductive inference machines. Consider a simple example to see whether the $V_{l_j}$ expression is sufficient to convey an expert's hypotheses to a machine so that rule refinement might be done by the machine.

Let there be two classes, $C_+$ and $C_-$, and two linear variables, $v_1$ and $v_2$, describing the events, each with cardinality 5. Let there be two initial hypotheses, $V_1$ and $V_2$, both for class $C_+$, and new sets of example events, $F_+$ and $F_-$, belonging to $C_+$ and $C_-$, respectively (see Figure 2).

Here, hypotheses $V_1$ and $V_2$ are described by the $V_{l_j}$ expression as two complexes, $[v_2 = 1..2]$, and $[v_1 = 2..4][v_2 = 4]$, respectively.

Let us consider the following five cases.

**Case 1**

Let the hypotheses be very "light" in weight, not representing many actual events by themselves (see Figure 3). This kind of situation occurs when an expert is inexperienced and is not sure of some of his claims, or is merely guessing the rule. Therefore, the expert wants the machine to generate a new rule mainly by the new event sets $F_+$ and $F_-$. Thus, machines should be able to detect the fact that these hypotheses are indeed unimportant or light, and hence might to generate new hypotheses by the given examples. In this case, the major cluster would emerge by capturing a cluster made of $C_+$ events.

**Case 2**

Let us consider the case when the hypothesis $V_1$ is heavy, but $V_2$ is light, compared with the actual number of $C_+$ example events (see Figure 4).

A major, newly generated hypothesis might be the original hypothesis except the part that a $C_-$ event resides inside the hypothesis $V_1$. Thus, this example shows how a hypothesis can be specialized by some exception events.

**Case 3**

Here, let $V_1$ be light, while $V_2$ is heavy. This is the reverse of the case 2, and therefore a major new hypothesis would be created around the hypothesis $V_2$.

As we can see in Figure 5, because two $C_+$ events are in the region where $V_2$ can be more generalized, a new hypothesis would be formed by merging $V_2$ with these two $C_+$ events.

Therefore, this case exemplifies how a hypothesis can be merged with some example events to be more generalized.
Case 4

Next, consider the case when both hypotheses \(V_1\) and \(V_2\) are equally heavy (see Figure 6). If there is no way to merge these two hypotheses to make a heavier hypothesis, new hypotheses would be created around original hypotheses.

But, in this specific example, as in the figure, if there is a way to merge these two together, and if the merged hypothesis is indeed heavier than each individual hypothesis, then a new, more important hypothesis can be made by the merge.

Case 5

Finally, let us reconsider the case 2, when hypothesis \(V_1\) is heavy, but \(V_2\) is not (see Figure 7). If the hypothesis \(V_1\) is heavy enough, then a flexible rule refinement scheme should be able to ignore a small number of exception events which would otherwise make the new hypothesis shrink too much. By doing so, the new hypothesis would be general enough to be a good description of the facts. This, in turn, is the problem of a rule refinement scheme having the capability of capturing complexes probabilistically.

V COMMUNICATION LANGUAGE

We have discussed so far only some of the cases that can occur during rule refinement process. There are other cases, such as the case when two hypotheses belonging to two different classes collide with each other. Then, a decision should be made by the user whether to divide an initial hypothesis to generate new hypotheses without any exception, or to accept the initial hypothesis without change if the collision is not severe. Of course, there should be some intermediate solutions between the two extremes, according to the user specification. Again, this is most naturally treated as a probabilistic rule refinement problem. Since the data might well be contaminated, being able to generate hypotheses probabilistically becomes important.

As we have seen, if an expert wished to describe his hypotheses to communicate with the machine, he would need to describe how important each hypothesis is, compared not only with other hypotheses, but also with the sets of example events.

Let us, for instance, say that there are two hypotheses, \([v_2 = 1..2]\) having a weight of 0.8 and \([v_1 = 2..4]\ [v_2 = 4]\) having a weight of 0.2, representing a total of 100 events. Then, we can express the initial hypotheses as:

\[
80[v_2 = 1..2] + 20[v_1 = 2..4] [v_2 = 4].
\]

Therefore, this expression contains not only the relative importance of each hypothesis, but also the relative importance of the total hypotheses to the sets of example events.

We will call this expression a weighted \(VL_1\) (\(WVL_1\)) expression.

VI PRG RULE REFINEMENT

The objective of the PRG rule refinement scheme is summarized below.

First, it should be able to consider all the interactions among hypotheses and sets of example events. In other words, it must be capable of merging hypotheses, resolving collisions between hypotheses belonging to different classes, generalizing hypotheses by adding some events, and specializing hypotheses by excluding some exception events.

Secondly, it should be flexible enough, hence be probabilistic, in generating new hypotheses according to user specification. For instance, a hypothesis can be very specific, explaining only the positive events, or be more general by ignoring some minor exceptions.

Because PRG is already probabilistic, the only problem left is to enter the \(WVL_1\) expression into PRG to be modified.

A simple solution is used in PRG rule refinement scheme. Since a human expert's description of a concept is often probabilistic, it is suitable for simulation in PRG.

PRG rule refinement scheme is described as:

PRG rule refinement;
begin
simulate each hypothesis by a set of randomly generated events in the subspace described by the hypothesis, making the number of events equal to the weight of the hypothesis;
end

*Notice that this expression is different from the weighted DVL expression in (Michalski and Chilausky, 1980), since weighted DVL expressions are concerned with the weighted selectors in a \(VL_1\) expression, rather than weighted complexes. We will call the coefficient in front of each complex, a "weight", as it represents the strength of evidence of each hypothesis.
Table 1. Result of Experiment 1.

Notes:
1. Major complex is created mainly from positive training events since hypotheses are lightly weighted.
2. The major complex comes from original hypothesis, \( V_1 \), by excluding one exception event.
3. The major complex is formed by merging the dominant hypothesis, \( V_2 \), with some positive training events.
4. A new, heavier hypothesis is created by merging hypotheses \( V_1 \) and \( V_2 \).
5. With specificity = 0.9 (which means that any subspace that has ratio of the number of positive events to the total number of events in it is greater than or equal to 0.9, is considered as an acceptable complex), a single exception event in hypothesis \( V_1 \) is ignored, permitting \( V_1 \) to be retained as the major complex.

<table>
<thead>
<tr>
<th>WVL, Hypotheses</th>
<th>Refined Rule</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_1 + V_2 )</td>
<td>4[v_1=0..1][v_2=2.4]+2[v_2=2]+1[v_1=4]</td>
<td>1</td>
</tr>
<tr>
<td>( 10V_1 + V_2 )</td>
<td>9[v_1=1..4][v_2=1.2]+5[v_1=0..1][v_2=2.4]+3[v_2=4]</td>
<td>2</td>
</tr>
<tr>
<td>( V_1 + 10V_2 )</td>
<td>12[v_2=4]+2[v_2=2]+4[v_1=0..1][v_2=2.4]</td>
<td>3</td>
</tr>
<tr>
<td>( 10V_1 + 10V_2 )</td>
<td>14[v_1=4][v_2=1..4]+12[v_2=4]+9[v_2=2]+5[v_1=0..1][v_2=2.4]</td>
<td>4</td>
</tr>
<tr>
<td>( 10V_1 + V_2 )</td>
<td>10[v_2=1..3]+3[v_2=4]+5[v_1=0..1][v_2=2.4]</td>
<td>5</td>
</tr>
</tbody>
</table>

Note: Class 3 where with hypotheses only \( (\lambda = \infty) \), the accuracy was only 19% but with \( \lambda = 2 \), the new events overcome the inaccuracy of the hypotheses and the resulting rules were nearly perfect (99% accuracy).

Class 2 exhibits more typical behavior, the accuracy rising monotonically (except for trivial noise fluctuations) from 71%, with hypotheses only, to 94% when only training data was used for the rules. Class 5 also exhibits accuracy growth that is monotonic, paralleling that of Class 2.

Only Class 1, which is known semantically as the noisiest, most poorly clustered class, shows strong erratic behavior. The anomalous-looking 5370 accuracy for hypotheses only is due to "overgeneralization" which becomes constrained by negative events as new data events are introduced. As hypothesis weight decreases, interaction between Class 0 and Class 1 manifests as non-monotonic accuracy increase.

Note: With hypotheses only \( (\lambda = \infty) \), the accuracy was only 19% but with \( \lambda = 2 \), the new events overcome the inaccuracy of the hypotheses and the resulting rules were nearly perfect (99% accuracy).

Class 2 exhibits more typical behavior, the accuracy rising monotonically (except for trivial noise fluctuations) from 71%, with hypotheses only, to 94% when only training data was used for the rules. Class 5 also exhibits accuracy growth that is monotonic, paralleling that of Class 2.

Only Class 1, which is known semantically as the noisiest, most poorly clustered class, shows strong erratic behavior. The anomalous-looking 5370 accuracy for hypotheses only is due to "overgeneralization" which becomes constrained by negative events as new data events are introduced. As hypothesis weight decreases, interaction between Class 0 and Class 1 manifests as non-monotonic accuracy increase.

Table 2. Rule Modification Experiment Result. Entries are % of events correctly classified \( (\lambda = \infty) \) in the case of hypothesis only. Here, specificity = 0.9, certainty = 0.9, and weight = 0.05.

<table>
<thead>
<tr>
<th>Class</th>
<th># New Events</th>
<th>( \lambda )-wt. of hypothesis/ wt. of new events</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>100 19 59 68 54 62</td>
</tr>
<tr>
<td>1</td>
<td>83</td>
<td>53 10 39 51 53</td>
</tr>
<tr>
<td>2</td>
<td>385</td>
<td>71 85 94 93 94</td>
</tr>
<tr>
<td>3</td>
<td>108</td>
<td>19 99 99 99 99</td>
</tr>
<tr>
<td>5</td>
<td>66</td>
<td>47 47 67 67 67</td>
</tr>
</tbody>
</table>

Total 742 52.4 72.0 80.6 81.5 82.7
VIII CONCLUSION

A system has been developed to permit communication between expert and machine using the following principles. Weighted $VL_1$ expressions are used by the expert to introduce hypotheses. Training events may be appended as new data, each event having weight 1. The hypotheses are expanded into a weight-equivalent number of data events, which are joined with the actual new events. Thus, neither hypotheses nor training events have special significance except through weighting.

The superset of events is then submitted to the Probabilistic Rule Generator which is capable of capturing major complexes in spite of moderate noise.

Experiments verify that the resulting rules may range from minor refinement of the hypotheses through various reorganizations of the hypotheses and on to rules which are completely dominated by the new training events in a continuous, systematic spectrum, controlled by assigned (confidence) weighting.

REFERENCES


