A Temporal Terminological Logic*

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Abstract

An attempt is made to integrate three well-known formalisms of knowledge representation: terminological logic in the tradition of KL-ONE, the temporal logic of Shoham, and Allen's interval calculus. Drawing on each of these sources, a temporal terminological logic is proposed which combines structural with temporal abstraction. A formal semantics is provided, and some hints are given for exploring the computational properties of reasoning in the formalism.

Introduction

Terminological logics in the tradition of KL-ONE [Brachman & Schmolze 1985] as well as temporal reasoning have both received considerable attention within the knowledge representation community in the last years. However, there has never been a serious attempt to integrate these two fields. Although in a number of projects using terminological logic (e.g. [Poesio 1988]) the problem of representing time has arisen, notably in the context of tense in natural language understanding, the approaches taken there and the partial solutions found have not culminated in a syntactically and semantically well-defined temporal variant of a terminological logic. The theoretical framework described in the following is the foundation of a (future) temporal extension of BACK, a knowledge representation system based on terminological logic being developed in our project [Peltason et al. 89].

The approach for integrating time into a terminological formalism which I am proposing here draws on three ingredients. First of all on terminological logic itself, the appealing features of which are completely preserved in the temporal variant. The temporal capabilities come straightforwardly by adding some new term-forming constructs. The model-theoretic semantics are accordingly amended, and remain unchanged for non-temporal terms. Compositionality is unaffected; there are no restrictions on the combination of temporal and non-temporal terms.

The second ingredient is concerned with the elementary combination of temporal and non-temporal objects. Following [Shoham 1987] I will keep temporal and non-temporal components of the language separate, giving time a special status in the formalism. The role of ‘TRUE’ in his logic is played by a new term-forming construct ‘(at interval concept)’ which denotes the set of all individuals that are in the denotation of concept at the time interval. Denotations of concepts are interpreted at intervals, not at points. Also as in Shoham’s approach there is no commitment with respect to the property/event/process trichotomy in the basic framework. The denotation of a concept at one interval is essentially unrelated to its denotation at other intervals.

Thirdly, for expressing temporal relationships and constraints, I rely on Allen’s interval calculus [Allen 83] extended by some additional constraint types for dealing with durations, absolute times, and the granularity of intervals. This restricts the range of expressible temporal constraints compared with a full-fledged temporal logic, but for this subset specialized algorithms are available making an efficient treatment at least for a broad range of ‘non-puzzle-mode’ cases conceivable, which is a prerequisite for a knowledge representation (KR) service.

Syntax and Introductory Examples

Figure 1 shows the syntax of some of the basic concept- and role-forming constructs common to most terminological logics of the KL-ONE family: and, all, atmost for concepts, and and, domain, range for roles. Note that the (restrict role concept) construct found in some systems is equivalent to (and role (range concept)) in our syntax. The new term-forming operators involving time are at, sometime, and alltime.1 The syntax for time intervals and time

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1I have deliberately included a fairly expressive set of role-forming constructs to demonstrate the expressive potential of the formalism, disregarding for the time being various possible trade-offs between expressivity and the
nets is found in Figure 2.

In standard terminological logics using the non-temporal constructs of the syntax concepts can be formed such as

\[(\text{and } \text{man} \quad (\text{atleast } 1 \quad (\text{and } \text{child} \quad (\text{range } \text{female})))) \quad (\text{all child grown-up})\]

with the intuitive meaning a man with atleast one child, which is a female, whose children are all grown-up. According to the model-theoretic semantics for the term-forming constructs, the extension of a term is strictly a function of the extensions of its subterms and of the extensions of the atomic concepts or roles it contains, in this example the concepts man, female, grown-up, and the role child.

An important feature that all KR systems based on terminological logics provide is the possibility of introducing new atomic concepts and roles by definition, for example

\[\text{daughter} := (\text{and } \text{child} \quad (\text{range } \text{female}))\]

After this definition, the new name can be used in other terms and definitions as an abbreviation for the defining term.

Before looking at some examples using the new constructs involving time, let us consider the meaning of non-temporal terms. For example, if we define

\[\text{car-owner} := (\text{atleast } 1 \quad (\text{and } \text{own} \quad (\text{range } \text{car})))\]

what is the meaning of car-owner within the temporal framework? The answer is that all terms have to be evaluated with respect to a particular interval of time. Consequently, car-owner does no more denote a set of individuals fulfilling the definition, but in fact a function that assigns such a set of individuals to every interval. Using the at-construct, we can bind the time of evaluation of a term to a specific interval. For example,

\[(\text{at } '\text{August 1990}' \quad \text{car-owner})\]

denotes the set of car-owners at the interval denoted by the interval constant 'August 1990'.

Every term can be rewritten as (at \text{NOW term}). \text{NOW} is the special reference interval which represents the index at which the term is evaluated. Thus, the index implicit in every non-temporal term can be made explicit. Every at-term creates an evaluation environment whereby every (explicit and implicit) occurrence of \text{NOW} in the embedded term is bound to the interval specified in the at-term. Taking the last example, if we expand the definition of car-owner and bind the implicit NOW's, we get

\[(\text{atleast } 1 \quad (\text{at } '\text{August 1990}' \quad \text{own}) \quad (\text{range} \quad (\text{at } '\text{August 1990}' \quad \text{car})))\]

Note that the outer at-terms are redundant (and have therefore been deleted in the example), since there are no more embedded \text{NOW}'s that the time interval could be bound to.

We can identify two classes of terms: time-indexed terms whose denotation depends on a particular value of \text{NOW} and universal terms whose denotation is the same for all values of \text{NOW}. Obviously, all terms that contain no unbound (implicit or explicit) occurrences of \text{NOW} are necessarily universal, and as a consequence an at-term embedding a universal term is redundant and can be replaced by the term itself.

With the at-construct and temporal constants alone, expressivity is very restricted. The potential for temporal abstraction comes with the ability to express abstract temporal patterns, which classify objects in terms of their pattern of change. In order to achieve

\[
\begin{align*}
\text{concept} & ::= \quad \text{atomic-concept} \\
& \quad (\text{and } \text{concept}^+) \\
& \quad (\text{all role concept}) \\
& \quad (\text{atleast min role}) \\
& \quad (\text{atmost max role}) \\
& \quad (\text{at interval concept}) \\
& \quad (\text{sometime} \quad (\text{interval-variable}^+) \quad \text{time-net concept}) \\
& \quad (\text{alltime} \quad (\text{interval-variable}^+) \quad \text{time-net concept})
\end{align*}
\]

\[
\begin{align*}
\text{role} & ::= \quad \text{atomic-role} \\
& \quad (\text{and } \text{role}^+) \\
& \quad (\text{domain concept}) \\
& \quad (\text{range concept}) \\
& \quad (\text{at interval role}) \\
& \quad (\text{sometime} \quad (\text{interval-variable}^+) \quad \text{time-net role}) \\
& \quad (\text{alltime} \quad (\text{interval-variable}^+) \quad \text{time-net role})
\end{align*}
\]

Figure 1: Syntax for Concepts and Roles with Temporal Structure
this, temporal variables and means of expressing constraints over these variables are necessary. 2 Temporal variables are introduced by the temporal quantifiers sometime and alltime together with a set of constraints, a time net, over these variables. Three kinds of constraints are allowed by the syntax for time nets according to Figure 2: relations between pairs of intervals using Allen's basic interval relations and disjunctions of these, metric constraints on single intervals, and granularity constraints requiring an interval to take values that are multiples of some time unit. Absolute bounds can be imposed on an interval by using interval constants in interval relations.

The following term denotes the set of individuals that were car-owners at an interval sometime before NOW:

\[ \text{former-car-owner} := (\text{sometime} (x) \text{ (before } x \text{ NOW) (at } x \text{ car-owner})) \]

former-car-owner can be equivalently expressed using a temporal role:

\[ \text{former-car-owner} := (\text{atleast } 1 \text{ have-owned-a-car}) \]

\[ \text{have-owned-a-car} := (\text{sometime} (x) \text{ (before } x \text{ NOW) (at } x \text{ (and own (range car))})) \]

Two individuals are related by have-owned-a-car if they were related by own at some interval before NOW and if at that time the second was an instance of car.

The following term correctly applies to an individual NOW if at a point during NOW he ceases to be a car-owner and becomes a bike-owner:

\[ \text{(and (sometime} (x y) \text{ (and (start } x \text{ NOW) (finishes } NOW y) (meets } x y)) \text{ (and (at } x \text{ car-owner) (at } y \text{ bike-owner})) (alltime} (x) \text{ (during } x \text{ NOW) (at } x \text{ (atmost } 1 \text{ (and own (range vehicle))))})) \]

It is important to realize that in the sometime-term there is nothing that expresses 'ceases to be' or 'becomes'; being car-owner and bike-owner at the same time throughout NOW would be perfectly consistent. It is only in conjunction with the alltime term, which restricts the number of role-fillers for the own-a-vehicle role to atmost one for all times during NOW, and with the assumption that car and bike are subsumed by vehicle, that this interpretation is ruled out.

In the previous examples, time nets only used interval relations to constrain temporal variables. The following time net additionally uses metric constraints on the duration of intervals, and granularity predicates:

\[ \text{(and (day } x \text{) (= } x \text{ '24h')) (day } y \text{) (= } y \text{ '24h'}) (meets } x y) ((\text{or starts finishes during equal} x \text{ NOW}) ((\text{or starts finishes during equal} y \text{ NOW})) \]

It constrains \( x \) and \( y \) to be consecutive days within NOW. The granularity constraint (day \( x \)) above restricts \( x \) to take only values that are started and fin-
ished by a day. Without this constraint, \( x \) could be any interval with a duration of 24 hours, due to the metric constraint \((= x 24h')\), but not necessarily coinciding with a full day of the calendar. On the other hand, leaving away the duration constraint, \( x \) could be any interval starting and ending with a full day.

**Semantics**

In terminological logics in the tradition of KL-ONE it has become customary to provide a model-theoretic account of the semantics (e.g. [Schmolze 1989, Nebel 1989]). I will follow this tradition. Before spelling out the semantics for concepts and roles, the semantics associated with temporal constraints must be clarified.

For the present purpose, I will assume a discrete time model and interpret all time intervals as pairs of integers, and define the domain of time intervals as follows:

\[
\mathcal{T} \overset{\text{def}}{=} \{(i, j) \mid i < j, i, j \in \text{Integer}\}.
\]

Thus, consecutive integers form the smallest, non-decomposable intervals, the *moments* in the sense of [Allen & Hayes 1985]. For time constraints according to Figure 2, I will assume a fixed model \( \mathcal{M} \) which maps interval constants to elements of \( \mathcal{T} \), duration constants to subsets of \( \mathcal{T} \), comparison operators and interval relations to sets of pairs of elements of \( \mathcal{T} \), and granularity predicates to subsets of \( \mathcal{T} \), such that the intuitive meaning of these constructs is adequately mirrored (for example, that \( \{\mathcal{M['August 1990']}, \mathcal{M['September 1990']}' \} \in \mathcal{M}\{\text{meets}\}, \mathcal{M['3/12/1990']}' \in \mathcal{M}\{\text{day}\}, etc.).

An *interpretation* of a time net \((\mathcal{T}, X)\), where \( \mathcal{T} \) is a set of constraints and \( X \) a set of variables, is a function \( \mathcal{I} : X \rightarrow \mathcal{T} \) which satisfies \( \mathcal{T} \) (for example, if \( \mathcal{I}(x) = \mathcal{I}(y) \in \mathcal{M}\{\text{meets}\} \), then \( (\mathcal{I}(x), \mathcal{I}(y)) \in \mathcal{M}\{\text{meets}\} \). The set of all interpretations of the time net \((\mathcal{T}, X)\) is denoted by \( \mathcal{I}^*((\mathcal{T}, X)) \). The set of all interpretations of a time net in which all of which \( x \) is mapped to the same value is denoted by \( \mathcal{I}^*((\mathcal{T}, X))_{x=1} \).

A model for a set of terms with temporal structure defined by the syntax in Figures 1 and 2 is a triple \((\mathcal{D}, \mathcal{T}, \mathcal{E})\) where \( \mathcal{D} \) is a set of individuals, \( \mathcal{T} \) is the set of time intervals, and \( \mathcal{E} \) is a function

\[
\mathcal{E} : \begin{cases} C \rightarrow (\mathcal{T} \rightarrow 2^\mathcal{D}) \\ R \rightarrow (\mathcal{T} \rightarrow 2^{2^\mathcal{D} \times \mathcal{D}}) 
\end{cases}
\]

where \( C \) are the concept terms and \( R \) are the role terms without free variables and after all definitions have been expanded. Thus, each concept (each role) is mapped to a function that assigns sets of individuals (sets of pairs of individuals) to each time interval. For \((\mathcal{D}, \mathcal{T}, \mathcal{E})\) to be a model, the conditions in Figure 3 that define the meaning of the syntactic constructs must be met for all \( t \in \mathcal{T} \).

Taking into account the extension at each time interval, subsumption can now be defined in the usual way:

*For all concepts and roles, \( c_1 \) subsumes \( c_2 \) iff for all extension functions \( \mathcal{E} \) (models) and all time intervals \( t \in \mathcal{T} \), \( \mathcal{E}[c_2]^t \subseteq \mathcal{E}[c_1]^t \).*

**Kinds of Time Dependency**

As already mentioned in the introduction, in the basic semantic framework as spelt out in the last section there are no built-in restrictions on the extensions of primitive concepts or roles. For example, if a pair of individuals are related by the primitive role \( \text{own} \) at one interval, they may or may not be related by that role in any subinterval. As a consequence, according to the semantics so far,

\( \text{(at 'June 1990' own)} \)

does not subsume

\( \text{(at '1990' own)} \),

and

\( \text{(at 'June 1990' own)} \)

does not subsume

\( \text{(at '1990' own)} \),

and

\( \text{(at 'June 1990' own)} \)

does not subsume

\( \text{(at '1990' own)} \),

and
does not subsume

(and (at 'June 1990' own)
 (at 'July 1990' own)

although intuitively if an own relation holds in an interval it should also be valid in all subintervals, and if it holds over two consecutive intervals it should hold over their union (which, in the second example, is then an interval longer than 35 days). Extra restrictions on extension functions are needed in order to legitimize the subsumptions above. Using the terminology in [Shoham 1987], the possible extensions of the primitive role own should be restricted to being downward-hereditary: for all t, t' ∈ T, t' subinterval of t, σ[own]t ⊆ σ[own]t', and concatenable: for all t, t', t'' ∈ T, t meets t', t' starts t'', t' finishes t''.

Of course, for other kinds of concepts and roles these restrictions are quite inadequate. For example, for a role average-temperature in a certain time interval, the first subsumption relation above should not hold since average temperature in a certain time interval, the first subsumption relation above should not hold since average remains identical for two consecutive intervals also applies for their union. The latter is certainly not true for integrated into our framework? The situation is quite similar to the problem of determining disjointness restrictions into terminological logics. Should the disjointness of e.g. male and female be treated as definitional and hence be used for validating certain subsumptions, or rather as assertional? On pragmatic grounds, the former alternative is generally chosen. Terminological systems allow disjointness restrictions for primitive concepts and use these for computing subsumption. The same approach could be adopted for restrictions on temporal extensions of primitive concepts or roles. So, in the example above, after declaring the primitive role own as downward-hereditary and concatenable, the subsumptions would be valid.

Computing Subsumption

Subsumption is the central semantic notion in terminological logics; designing sound (and possibly complete) algorithms that compute subsumption is the central issue for providing practical KR services. Until now, no algorithms are available for the temporal variant presented here, but at least some preliminary hints as to what is involved can be given. Assume CT' is of the form

\[(\text{sometime } X' \text{ TC}'' \text{ (and ... (at } x'_j c'_j \text{ ...) })\],

\[j \in J, x'_j \in X', c'_j \in C'\]

and CT is of the form

\[(\text{sometime } X \text{ TC (and ... (at } x_i c_i \text{ ...) })\]

\[t \in I, x_i \in X, c_i \in C\]

Under what conditions does CT' subsume\(^3\) CT? Intuitively, CT' is a more general concept than CT, if its temporal variables are less constrained than those of CT, and for each of its temporal variables there is a corresponding variable in CT such that the associated concept of that variable subsumes the associated concept in CT. Obviously, CT can have additional temporal variables and associated concepts, which specialize it further.

To formalize this notion, let S : J → I be a function from indices of CT' to indices of CT. Sx : X' → X is defined as Sx(x'_j) = x(S(j)), and SC : C' → C is defined as SC(c'_j) = c(S(j)). S must always map NOW to NOW, i.e., S(NOW) = NOW.

The notion of 'less constrained temporal variables' is captured by the following definition: A time net (TC',X') subsumes another time net (TC,X) wrt a variable mapping Sx : X' → X is defined as Sx(x'_j) = x(S(j)), and SC : C' → C is defined as SC(c'_j) = c(S(j)). S must always map NOW to NOW, i.e., S(NOW) = NOW.

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Additional restrictions as mentioned in the last section are not taken into account in the following.

\(^3\) Additional restrictions as mentioned in the last section are not taken into account in the following.
Conclusion

Of course complete and tractable subsumption algorithms for the whole language and for the standard semantics presented here cannot be expected. In Allen’s interval calculus on its own, which is a subset of our temporal constraint language, determining all consequences of a set of constraints is NP-hard [Vilain & Kautz 1986]. And even for moderately expressive languages the non-temporal part is intractable [Nebel 1988]. That does not render these formalisms useless. On the one hand, it remains to be seen to what extent normal cases in practical applications can be handled even by complete algorithms. On the other hand, algorithms for computing subsumption in terminological logics that are incomplete with respect to standard semantics are increasingly being characterized as complete with respect to a weakened semantics [Schild 1988, Patel-Schneider 1988]; approximative algorithms are also studied in the field of temporal reasoning [van Beek 1989]. These developments are a reasonable starting point for developing subsumption algorithms for temporal terminological logics.

References