A Logic for Revision and Subjunctive Queries

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Abstract
We present a logic for belief revision in which revision of a theory by a sentence is represented using a conditional connective. The conditional is not primitive, but rather defined using two unary modal operators. Our approach captures and extends the classic AGM model without relying on the Limit Assumption. Reasoning about counterfactual or hypothetical situations is also crucial for AI. Existing logics for such subjunctive queries are lacking in several respects, however, primarily in failing to make explicit the epistemic nature of such queries. We present a logical model for subjunctives based on our logic of revision that appeals explicitly to the Ramsey test. We discuss a framework for answering subjunctive queries, and show how integrity constraints on the revision process can be expressed.

Introduction
An important and well-studied problem in philosophical logic, artificial intelligence and database theory is that of modeling theory change or belief revision. That is, given a knowledge base KB, we want to characterize semantically the set that results after learning a new fact α. However, the question of how to revise KB is important not just in the case of changing information or mistaken premises, but also when we want to investigate questions of the form “What if α were true?” A subjunctive conditional A > B is one of the form1 “If α were the case then B would be true.” Subjunctives have been widely studied in philosophy and it is generally accepted that (some variant of) the Ramsey test is adequate for evaluating the truth of such conditionals:

First add the antecedent (hypothetically) to your stock of beliefs; second make whatever adjustments are required to maintain consistency (without modifying the hypothetical belief in the antecedent); finally, consider whether or not the consequent is true. (Stalnaker 1968, p.44)

The connection to belief revision is quite clearly spelled out in this formulation of the Ramsey test: to evaluate a subjunctive conditional A > B, we revise our beliefs to include A and see if B is believed. If we take KB to represent some initial state of knowledge, a characterization of subjunctive reasoning must include an account of how to revise KB with new information.

In this paper, we will develop a logic for belief revision using a conditional connective %, where A % B is interpreted roughly as “If KB were revised by A, then B would be believed.” The connective will not be primitive however; instead it is defined in terms of two unary modal operators, which refer to truth at accessible and inaccessible worlds. Our model of revision will satisfy the classic AGM postulates and will be general enough to represent any AGM revision function. However, our approach is based on a very expressive logical calculus rather than extra-logical postulates, and can be used to express natural constraints on the revision process. Furthermore, this is accomplished without reliance on the Limit Assumption. We will then use this logic to develop a framework in which subjunctive queries of a knowledge base can be answered, and show that it improves on existing subjunctive logics and systems in several crucial respects. Finally, we provide a semantic characterization of integrity constraints suitable for this type of subjunctive reasoning.

Revision and the AGM Postulates Recently, work on the logic of theory change has been adopted by the AI community for use in the task of belief revision. By far the most influential approach to revision has been that of Alchourrón, Gärdenfors and Makinson (1985; 1991), which we refer to as the AGM theory of revision. We assume beliefs sets to be deductively closed sets of sentences, and for concreteness we will assume that the underlying logic of beliefs is classical propositional, CPL. We let ⊨ and Cn denote classical entailment and consequence, respectively, and use K to denote arbitrary belief sets. If K = Cn(KB) for some finite set of sentences KB, we say K is finitely specified by KB.

Revising a belief set K is required when new information must be accommodated with these beliefs. If K ⊭ ¬A, learning A is relatively unproblematic as the

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1 At least, in “deep structure.”
A Conditional for Revision

The Modal Logic CO* We now present a modal logic for revision in which we define a conditional connective $KB$. A $KB$ is read "if (implicit theory) $KB$ is revised by $A$, then $B$ will be believed." The modal logic CO is based on a standard propositional modal language (over variables $P$) augmented with an additional modal operator $\square$. The sentence $\square \alpha$ is read "$\alpha$ is true at all inaccessible worlds" (in contrast to the usual $\square \alpha$ that refers to truth at accessible worlds). A CO-model is a triple $M = (W, R, \varphi)$, where $W$ is a set of worlds with valuation $\varphi$ and $R$ is an accessibility relation over $W$. We insist that $R$ be transitive and connected. Satisfaction is defined in the usual way, with the truth of a modal formula at a world defined as:

1. $M \models_w \square \alpha$ if for each $v$ such that $wRv$, $M \models_v \alpha$.
2. $M \models_w \boxdot \alpha$ if for each $v$ such that not $wRv$, $M \models_v \alpha$.

We define several new connectives as follows: $\boxdot \alpha \equiv_{df} \neg \boxdot \neg \alpha$; $\alpha \equiv_{df} \neg \square \neg \alpha$; $\boxdot \alpha \equiv_{df} \square \alpha \land \square \alpha$; and $\alpha \equiv_{df} \boxdot \alpha \lor \boxdot \alpha$. It is easy to verify that these connectives have the following truth conditions: $\alpha (\boxdot \alpha)$ is true at a world if $\alpha$ holds at all accessible (inaccessible) world; $\square \alpha (\boxdot \alpha)$ holds if $\alpha$ holds at all (some) worlds. The following set of axioms and rules is complete for CO (Boutilier 1991):

2 $R$ is (totally) connected if $wRv$ or $vRw$ for any $v, w \in W$ (this implies reflexivity). CO was first presented in (Boutilier 1991) to handle the problem of irrelevance in default reasoning.
Given these restrictions, we can use CO*-structures to represent the revision of a theory $K$. However, arbitrary CO*-models are inappropriate, for we must insist that those worlds consistent with our belief set $K$ should be exactly those minimal in $R$. That is, $v R w$ for all $v \in W$ iff $M \models_w K$. This condition ensures that no world is more plausible than any world consistent with $K$, and that all $K$-worlds are equally plausible. Such a constraint can be expressed in our language as

$$\boxed{\forall (KB \supset (\Box KB \land \neg \Box KB))}$$

for any $K$ that is finitely expressible as $KB$. This ensures that any $KB$-world sees every other $KB$-world $(\neg \Box KB)$, and that it sees only $KB$-worlds ($\neg \Box KB$). All statements about revision are implicitly evaluated with respect to $KB$. We abbreviate sentence (1) as $O(KB)$ and intend it to mean we "only know" $KB$. Such models are called $K$-revision models.

Given this structure, we want the set of $A$-worlds minimal in $R$ to represent the state of affairs believed when $K$ is revised by $A$. These are the most plausible worlds, the ones we are most willing to adopt, given $A$. Of course such a minimal set may not exist (consider an infinite chain of more and more plausible $A$-worlds). Still, we can circumvent this problem by adopting a conditional perspective toward revision. Often when revising a belief set, we do not care to characterize the entire new belief state, but only certain consequences of interest of the revised theory (i.e., conditionals).

The sentence $A \xrightarrow{KB} B$ should be true if, at any point on the chain of decreasing $A$-worlds, $B$ holds at all more plausible $A$-worlds (hence, $B$ is true at some hypothetical limit of this chain). We can define the connective $\xrightarrow{KB}$ as follows:

$$A \xrightarrow{KB} B \equiv_{df} \neg A \lor \Box (A \land \Box (A \supset B)).$$

This sentence is true in the trivial case when $A$ is impossible, while the second disjunct states that there is some world $w$ such that $A$ holds and $A \supset B$ holds at all worlds still more plausible than $w$. Thus $B$ holds at the most plausible $A$-worlds (whether this is a "hypothetical" or actual limit). In this manner we avoid the Limit Assumption (see below). It is important to note that $\xrightarrow{KB}$ is a connective in the usual sense, not a family of connectives indexed by "KB". Given the Ramsey test, $\xrightarrow{KB}$ is nothing more than a subjunctive conditional.

We can define for any propositional $A \in L_{CP}$, the belief set resulting from revision of $K$ by $A$ as

$$K_A^M = \{ B \in L_{CP} : M \models A \xrightarrow{KB} B \}. $$

**Theorem 1** If $M$ is a $K$-revision model for any $K$, then $^*M$ satisfies postulates (R1)–(R8).

**Theorem 2** Let $\ast$ be any revision operator satisfying (R1) through (R8). Then there exists a $K$-revision model, for any theory $K$, such that $\ast = ^*M$.

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3 For all $w \in W$, $w^*$ is defined as the map from $P$ into $\{0, 1\}$ such that $w^*(A) = 1$ iff $w \in \varphi(A)$; in other words, $w^*$ is the valuation associated with $w$.

4 See (Lewis 1973; Stalnaker 1984) on this notion.

5 In (Boutilier 1992a) we develop this minimal logic in the context of preorder revision.
Thus, \( CO^* \) is an appropriate logical characterization of AGM revision and, in fact, is the first logical calculus of this type suited to the AGM theory. However, the modal approach suggests a number of generalizations of AGM revision, for example, by using CO or dropping connectedness (Boutilier 1992a). It also provides considerable expressive power with which we can constrain the revision process in novel ways.

The Limit Assumption Our approach to revision makes no assumption about the existence of minimal \( A \)-worlds, which Grove (1988) claims forms an integral part of any model of revision. As Lewis (1973) emphasizes, there is no justification for such an assumption other than convenience. Consider a \( KB \) that contains the proposition “I am 6 feet tall.” Revising by \( A = \{ \text{I am over 7 feet tall} \} \) would allow one to evaluate Lewis’s classic counterfactual “If I were over 7 feet tall I would play basketball.” However, it doesn’t seem that there should exist a most plausible \( A \)-world, merely an infinite sequence of such worlds approaching the limit world where “I am 7 feet tall” is true. Our model allows one to assume the truth of the counterfactual \( A \models KB, B \) if the consequent \( B \) (“I play basketball”) is strictly implied (i.e., if \( \Box (A \models B) \) holds) at any world in this sequence of \( A \)-worlds. In this respect, we take Lewis’s semantic analysis to be appropriate.

Models of revision that rely on the the Limit Assumption (e.g., (Grove 1988; Katsuno and Mendelzon 1991b)) would also make \( A \models KB, B \) true, but for the wrong reasons. It holds vacuously since there are no minimal \( A \)-worlds. Of course, \( A \models KB, \neg B \) is true in this case too, which is strongly counterintuitive. How can this be reconciled with Theorems 1 and 2, which show that our notion of revision is equivalent to the AGM version, including those that make the Limit Assumption?

In fact, this points to a key advantage of the modal approach, its increased expressive power. We can easily state that worlds of decreasing height (ht), down to my actual height of 6 feet, are more plausible using:

\[
\exists y \left[ 6 < y \land (\text{ht(Me)} = x \land \Box (\text{ht(Me)} \neq y) \right]. \tag{4}
\]

Not only can we constrain the revision process directly using conditionals, but also indirectly using such intensional constraints. Of course, there must be some AGM operator that has \( \beta \in K_0 \) exactly when our model satisfies \( \alpha \models KB, \beta \), including \( B \in K_0 \) (the basketball counterfactual). But models making the Limit Assumption do not reflect the same structure as our \( CO^* \)-model. They cannot express nor represent the constraint relating plausibility to height. In order to ensure \( B \in K_0 \), they must violate sentence (4). The modal language also improves on AGM revision in general, where such a constraint can only be expressed by some infinite set of conditions \( \beta \in K_0 \). In (Boutilier 1992a) we examine the Limit Assumption and intensional constraints in more detail. In particular, we show how constraints on the entrenchment and plausibility of sentences and beliefs can also be expressed at the object level.

When reasoning about the revision of a knowledge base \( KB \), we require a background theory with the sentence \( O(KB) \), which implicitly constrains the conditional connective to refer to \( KB \), and a set of conditional assertions from which we can derive new revision assertions. For instance, to take an example from default reasoning, if one asserts

\[
\{ \text{bird} \models KB, \text{fly}, \text{penguin} \models KB, \neg \text{fly}, \text{penguin} \models KB \} \models \text{bird}
\]

then the conclusion \( \text{bird} \models KB, \neg \text{fly}, \text{penguin} \models KB \) can be derived. Indeed this should be the case as penguins are a specific subclass of birds and properties associated with them should take precedence over those associated with birds. Beliefs in \( KB \) can influence revision as well. If we take \( KB \) to be \( \{ A \cup B, C \cup D \} \) (where \( A, B, C, D \) are distinct atoms) then from \( O(KB) \) we can infer, for instance, \( A \models KB, B \land A \cup C \models KB \land A \cup D \). We can also derive \( \neg (A \models KB, C) \) since revision by \( A \) does not force acceptance of \( C \). Other derived theorems include (see (Boutilier 1992a) for more examples):

- \( (A \models KB, B) \land (A \models KB, C) \models (A \models KB, B \land C) \)
- \( (A \models KB, C) \land (B \models KB) \models (A \land B \models KB, C) \)
- \( (A \models KB, B) \models ((A \land B \models KB, C) \models (A \models KB, C)) \)
- \( (A \models KB, B) \land (A \models KB, C) \models (A \land B \models KB, C) \)
- \( (A \models KB, C) \models (A \land B \models KB, C) \models A \models KB \land \neg B \)

We now turn our attention to a systematic framework for representing knowledge about belief revision.

A Framework for Subjunctive Queries

Let \( KB \) be as usual a set of beliefs representing our knowledge of the world. We also expect there to be some conditional beliefs among these that constrain the manner in which we are willing to revise our (objective) beliefs. These take the form \( \alpha \models KB, \beta \) (or \( \alpha > \beta \) when we intend Lewis’ connective), and will be referred to as subjunctive premises. By a subjunctive query we mean something of the form “If \( A \) were true, would \( B \) hold?” In other words, is \( A > B \) a consequence of our beliefs and subjunctive premises?

Given the connection between VC and belief revision, and assuming the Ramsey test is an appropriate truth test for subjunctives, it would appear that VC is exactly the logical calculus required for formulating subjunctive queries. However, we have misrepresented the Gärdenfors result to a certain degree; in fact, his
semantics does not account for the postulate of consistent revision (R4). It is excluded because it results in triviality (see (Gärdenfors 1988)) and, together with the other postulates, is far too strong to be of use. Because (R4) is unaccounted for in VC, it is inadequate for the representation of certain subjunctive queries.

**Example** Suppose $KB = \{B\}$, a belief set consisting of a single propositional letter. If we were to ask "If $A$ then $B$?" intuitively we would expect the answer YES, when $A$ is some distinct atomic proposition. With no constraints (such as $A \triangleright B$), the postulate of consistent revision should hold sway and revising by $A$ should result in $KB' = \{A, B\}$. Hence, $A \triangleright B$ should be true of $KB$. Similarly, $(A \triangleright C)$ should also be true of $KB$ for any distinct atom $C$.

In VC there is no mechanism for drawing these types of conclusions. At most one could hope to assert $B$ as a premise and derive $A \triangleright B$ or $(A \triangleright C)$, but neither of $B \vdash_{VC} A \triangleright B$ or $B \vdash_{VC} (A \triangleright C)$ is true, nor should they be. It should be the case that if $A$ is consistent with our beliefs that $A \triangleright B$ holds, but merely asserting $B$ doesn't carry this force. When $B$ is a premise we mean "$B$ is believed;" but this does not preclude the possibility of $A$, $\neg A$, $C$, or anything else being believed. When $KB = \{B\}$ we intend something stronger, that "$B$ is all that is believed." Because $B$ is the only sentence in $KB$, we convey the added information that, say, neither $A$ nor $\neg A$ is believed. In Levesque's (1990) terminology, we only know $KB$. To only know some sentence is to both know (or believe) $B$ and to know nothing more than $B$. To know $B$ is to restrict one's set of epistemic possibilities to those states of affairs where $B$ is true. If some $\neg B$-world were considered possible an agent could not be said to know $B$, for the possibility of $\neg B$ has not been ruled out. To know nothing more than $B$ is to include all possible $A$-worlds among one's set of epistemic possibilities. Adding knowledge to a belief set is just restricting one's set of epistemic possibilities to exclude worlds where these new beliefs fail, so if some $B$-world were excluded from consideration, intuitively an agent would have some knowledge other than $B$ that ruled out this world.

In our logic $CO^*$ we have precisely the mechanism for stating that we only know a $KB$. We consider the set of minimal worlds to represent our knowledge of the actual world. Exactly those possible worlds consistent with our beliefs $KB$ are minimal in any $KB$-revision model. This is precisely what the sentence $O(KB)$ asserts, that $KB$ is believed (since only $KB$-worlds are minimal) and that $KB$ is all that is believed (since only minimal worlds are $KB$-worlds).

Returning to the query $A \rightarrow KB\ B$, this analysis suggests that $B \vdash CO^* A \rightarrow KB\ B$ is not the proper formulation of the query. This derivation is not valid (just as it is not in VC). Rather, we ought to ask if $A \rightarrow KB\ B$ holds if we only know $B$. In fact, both

$$O(B) \vdash CO^* A \rightarrow KB\ B$$

and

$$O(B) \vdash CO^* \neg(A \rightarrow KB\ C)$$

are legitimate derivations.

This leads to an obvious framework for subjunctive query answering, given a set of beliefs $KB$. Our knowledge of the world is divided into two components, a set $KB$ of objective (propositional) facts or beliefs, and a set $S$ of subjunctive conditionals acting as premises, or constraints on the manner in which we revise our beliefs. To ask a subjunctive query $Q$ of the form $\alpha \rightarrow KB\ \beta$ is to ask if $\beta$ would be true if we believed $\alpha$, given that our only current beliefs about the world are represented by $KB$, and that our deliberations of revision are constrained by subjunctive premises $S$.8 The expected answers YES, NO and UNK (unknown) to $Q$ are characterized as follows.

$$ASK(Q) = \begin{cases} YES & \text{if } O(KB) \cup S \models CO^* Q \\ NO & \text{if } O(KB) \cup S \models CO^* \neg Q \\ UNK & \text{otherwise} \end{cases}$$

Objective queries about the actual state of affairs (or, more precisely, about our beliefs) can be phrased as $\tau KB\ \beta$ where $\beta$ is the objective query of interest. It's easy to see that for such a $Q$

$$ASK(Q) = YES \iff \vdash CO^* KB\ \beta.$$  

The ability to express that only a certain set of sentences is believed allows us to give a purely logical characterization of subjunctive queries of a knowledge base. The logic VC seems adequate for reasoning from subjunctive premises and for deriving new conditionals, but it cannot account for the influence of factual information on the truth of conditionals in a completely satisfying manner; for it lacks the expressive power to enforce compliance with postulate (R4).9 The approaches of Ginsberg and Jackson take VC to be the underlying counterfactual logic. Indeed, their approaches (under certain assumptions) satisfy the Lewis axioms. However, they recognize that the ability to only know a knowledge base is crucial for revision and subjunctive reasoning, an expressive task not achievable in VC. Therein lies the motivation for their extra-logical characterizations, and the underlying idea that $KB$ is representable as a set of sentences or set of possible worlds from which we construct new sets in the course of revision. $CO^*$ can be viewed as a logic in which one can capture just this process.10

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8We note that $S$ can be part of $KB$, lying within the scope of $O$, but prefer to keep them separate for this exposition (see (Boutilier 1992b)).

9In fact, it is not hard to verify that the axioms for VC are each valid in $CO^*$ if we replace nonsubjunctive information (say $\alpha$) by statements to the effect that $\alpha$ is believed, also expressible in $CO^*$; see (Boutilier 1992b).

10Other distinctions exist (see Section 1), e.g., Ginsberg's proposal is syntax-sensitive, while Jackson's system is committed to specific minimality criteria.
Integrity Constraints

Often it is the case that only certain states of knowledge, certain belief sets, are permissible. The concept of integrity constraints, widely studied in database theory, is a way to capture just such conditions. For a database (or in our case, a belief set) to be considered a valid representation of the world, it must satisfy these integrity constraints. For instance, we may not consider feasible any belief set in which certain commonsense laws of physics are violated; or a database in which there exists some student with an unknown student number may be prohibited.

This apparently straightforward concept actually has several distinct interpretations. Reiter (1990) surveys these and proposes the definition we favor, which essentially asserts that an integrity constraint $C$ should be entailed by $KB$. The distinguishing characteristic of Reiter's definition is that integrity constraints can be phrased using a modal knowledge operator, which refers to "what is known by the database." We will assume constraints are propositional and that $KB$ satisfies $C$ just when $KB$ entails $C$.

As emphasized in (Fagin, Ullman and Vardi 1983) and (Winslett 1990), integrity constraints are particularly important when updating a database. Any new database (or belief set) should satisfy these constraints, therefore any reasonable model of update or revision must explicitly account for integrity constraints.

Example Let the constraint $C$, that a department has only one chair, be expressed as

$$\text{chair}(x, d) \land \text{chair}(y, d) \supset x = y$$

Suppose we update $KB$ with $\text{chair}(\text{Ken}, \text{DCS}) \lor \text{chair}(\text{Maria}, \text{DCS})$, and assume

$$KB = \{\text{chair}(\text{Derek}, \text{DCS})\}$$

so this new fact in inconsistent with the existing $KB$ (assuming Unique Names). The constraint can not be enforced in the updated $KB'$, for nothing about $C$ says it must be true in the revised state of affairs, even if it is an explicit fact in the original $KB$.

This example illuminates the need for integrity constraints to be expressed intentionally. They refer not only to the actual world, but to all (preferred) ways in which we view the world.

We can ensure a revised belief set or database satisfies a constraint $C$ by asserting $\Box C$ as a premise in our background theory (on the same level as $O(KB)$). This has the effect of ensuring any possible worlds ever considered satisfy $C$ (thus requiring the logic CO). However, this may be too strong an assertion in many applications. Such a statement will force any revision of $KB$ by a fact inconsistent with $C$ to result in the inconsistent belief set $Cn(\bot)$. In certain (maybe most) circumstances, we can imagine the violation of a constraint $C$ ought not force us into inconsistency.

Instead of abolishing $\neg C$-worlds outright, we'd like to say all $C$-worlds are "preferred" to any world violating the constraint. Such a condition is expressible as $\Box (C \supset \Box C)$. To see this, imagine some $\neg C$-world $v$ is more plausible than some $C$-world $w$. Then $w \models u$ and $M \not\models (C \supset \Box C)$. We denote this formula by $WIC$. Since we are often concerned with a set of constraints $C = \{C_1, \ldots, C_n\}$, in such a case we use $C$ to denote their conjunction, and

$$WIC = \Box (C \supset \Box C) \quad \text{where} \quad C = \bigwedge_{i\leq n} C_i.$$

Definition $M$ is a revision model for $KB$ with weak integrity constraints $C$ iff $M$ is a revision model for $KB$ and $M \models \Box (C \supset \Box C)$.

Theorem 3 Let $M$ be a revision model for $KB$ with weak integrity constraints $C$. Then $KIM^M C$ for all $A$ consistent with $C$.

Thus we validate the definition of integrity constraint.

Example Let $KB = \{\text{chair}(\text{Derek}, \text{DCS})\}$ and $C$ be the previous constraint (5). If we update $KB$ with $\text{chair}(\text{Ken}, \text{DCS}) \lor \text{chair}(\text{Maria}, \text{DCS})$, then from $\{O(KB)\} \cup WIC$ we can derive in $CO^*$

$$\text{chair}(\text{Ken}, \text{DCS}) \lor \text{chair}(\text{Maria}, \text{DCS}) \supset \text{chair}(\text{Ken}, \text{DCS}) \lor \text{chair}(\text{Maria}, \text{DCS}).$$

This definition of integrity constraint has the unappealing quality of being unable to ensure that as many constraints as possible be satisfied. For instance, if some update $A$ violates some $C_i$ of $C$, then revision by $A$ is not guaranteed to satisfy other constraints. In (Boutilier 1992a) we introduce strong constraints that accomplish this. We can also prioritize constraints, assigning unequal weight to some constraints in $C$. In (Boutilier 1992a) we introduce strong constraints that accomplish this. We can also prioritize constraints, assigning unequal weight to some constraints in $C$. Fagin, Ullman and Vardi (1983) have argued that sentences in a database can have different priorities and that updates should respect these priorities by "hanging on to" sentences of higher priority whenever possible during the course of revision. Consider two constraints asserting that a department has one chair and that the chair of Computer Science (CS) is the only person without a course to teach. It might be that certain information cannot satisfy both of the constraints, but could satisfy either one singly — for example, when we learn that Maria is the chair and Ken has no course load. We may also prefer to violate the constraint that a non-chair faculty member teaches no course, deferring to the fact that CS has only one chair.

We can express constraints involving a knowledge modality (see (Boutilier 1992b) for details).
Suppose that the set \( C = \{ C_1, \ldots, C_n \} \) is now an ordered set of integrity constraints with \( C_i \) having higher priority than \( C_j \) whenever \( i < j \). We prefer \( C_i \) when a conflict arises with \( C_j \). Let \( P_i \) denote the conjunction of the \( i \) highest priority integrity constraints

\[
P_i = C_1 \land C_2 \land \cdots \land C_i.
\]

The set of prioritized integrity constraints is

\[
ICP = \{ \Box (P_i \lor \square P_i) : i \leq n \}.
\]

**Definition** \( M \) is a revision model for \( K \) with prioritized integrity constraints \( C_1, \ldots, C_n \) iff \( M \) is a revision model for \( K \) and \( M \models ICP \).

**Theorem 4** Let \( M \) be a revision model for \( K \) with prioritized integrity constraints \( C_1, \ldots, C_n \). If \( A \) is consistent with the conjunction \( P_j \) of all constraints \( C_i, i \leq j \), then \( K^M_A \models C_i \) for all \( i \leq j \).

**Example** Let \( KB = \{ \text{chair}(Derek, DCS) \} \) and \( C = \{ C_1, C_2 \} \), where \( C_1 \) is constraint (5) and

\[
C_2 = \text{chair}(x, DCS) \equiv \text{teachnocourse}(x).
\]

From \( \{ O(KB) \} \cup ICP \) we can derive in \( CO^* \)

\[
\text{teachnocourse}(Ken) \land \text{chair}(Maria, DCS) \Rightarrow \text{chair}(x, DCS) \equiv x = \text{Maria}.
\]

**Concluding Remarks**

We have presented a modal logic for revision and subjunctive reasoning that, unlike current logics, can account for the effect of only knowing a knowledge base. \( CO^* \) can be viewed as a logical calculus for AGM revision. Furthermore, it characterizes these processes without making the Limit Assumption, as advocated by Lewis (1973), and allows integrity constraints to be expressed naturally within the logic. In (Boutilier 1992b) we show that \( CO^* \), with its ability to reason about knowledge, can be viewed as a generalization of autoepistemic logic, and that our subjunctive bears a remarkable similarity to the normative conditionals postulated for default reasoning. Indeed, we show that default reasoning can be viewed as a special case of revision by using our subjunctive. Furthermore, we show that the triviality result of Gärdenfors (1988) is avoided in our logic at the expense of postulate (R4), which we claim cannot be correct for belief sets that include conditionals. The triviality result has also led to a distinction between revision and update (Winslett 1990; Katsuno and Mendelzon 1991a), which has also been used to define subjunctives (Grahne 1991). An interesting avenue of research would be to pursue the connection between the two, examining the extent to which update can be captured by unary modal operators, and the extent to which either problem subsumes the other. The generalizations of revision afforded by the modal approach may also apply to update.

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**References**


