Decidability of Contextual Reasoning

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Contexts were first suggested in McCarthy's Turing Award Paper, (McCarthy 1987), as a possible solution to the problem of generality in AI. McCarthy's concern with the existing AI systems has been that they can reason only about some particular, predetermined task. When faced with slightly different circumstances they need to be completely rewritten. In other words, AI systems lack generality. Cyc (Guha & Lenat 1990), a large common-sense knowledge-base currently being developed at MCC, is one example of where contexts have already been put to use in attempt to solve the problem of generality. Because of the complexity of the problem of generality, it has been speculated that any reasoning system which would be able to solve this problem would itself be computationally unacceptable. The purpose of this paper is to show that propositional contextual reasoning is decidable.

Propositional logic of context extends classical propositional logic with a new modality, $\mathtt{ist}(c, \phi)$, used to express that the sentence, ϕ, is true in the context c. We first give a short sketch the syntax and the semantics of the language of context, as proposed in (McCarthy 1993) and formalized in (Buvač & Mason 1993).

To define the syntax, we begin with two distinct countable sets: $\mathbb{K}$ the set of all contexts, and $\mathbb{P}$ the set of propositional atoms. The set, $\mathbb{W}$, of well-formed formulas is built up from the propositional atoms, $\mathbb{P}$, using the usual propositional connectives (negation and implication) together with the ist modality: $\mathbb{W} = \mathbb{P} \cup (\neg\mathbb{W}) \cup (\mathbb{W} \to \mathbb{W}) \cup \mathtt{ist}(\mathbb{K}, \mathbb{W})$.

To define the semantics we first need to introduce some mathematical notation. If X is a set then $\mathbf{P}(X)$ is the set of subsets of X. X^* is the set of all finite sequences, and we let $\bar{x} = [x_1, \ldots, x_n]$ range over X^*. The infix operator $*$ is used for appending sequences. Drawing on the intuition that a context describes a state of affairs, and that the nature of the context may itself be context dependent, i.e. that a context may appear different when viewed from different contexts, a model, $\mathfrak{M}$, is defined to be a function which maps a context sequence to a set of truth assignments. Formally, $\mathfrak{M} : \mathbb{K}^* \to \mathbf{P}(\mathbb{P} \to 2)$. Satisfaction is a relation on $<\mathfrak{M}, \nu, \bar{c}, \phi>$, written as $\mathfrak{M}, \nu \models_{\bar{c}} \phi$, and defined inductively by:

$$\mathfrak{M}, \nu \models_{\bar{c}} \rho \text{ iff } \nu(\rho) = 1, \quad \rho \in \mathbb{P}$$

$$\mathfrak{M}, \nu \models_{\bar{c}} \mathtt{ist}(c_1, \phi) \text{ iff } \forall \nu_1 \in \mathfrak{M}(\bar{c} * c_1) \quad \mathfrak{M}, \nu_1 \models_{\bar{c}*c_1} \phi$$

The clauses for $\neg$ and $\to$ are defined in the usual way. We write $\mathfrak{M} \models_{\bar{c}} \phi$ iff $\forall \nu \in \mathfrak{M}(\bar{c}) \quad \mathfrak{M}, \nu \models_{\bar{c}} \phi$; we say that ϕ is valid in $\bar{c}$ iff $\forall \mathfrak{M} \quad \mathfrak{M} \models_{\bar{c}} \phi$.

We proceed to define some notation, needed for the decidability results. The *vocabulary* of a sentence ϕ in given in $\bar{c}$, $\mathrm{Vocab}(\bar{c}, \phi)$, is a relation on a context sequence and the atoms which occur in the scope of that context sequence:

$$\mathrm{Vocab}(\bar{c}, \phi) = \begin{cases} \{<\bar{c}, \phi>\} & \phi \in \mathbb{P} \\ \mathrm{Vocab}(\bar{c}, \phi_0) & \phi \text{ is } \neg\phi_0 \\ \mathrm{Vocab}(\bar{c} * c, \phi_0) & \phi \text{ is } \mathtt{ist}(c, \phi_0) \\ \mathrm{Vocab}(\bar{c}, \phi_0) \cup \mathrm{Vocab}(\bar{c}, \phi_1) & \phi \text{ is } \phi_0 \to \phi_1 \end{cases}$$

The restriction of a truth assignment, ν, with respect to $\mathrm{Vocab}(\bar{c}_0, \phi)$ is defined to be the unique truth assignment ν' such that

$$\nu'(p) = \begin{cases} \nu(p) & <\bar{c}, p> \in \mathrm{Vocab}(\bar{c}_0, \phi) \\ 0 & <\bar{c}, p> \notin \mathrm{Vocab}(\bar{c}_0, \phi). \end{cases}$$

The definition extends in the natural way to $\mathfrak{M}_{\mathrm{Vocab}(\bar{c}_0, \phi)}$, the restriction of the model $\mathfrak{M}$ with respect to the vocabulary $\mathrm{Vocab}(\bar{c}_0, \phi)$.

Theorem (Finite Model Property): $\mathfrak{M} \models_{\bar{c}_0} \phi$ iff $\mathfrak{M}_{\mathrm{Vocab}(\bar{c}_0, \phi)} \models_{\bar{c}_0} \phi$.
The theorem is proved by induction on the structure of the formula ϕ.

Corollary (Decidability): There is an effective procedure which will determine whether or not a formula given in some context is valid.

References

Buvač, S., and Mason, I. 1993. Propositional logic of context. In *AAAI 93*.

Guha, R. V., and Lenat, D. B. 1990. Cyc: A midterm report. *AI Magazine* 11(3):32–59.

McCarthy, J. 1987. Generality in artificial intelligence. *Comm. of ACM* 30(12):1030–1035.

McCarthy, J. 1993. Notes on formalizing context. In *IJCAI 93*.