MarketBayes: A Distributed, Market-Based Bayesian Network

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This paper presents initial work on a system called *MarketBayes*, a computational market economy where distributed agents trade in uncertain propositions. For any Bayesian network, we have defined a corresponding economy of goods, consumers and producers that essentially "computes" the same information. Although our research thus far has only verified the *existence* of a market structure capable of Bayesian calculations, our hope is that such a system may address a variety of interesting problems of distributed uncertain reasoning. For example, the economic framework should be well suited for *belief aggregation*, since the bids of numerous agents with varying beliefs, confidence levels and wealth are concisely "summarized" in the going prices of goods.

A Bayesian network structure consists of a set of related propositions with information about how the probabilities of the propositions depend on one another. In a MarketBayes economy, the goods to be bought and sold correspond to these propositions. If a proposition is true, the corresponding good is worth one "dollar"; if the proposition is false, it is worth nothing. Then if the proposition is uncertain, its worth should be exactly the probability that it is true (Hanson 1995), assuming risk neutrality. A MarketBayes economy is a set of goods along with a mix of consumers and producers that trade in these goods. After equilibrium is reached, the prices of the propositions should equal the probabilities that the propositions are true.

In a Bayesian network, links between propositions encode conditional probabilities. For example a single link from proposition A to proposition B is accompanied by the information $P(B|A) = k$ where k is some probability. The same equation can be rewritten as:

$$\Pr(AB) = k \Pr(A) \quad (1)$$

In a MarketBayes economy, the *consumers* effectively implement equations of the form (1). AB and A are propositions or goods, and the consumer's preference for AB is k times that of A. If the ratio of the prices $\Pr(AB)/\Pr(A)$ diverges from k, the consumer will buy or sell according to its preference, driving the ratio toward k.

In a Bayesian network, the laws of probability are inherent in the inference mechanism. In a MarketBayes economy, *producers* ensure that the laws of probability are not violated. For example, the following is an identity in probability theory:

$$\Pr(A) = \Pr(AB) + \Pr(A\bar{B}) \quad (2)$$

Equations of the form (2) are enforced by producers that have the technology to "transform" one A into one AB and one $A\bar{B}$, and vice versa. If the price $\Pr(A)$ diverges from the price $\Pr(AB) + \Pr(A\bar{B})$, a producer will transform one good into the other in order to capitalize on the potential profits—thus driving the two prices together. This type of producer is an *arbitrageur* since it capitalizes on inconsistencies between related prices.

We have found that consumers of the form (1) and producers of the form (2) are sufficient to encode any Bayesian network with binary propositions.

We have built the initial MarketBayes system on top of a distributed auction mechanism called WALRAS (Wellman 1993). Our next research goal is to better characterize any advantages that a market-based probabilistic reasoning mechanism may have over traditional Bayesian networks. We conjecture that the market system will offer a concise and principled way to aggregate beliefs of multiple distributed agents.

References

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