Abstract

We present a theory of qualitative rigid body mechanics and describe a program that uses this theory to compute qualitative dynamic simulations.

The program works directly from a qualitative representation of geometry (qc-space). It employs a new qualitative representation for forces that reduces ambiguity in force sums and hence reduces branching.

Introduction

Our work is motivated by the task of interpreting rough sketches of mechanical devices of the sort shown in Figure 1. These kinds of sketches are commonly used in mechanical engineering both to record and to communicate design information during the conceptual phase of design. By “interpreting a sketch” we mean achieving something of the understanding of the device that a human engineer would get, including the ability to simulate the device’s behavior in the mind’s eye (i.e., qualitatively, before knowing the quantitative details of the device’s design) and the ability to understand how the geometry of the device produces its behavior (e.g., how the dimensions of the device are constrained by the need to produce the desired behavior). Software capable of interpreting sketches will provide a natural user interface for the next generation of mechanical CAD software.

We abstract the sketch into a novel representation called qualitative configuration space (qc-space) which enables a program both to generalize a single instance of a design (i.e., the sketch) into multiple families of new designs and to compensate for certain kinds of inaccuracies in the sketch [6]. Qc-space is an appropriate representation for conceptual design in part because it attempts to capture only what is known at this stage, i.e., the constraints among dimensions rather than their quantitative values. There are, however, no existing qualitative simulators capable of working from a qc-space.

Our task, therefore, was to develop a new qualitative simulator. A guiding principle in the design of our system was the desire to minimize branching of the simulation, as intractable branching is a common problem in qualitative simulation [3]. We did this by developing a new qualitative representation for forces.

Context

This section provides a brief overview of qc-space; see [6] for a more detailed discussion.

Figure 2 shows the qc-space for the circuit breaker sketched in Figure 1. The interaction between each pair of parts is described with a plane called a qcs-plane (e.g., the hook-position vs. level-angle plane in Figure 2a). The bold lines in each plane, called qcs-curves, describe the configurations of the device in which a particular pair of faces touch. For example, the “cam-follower” qcs-curve in Figure 2a describes the configurations in which the sloped face at the end of the lever touches the sloped face at the end of the hook. The shaded region “behind” each curve represents blocked space, configurations in which the part faces would penetrate. The empty space “in front” of the curves, called free space, represents configurations in which the faces do not touch. Although we depict them as straight lines, diagonal qcs-curves represent families of diagonal, monotonic curves.

Figure 1: A sketch of a circuit breaker.
describes the motions of the lever and hook that occur when the circuit breaker trips. (The hook is a bimetallic strip; during overload the hook heats and bends.) As the trajectory moves down the lever-stop curve the hook is disengaging the lever. Once they disengage, the hook continues to move toward its hot neutral position (“hook=hot” boundary) while the lever is pushed to the right by the lever-spring.2

The trajectory in Figure 2b provides another view of the tripping action of the circuit breaker, this time describing the motions of the lever and pushrod: The pushrod remains against its stop, thus the trajectory follows the pushrod-stop qcs-curve while the lever is pushed to the right by its spring. Eventually the trajectory terminates by colliding with the push-pair qcs-curve, corresponding to the lever coming to rest against the end of the pushrod.

**Representing Forces**

As we know from Newton's laws of motion, to compute the motion of a body one must first compute the net force on it. Unfortunately in a qualitative world this can be a difficult task. Consider, for example, the three blocks in Figure 3. The spring pushes block A to the right, the actuator pushes block C to the left. Block B, the block in the middle, experiences two forces, one from A and one from C. Because the forces are in opposite directions, their qualitative sum is ambiguous.

However, we know that B will move to the left. Why? We know that the force C applies to B is whatever force is necessary for C to achieve its assigned motion (the actuator imparts a known motion to C). Apparently, a qualitative force representation that captures only the direction of a force is inadequate to support this kind of conclusion. In response we developed a force representation with four properties: direction, magnitude, type, and interface motion, and use it to substantially reduce ambiguity. We discuss each of these in turn, but begin by considering another ambiguity reducing insight.

**Simplifying Forces**

Forces in 3-space are commonly described by their vector components, i.e., their projections on each of the coordinate axes. But the only components of a force that have any effect on the motion of a body are the projections of the force onto the body's degrees of freedom. Because we consider only fixed-axis bodies, i.e.,

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2We model the bimetallic strip as a spring having one neutral position when it is cold, and another when it is hot.
those with only one degree of freedom, we need consider only one projection. This results in a substantial reduction in potential ambiguity compared to previous qualitative force representations (e.g., [2]) which consider all three vector components.

Direction and Magnitude
Having focused on one single projection of the force that matters, we describe direction with one of the three standard (qualitative) values +, -, or 0. The direction is + if the force has a component in the direction of positive motion, - if the force has a component in the direction of negative motion, and 0 if the force is perpendicular to the direction of motion. We describe magnitude as 0 if the force has zero magnitude and 1 otherwise.

Force Type
The qualitative property we call “force type” was inspired by the situation depicted in Figure 3 in which C applies what we call a “motion constrained engagement force,” a force that constrains the motion of the body to which it is applied.

According to Hooke’s law, the spring’s deflection determines the magnitude of the spring’s force on A. If inertia is negligible, A will transmit the spring force to B.4 Thus, A applies a force of known magnitude to B. In contrast to a motion constrained engagement force which assigns a known motion, this kind of force assigns a known magnitude. We call this type of force a “compliant engagement force” because it has a known magnitude in the same way that a compliant member (e.g., a spring) produces a force of known magnitude.

This distinction enables one of our basic principles: a motion constrained engagement force overpowers a compliant engagement force. Because there is a motion constrained engagement force pushing B to the left and a compliant engagement pushing B to the right, B must move to the left in Figure 3. In our experience, this principle has proven very useful in resolving ambiguities in force sums.

Interface Motion
The fourth component of our qualitative representation of forces is interface motion. To see what we mean by this, consider that in the example of Figure 3 we decided that block B will move to the left because there is a motion constrained engagement force pushing it in that direction. Now imagine that the actuator attached to block C pushes C to the right instead of the left. Because C will still apply a position constraint to B, the force C applies to B will still be a motion constrained force whose direction is still toward the left. But in this case block B will move to the right.

A convenient way to describe the difference between these two situations is in terms of the relationship between the direction of C’s motion and the direction of the force it applies to B. With the actuator moving to the left, they are in the same direction. In this case we say that C is applying an “advancing force.” With the actuator moving to the right, C’s motion and the force C applies to B are in opposite directions, and we say that C is applying a “retreating force.” (If the actuator holds C fixed, we say that C is applying a “stationary force.”)

We now have four properties with which to describe an engagement force: the direction of the force—[+, -, or 0], its magnitude—[0 or 1], its type—[motion constrained or compliant], and the interface motion—[advancing, retreating, or stationary].

We stated previously that a motion constrained engagement force always overpowers a compliant one. By considering the interface motion of a force we can state this principle more precisely: An advancing motion constrained force causes movement in the direction of the force; a stationary motion constrained force prevents movement opposite the force; and a retreating motion constrained force allows movement opposite the force, not faster than the speed of retreat. In our experience, this more precise version of the principle is quite effective in resolving ambiguities in force sums.

Computing Motion
Because we use a qualitative representation, and because bodies are fixed-axis, we describe the motion (velocity) of a body with the standard values of +, -, and 0. Because we assume motion is inertia-free, the motion is always in the direction of the net force.

To compute the net force the program must first compute the values of the four qualitative properties of each force. Our program begins by determining the qualitative direction and type of each force.

The qualitative direction of a force can easily be obtained by direct examination of the qcs-curve: the direction is given by the outward normal to the qcs-curve. The other three properties are determined by constraint propagation techniques.

The program uses constraint propagation to systematically identify all of the motion constrained forces, then labels the remaining forces as compliant because there is no other choice.

The program begins by identifying all of the obvious motion constrained forces: The engagement forces produced by stationary bodies and the engagement forces produced by bodies positioned by actuators are motion constrained. Then the program uses a constraint

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3This works equally well if the body rotates instead of translates. In this case we compute the moment of the force about a point on the axis of rotation, then project this moment onto the axis.

4We assume motion is inertia-free for reasons described below.

5With inertia, the acceleration is in the direction of the net force but the velocity need not be.
The propagation technique is based on the following rule: Body-1 applies a motion constrained force to body-2 if and only if the force that body-2 applies to body-1 is resisted by some other motion constrained force applied to body-1. For example, in Figure 4 force $F$ is motion constrained because force $M$ is resisted by force $P$, a motion constrained force applied by the fixed surface.

The simulator applies this rule repeatedly until no new motion constrained forces are identified. If all of the obvious cases are identified first, this propagation technique is guaranteed to identify all of the motion constrained forces.

To illustrate this procedure, we apply it to the blocks in Figure 3. We begin by labeling the obvious case: C applies a motion constrained force to B because C is positioned by an actuator. Using the propagation rule, we determine that B applies a motion constrained force to A. The rule cannot be applied to any other engagements, thus the remaining forces are compliant.

Having determined the direction and type of the forces, we turn next to magnitude and interface motion. We need to solve for these two properties and the motion all at the same time. To see why, note that if two bodies are touching but the instantaneous motions are causing the bodies to separate, the magnitudes of the engagement forces will be 0. However, if the instantaneous motions cause the bodies to remain in contact, the magnitudes may be non-zero. Similarly, to compute the motion of a set of bodies one must first know the interface motion of each of the forces, but to determine the interface motion of the forces one must first know the motion of each of the bodies.

Here we use another version of constraint propagation to simultaneously solve for the magnitude and interface motion of the forces and the motion of the bodies. Our approach, variously referred to as "opportunistic search" or "constraint propagation with assumed states and backtracking" [7], propagates motion from one body to the next. Our program starts the propagation process by determining the motion of each body positioned by an actuator, which is simply the motion of the actuator. The simulator then attempts to propagate motion from the bodies with known motion to those they touch, repeating the propagation until it subsides.

Propagation can stall before we have computed the motion of all of the bodies. To prevent this, we use a heuristic to guess values for motion: if all of the forces have the same direction, we guess the motion will be in this direction. (This assumption will be wrong if none of the forces have advancing interface motion and non-zero magnitude.) After we have computed or guessed a value for the motion of each body, the program uses Table 1 to check whether all the motions are consistent with the forces. If not, the program chooses different motions for the bodies whose motion was inconsistent with the applied forces.

Our heuristic can even be applied to some situations in which the forces do not all have the same direction. The additional insight is that any forces whose interface motion is known to be retreating can safely be ignored, because they have no effect on the motion of a body. If, when retreating forces are ignored the result...

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**Table 1**: A body's motion is consistent with the applied forces if the motion and the forces match one of these rules. Any combination not listed is inconsistent.

<table>
<thead>
<tr>
<th>spring,</th>
<th>engagement force</th>
<th>motion constrained</th>
<th>compliant</th>
<th>motion</th>
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<td>$+$</td>
<td>$-$</td>
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For example, because block B in Figure 3 experiences a motion constrained, --, advancing engagement force (from C) and a compliant, +, retreating engagement force (from A), the eighth row of the table tells us that B's motion is $+$. 
maining forces all have the same direction, then the original heuristic applies.

While this augmented heuristic was sufficient to prevent blocking in all of our examples, there are likely to be situations in which the heuristic will be insufficient. In those cases we would have to randomly choose the motion for one body each time the propagation blocks, then, if necessary, backtrack as before.

**The Next Event**

Our simulator is event-oriented: each time step continues until there is a change in the nature of the forces, which in turn causes a change in the motion of the bodies. Four kinds of events can cause such a change: an engagement is broken (a pair of faces separate), an engagement is made (two faces collide), a spring passes through its neutral position, or an actuator reaches its limit of motion. The first kind of event occurs when a trajectory following a qcs-curve reaches the end of that curve; the other kinds of events occur when the trajectory hits a new qcs-curve. Hence, to determine the next event, we must examine the trajectory through qcs-space.

We make a number of assumptions about devices that greatly simplify the task of reasoning about trajectories. We assume that qcs-curves are monotonic, that motion is inertia-free, and that collisions are inelastic. As a result, until the forces on a body change, the motion of the body remains either strictly positive, strictly negative, or zero. Consequently, all trajectories in qcs-space are monotonic over any given time step.

We assume also that devices are fixed-axis and as a result we can examine the trajectory through a multi-dimensional qcs-space by examining the trajectories through 2D projections of the space, i.e., the qcs-planes. We compute the next event, and hence the next state of the device, by first determining which events would happen if the trajectory in each qcs-plane continued until an event occurs in that plane, then using constraint propagation to determine which of the events predicted by the individual planes can happen first.

**Events in the Plane**

If the trajectory follows a qcs-curve, the next event occurs when the trajectory reaches the end of that curve or when the trajectory reaches the intersection of that curve and another curve. For example, if the pushrod is against its stop and the lever is moving to the right

7If the trajectory leaves perpendicular to the qcs-curve rather than reaching the end of the curve, there is no change in the forces. In this case, the engagement forces were retreating and hence had a zero magnitude even before the pair of faces separated.

8For the sake of brevity, we use the term "qcs-curve" to refer to both qcs-curves and the boundaries that represent spring neutral positions and the motion limits of actuators.

(Figure 5), the trajectory in the lever-pushrod qcs-plane will follow the pushrod-stop curve (Figure 6). The next event in that plane occurs when the trajectory reaches the intersection of the pushrod-stop and push-pair curves, i.e., when the lever hits the end of the pushrod.

![Figure 5: Lever-spring pushes lever counterclockwise and hook moves towards its cold neutral position. Hook is high enough that lever cannot miss it.](image)

![Figure 6: Next event occurs when trajectory (thick arrow) hits push-pair. L₀ is start point of trajectory.](image)

If the trajectory in a qcs-plane is horizontal or vertical (and does not follow a qcs-curve), the simulator determines the next event by looking along the direction of motion to see which qcs-curve is reached first. If the trajectory does not intersect any qcs-curves, then no event will occur in this plane (and thus the motion will continue without bound).

If the trajectory is diagonal (and does not follow a qcs-curve), the simulator must perform more complicated geometric reasoning to determine what event happens next. The simulator uses a process of elimination to identify which qcs-curves can be reached: it determines geometrically all of the qcs-curves that cannot be reached; any remaining qcs-curves can be reached.

Consider, for example, the diagonal trajectory in Figure 7 which corresponds to motion of the lever and hook shown in Figure 5. The simulator begins by defining a new coordinate system with its origin at the

9As we will see, this event is prevented by an event in Figure 7.
Figure 7: Trajectory in qc-space (thick arrow) is diagonal. The active region of the qcs-plane is shaded.

starting point of the trajectory. Because the trajectory is monotonic, it will remain entirely in one quadrant of the new coordinate system; we call this the active quadrant. Any qcs curve that lies completely outside the active quadrant cannot be reached by the trajectory.

Now imagine a “chain” of qcs-curves that divides the active quadrant into two regions, only one of which includes the origin. We call the region that includes the origin the active region. Figure 7 shows an example in which the chain is composed of the lever-stop and the “hook—cold” qcs-curves; the active region is shown as a shaded rectangle. The trajectory cannot reach any qcs-curve that lies completely outside the active region.

There may be more than one chain that separates the active quadrant into two regions. For example, in Figure 7, there are three chains: the lever-stop and “hook=cold” chain, the cam-follower and “hook=cold” chain, and the lever-spring and “hook=cold” chain. When there are multiple chains, any qcs-curve that lies completely outside the smallest chain cannot be reached.\(^\text{10}\)

The trajectory can reach any qcs-curve inside the smallest chain and can reach all of the curves that compose the chain. In the example of Figure 7 for instance, only the “hook=cold” and the lever-stop curves can be reached. In this case, there are three possible events: The trajectory can reach the lever-stop curve, in which case the lever strikes the hook; the hook can reach the “hook=cold” curve, in which case the hook relaxes; and the trajectory can reach the intersection of these two curves, in which case the lever strikes the hook at the same instant the hook relaxes.

The Next State

To compute the possible next states of the device, the simulator picks one event, possibly a null event (i.e., no event occurs and there is no changes in the forces), for each qcs-plane. It then checks if this set of events (one from each plane) places consistent constraints on the positions of the bodies. If the constraints are consistent (i.e., the bodies can all be in the required locations at the same time), this set of events can happen simultaneously and hence represents a possible next state of the device. The simulator repeats this process for each possible combination of events obtained by choosing one event for each plane.\(^\text{11}\) If there is more than one consistent set of events, i.e. more than one possible next state, the simulator generates them all, producing an envisionment.

We illustrate this process by showing how the program determines the next states from the events predicted in Figure 6 and Figure 7.\(^\text{12}\)

The first step is to express the position constraints imposed by an event as simple equalities and inequalities between landmarks. For example, for the next event in Figure 6 to occur the position \(L\) of the lever must satisfy the constraints: \(L = B\). Likewise, the null event is described by: \(L < L < B\).

In the same fashion the program describes the three possible events in Figure 7 with the constraints: \([L = A, H_0 < H < C]\), \([L_0 < L < A, H = C]\), and \([L = A, H = C]\), where \(H\) is the position of the hook.

When the trajectory is diagonal, as it is in Figure 7, we use a tiling procedure to construct the constraints for the null event. The simulator first computes the smallest chain as described above, then tiles the active region of this chain with rectangular tiles as the example in Figure 8 illustrates.\(^\text{13}\) The null event corresponds to the trajectory terminating in any tile that does not contain a qcs-curve (these tiles are shaded in the figure). In Figure 7 there is only one tile and the null event is given by: \([L_0 < L < A, H_0 < H < C]\).

With all of the events from the individual planes expressed as constraints, the simulator is ready to enumerate possible next states of the device (i.e., sets of globally consistent next events). Because there are two possible events in Figure 6 (including the null event) and four in Figure 7 (also including the null event), the simulator must consider a total of seven sets of next events (eliminating the set consisting entirely of null events). One set of events, for example, consists of the lever hitting the pushrod in Figure 6 (\(L = B\)) and the lever hitting the hook in Figure 7 (\([L = A, H_0 < H < C]\)). This set of events places inconsistent constraints on the position of the lever \((A \neq B)\) and hence is not a possible next state. A second set

\(^{10}\)The smallest chain is determined by comparing landmark values: in this case its the one with the smallest maximum \(x\)-coordinate.

\(^{11}\)The simulator does, however, throw out the combination that consists only of null events, because that combination corresponds to the current state.

\(^{12}\)Because the circuit breaker has 3 degrees of freedom, there are 3 qcs planes. However, in this case the third plane provides no additional information about the next event.

\(^{13}\)The program places artificial boundaries at + and - infinity to ensure there is always a chain.
Figure 8: Tiling the active region. There is a column of tiles for each landmark along the abscissa, and a row for each landmark along the ordinate. The shaded tiles constitute the null event.

The simulator of Forbus et. al. [2] is more similar to our work. Theirs works from a representation they call "place vocabulary" [1], where a place is a region of uniform contact or a subdivision of free space. Each place contains a mapping from qualitative motions to possible place transitions. The subdivisions of free space are selected to reduce ambiguity in place transitions. They precalculate the allowed transitions between places, while we compute the possible next events for each qcs-plane on the fly. They calculate the places and the possible transitions by quantitative analysis of the configuration space, we compute the next events directly from a qualitative representation.

**Conclusion**
We have developed a qualitative simulator capable of working directly from a qualitative configuration space. The simulator employs a new representation of forces that reduces ambiguity in force sums and hence reduces branching of the simulation.

The simulator assumes fixed-axis parts, inertia-free motion, and inelastic collisions. Sacks and Joskowicz [4] demonstrate that these assumptions encompass a useful class of mechanical devices.

The simulator has enabled us to build a program that interprets rough sketches of mechanical devices.

A long standing goal of mechanical engineering CAD research is the development of analysis tools that are useful early in the design process when much of the detail is yet to be specified. Our simulator is one step toward this goal: It can provide approximate dynamic analysis given only a rough sketch.

**References**