Bargaining with Deadlines

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Abstract
This paper analyzes automated distributive negotiation where agents have firm deadlines that are private information. The agents are allowed to make and accept offers in any order in continuous time. We show that the only sequential equilibrium outcome is one where the agents wait until the first deadline, at which point that agent concedes everything to the other. This holds for pure and mixed strategies. So, interestingly, rational agents can never agree to a nontrivial split because offers signal enough weakness of bargaining power (early deadline) so that the recipient should never accept. Similarly, the offerer knows that it offered too much if the offer gets accepted: the offerer could have done better by out-waiting the opponent. In most cases, the deadline effect completely overrides time discounting and risk aversion: an agent's payoff does not change with its discount factor or risk attitude. Several implications for the design of negotiating agents are discussed. We also present an effective protocol that implements the equilibrium outcome in dominant strategies.

1 Introduction
Multiagent systems for automated negotiation between self-interested agents are becoming increasingly important due to both technology push and application pull. For many-to-many negotiation settings market mechanisms are often used—and for one-to-many negotiation auctions are often appropriate. The competitive pressure on the side with many agents often reduces undesirable strategic effects. On the other hand market mechanisms often have difficulty in "scaling down" to small numbers of agents (Osborne & Rubinstein 1990). In the limit of one-to-one negotiation strategic considerations become prevalent. At the same time one-to-one negotiation settings that crave software agents are ubiquitous. Consider for example two scheduling agents negotiating meeting times on behalf of their users or any e-commerce application where agents negotiate the final price of a good or a scenario where agents representing different departments bargain over the details of a service which they provide jointly.

One-to-one negotiation generally involves both integrative and distributive bargaining. In integrative bargaining the agents search for Pareto efficient agreements i.e. deals such that no other deal exists for making one agent better off without making the other worse off. Intuitively integrative bargaining is the process of making the joint cake as large as possible. Enumerating and evaluating the Pareto efficient deals can be difficult especially in combinatorially complex settings. Automated negotiating agents hold significant promise in this arena due to their computational speed (Sandholm 1993).

In distributive bargaining the focus of this paper the agents negotiate on how to split the surplus provided by the deal i.e. how to divide the cake. A continuum of splits is possible at least if the agents can exchange sidepayments. We call any split where each agent gets a nonnegative benefit from the deal individually rational i.e. each agent would rather accept the deal than no deal. Splitting the gains of an optimal contract in an individually rational way can be modeled generically as follows. Without loss of generality the surplus provided by the contract is normalized to 1 and each agent's fallback payoff that would occur if no contract is made is normalized to 0. Then distributive bargaining can be studied as the process of "splitting-a-dollar". This paper focuses on designing software agents that optimally negotiate on the user's behalf in distributive bargaining.

The designer of a multiagent system can construct the interaction protocol (aka. mechanism) which determines the legal actions that agents can take at any point in time. Violating the protocol can sometimes be made technically impossible e.g. disallowing a bidder from submitting multiple bids in an auction—or illegal actions can be penalized e.g. via the regular legal system. To maximize global good the protocol needs to be designed carefully taking into account that each self-interested agent will take actions so as to maximize its own utility regardless of the global good. In other words the protocol has to provide the right incentives for the agents. In the extreme the protocol could specify everything i.e. give every agent at most one action to choose from at any point. However in most negotiation settings the agents can choose whether to participate or not. So to have the protocol useful the designer has to provide incentives for participation as well. We will return to this question in Section 8.

The most famous model of strategic bargaining is the infinite horizon alternating offers game (Rubinstein 1982). Since it has a unique solution where agents agree on a split immediately it seems attractive for automated negotiation see e.g. (Kraus Wilkenfeld &
Our model resembles war of attrition games where two agents compete for an object and winner-takes-all when one concedes. Those games exhibit multiplicity of equilibria. If the value of the object is common knowledge there is a symmetric equilibrium where one agent concedes immediately. There is also a symmetric equilibrium where agents concede at a rate which depends on the value of the object. (Hendricks & Wilson 1988) study the general class of war of attrition games with perfect information. (Hendricks & Wilson 1989) study the case of incomplete information normally about the object’s value (see also (Riley 1980)). The game was introduced by (Smith 1974) in a biological context and has been applied in industrial economics (Fudenberg & Tirole 1983) (Fudenberg & Tirole 1985) (Kreps & Wilson 1982).

Our model differs from the war of attrition. Agents are allowed to split the dollar instead of winner-takes-all. One would expect that to enlarge the set of equilibria and equilibrium outcomes. This intuition turns out to be false. We show that the only equilibrium outcome is one where agreement is delayed until one of the deadlines is reached and then one agent gets the entire surplus.

We show that there exists a sequential equilibrium where agents do not agree to a split until the first deadline is at which time the agent with the later deadline receives the whole surplus. Conversely we show that there do not exist any other Bayes-Nash equilibria where agents agree to any other split at any other time. Therefore both our positive and negative results are strong with respect to the degree of sequential rationality agents are assumed to have. Intuitively speaking these equilibria the agents update their beliefs rationally and neither agent is motivated to change its strategy at any point of the game given that the other agent does not change its strategy.

Our results are robust in other ways as well. First there does not exist even a mixed strategy equilibrium where an agent concedes at any rate before its deadline. This is in contrast with the usual equilibrium analysis of war of attrition games. Second the results hold even if the agents discount time in addition to having deadlines. Third even if agents have different risk attitudes they will not agree to any split before their deadline. That is even risk averse agents will refuse safe and generous offers and will instead prefer to continue to the risky “waiting game”.

The rest of the paper is organized as follows. Section 2 describes our formal model of bargaining with deadlines. Section 3 presents our main results for pure-strategy equilibria. Section 4 extends them to mixed strategies. Sections 5 and 6 present the results with time discounting and with agents with risk attitudes. Section 7 describes the entailed insights for designing automated negotiating agents. Section 8 discusses implications for the design of interaction protocols. Finally Section 9 concludes.

2 Our model of bargaining under deadlines

Our bargaining game $\Gamma(a, b)$ has two agents $1$ and $2$. The type of agent $1$ is its deadline $d_1$. The type of agent $2$ is its deadline $d_2$. The types are private infor-
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Proof. Assume for contradiction that there exist types \( d_1 > 0 \) and \( d_2 > 0 \) and a pure strategy Bayes-Nash equilibrium \((g_1, g_2)\) where the agents agree to a split \((x_0, x_1) = (x, 1-x)^t\) where \( x \in (0, 1) \) at time \( t \geq 0 \). We assume without loss of generality that agent 1 receives at least one half i.e. \( x \geq \frac{1}{2} \). We can therefore write \( x = \frac{1}{2} + \epsilon \) where \( \frac{1}{2} > \epsilon \geq 0 \).

Let \( g_0(d_1) \) denote agent 1's beliefs about \( d_1 \) at time \( t \). Similarly, let \( f_0(d_1) \) denote agent 2's beliefs about \( d_1 \) at time \( t \). Denote by \( G(d_2) \equiv \int_{d_2} f_0(d_2)dd_2 \) the cumulative distribution of \( f_0 \) and by \( F(d_1) \equiv \int_{d_1} f_0(d_1)dd_1 \) the cumulative distribution of \( f_0 \).

In equilibrium, agent 2 will accept \( 1 - x \) only if she does not expect to receive more by unilaterally moving to the waiting game. The expected payoff from the waiting game is simply agent 2's subjective probability of winning the waiting game with probability one so the split \((x, 1-x)\) could not occur. The second case occurs if \( d_2 > 0 \). Contradiction.

Based on these updated beliefs \((g_2)\) agent 2 would accept only if

\[
\frac{1}{2} - \epsilon \geq \int_{d_2} g_0(d_2)dd_2 = F(d_2) - F(\epsilon) = F(d_2) \quad (3)
\]

In other words, agent 2's type \( d_2 \) must not be too high. Let \( \alpha(y) \equiv \inf[d_2 \mid y \geq F(d_2)] \). With this notation, \((3)\) can be rewritten as \( d_2 \leq \frac{1}{2} - \epsilon \).

Now, agent 1 will only accept this offer if it will give her an expected payoff at least as large as that of the waiting game which equals her subjective probability of winning the waiting game. There are two cases. First, if \( d_1 > \frac{1}{2} - \epsilon \) agent 1 knows that she will win the waiting game with probability one i.e. the split \((x, 1-x)\) could not occur. The second case occurs when \( d_2 \leq \beta(\frac{1}{2} - \epsilon) \). Agent 2 can use the fact that \( d_1 \leq \beta(\frac{1}{2} - \epsilon) \) to update her beliefs about agent 1's deadline as follows:

\[
f_1(d_1) = 0 \quad \text{if} \quad (d_1 > \beta(\frac{1}{2} - \epsilon)) \].

\[
f_1(d_1) = g_0(d_2) \cdot \frac{\int_{\alpha(1-\epsilon)}^{1} g_0(d_2)\,dd_2}{\int_{\alpha(1-\epsilon)}^{1} g_0(d_2)\,dd_2}
= g_0(d_2) \cdot \left[ 1 + \frac{\int_{\alpha(1-\epsilon)}^{1} g_0(d_2)\,dd_2}{\int_{\alpha(1-\epsilon)}^{1} g_0(d_2)\,dd_2} \right]
\]

\( \geq 2 \) because \( \epsilon \leq \beta(\frac{1}{2} - \epsilon) \).

Based on these updated beliefs, agent 1 would accept only if

\[
\frac{1}{2} + \epsilon \geq \int_{d_1} g_0(d_1)dd_1 \geq 2[G(d_1) - G(\epsilon)] = 2G(d_1) \quad (5)
\]

In other words, agent 1's type \( d_1 \) must also not be too high. Let \( \beta(y) \equiv \inf[d_1 \mid y \geq G(d_1)] \). With this notation, \((5)\) can be rewritten as \( d_1 \leq \beta(\frac{1}{2} - \epsilon) \).

In equilibrium, agent 2 only accepts if it gives her an expected payoff at least as large as that of the waiting game which equals her subjective probability of winning the waiting game. There are two cases. First, if \( d_2 > \beta(\frac{1}{2} - \epsilon) \) agent 2 knows that she will win the waiting game with probability one i.e. the split \((x, 1-x)\) could not occur. The second case occurs when \( d_2 \leq \beta(\frac{1}{2} - \epsilon) \). Agent 2 can use the fact that \( d_1 \leq \beta(\frac{1}{2} - \epsilon) \) to update her beliefs about agent 1's deadline as follows:

\[
f_1(d_1) = 0 \quad \text{if} \quad (d_1 > \beta(\frac{1}{2} - \epsilon)) \].

\[
f_1(d_1) = \frac{\int_{\beta(\frac{1}{2} - \epsilon)}^{1} f_0(d_1)\,dd_1}{\int_{\beta(\frac{1}{2} - \epsilon)}^{1} f_0(d_1)\,dd_1}
= \frac{f_0(d_1) \cdot \left[ 1 + \frac{\int_{\beta(\frac{1}{2} - \epsilon)}^{1} f_0(d_1)\,dd_1}{\int_{\beta(\frac{1}{2} - \epsilon)}^{1} f_0(d_1)\,dd_1} \right]}{\int_{\beta(\frac{1}{2} - \epsilon)}^{1} f_0(d_1)\,dd_1}
\]

\( \geq 2 \) because \( \beta(\frac{1}{2} - \epsilon) \leq \text{median}(f_0) \).

Based on these updated beliefs, agent 2 would accept only if

\[
\frac{1}{2} - \epsilon \geq \int_{d_2} f_0(d_2)dd_2 \geq 2[F(d_2) - F(\epsilon)] = 2F(d_2) \quad (7)
\]

\( i.e. \) \( d_2 \leq \alpha(\frac{1}{2} - \epsilon) \).

This process of belief update and acceptance threshold resetting continues to alternate between agents. After \( r \) rounds of this alternation, all types have been eliminated except those that satisfy \( d_1 \leq \beta(\frac{1}{2} - \epsilon) \) and \( d_2 \leq \alpha(\frac{1}{2} - \epsilon) \). This process can continue for an unlimited number of steps i.e. the upper bounds approach zero. Therefore the equilibrium cannot exist if \( d_1 > 0 \) or \( d_2 > 0 \). Contradiction. \( \square \)

4 Mixed strategy equilibria

We now strengthen our impossibility result by showing that it holds for mixed strategies as well i.e. that there is no other rational way of playing the game than "sit-and-wait" even if randomization is possible. This is yet another difference between our setting and war of attrition games. In the latter, mixed strategies play an important role: typically the unique symmetric equilibrium has concession rates that are mixed strategies.

**Theorem 4.1** If \( d_1 > 0 \) or \( d_2 > 0 \), there does not exist a mixed strategy Bayes-Nash equilibrium of \( (\Gamma, (a, b)) \), where the agents agree to a split other than \((1,0)\) or \((0,1)\) with positive probability.

**Proof.** Assume for contradiction that there exist types \( d_1 > 0 \) and \( d_2 > 0 \) and a mixed strategy Bayes-Nash equilibrium where there is positive probability of an agreement other than \((1,0)\) or \((0,1)\). Now there has to exist at least one point in time \( t \) where there is positive probability of an agreement other than \((1,0)\) or \((0,1)\). We analyze the equilibrium at such a time. Recall \( f(t, \Gamma, (\alpha, \beta)) \) and \( \beta \) from the proof of Thrm. 3.1.

Agent 1 will accept an agreement if she gets a share \( x > a_1 \Gamma \) where \( a_1 \) is her acceptance threshold. That
threshold depends on her type. Since we are analyzing a mixed strategy equilibrium the threshold can also depend on randomization. We therefore say that $a_1$ is randomly chosen for time $t$ from a probability density function $n(a_1)$. Similarly, agent 2 will accept an agreement if she has to offer a share of $x < a_2$ where $a_2$ is agent 2's offering threshold. We say that $a_2$ is chosen for time $t$ from a probability density function $n(a_2)$.

Without loss of generality, we assume that there is positive probability that the agreement is made in the range $x > \frac{1}{2}$. This implies that there is positive probability that $a_2 > \frac{1}{2}$.

Let $a_{L}$ be the smallest $a_1$ in the support of $m$ (alternatively let $a_{L}$ be the infimum of $m$). The assumption that there is positive probability of an agreement other than $(1,0)$ or $(0,1)$ means that $a_1 = 1 - \epsilon$ for some $\epsilon > 0$.

Because the strategies are in equilibrium, $m$ and $n$ must be best responses to each other. For $n$ to be a best response, each threshold $a_2$ in the support of $n$ has to give agent 2 at least the same payoff as she would get by going to the waiting game. Focusing on those $a_2$ for which $a_2 > \frac{1}{2}$, this means

$$\frac{1}{2} \geq E[a_{2 \text{wait}}] = \int_{t}^{d_2} f_0(d_1) dd_1 = F(d_2) - F(t) = F(d_2)$$

So $\Gamma_2 \leq \alpha(\frac{1}{2})$.

Now, in equilibrium, every strategy in the support of $m$ has to give agent 1 at least the same payoff that she would get by going to the waiting game. There are two cases. First, if $d_1 > \alpha(\frac{1}{2})$, agent 1 knows that she will win the waiting game with probability one, so the split $(x, 1-x)$ could not occur in equilibrium. The second case occurs when $d_1 \leq \alpha(\frac{1}{2})$. Agent 1 can use the fact that $d_2 \leq \alpha(\frac{1}{2})$ to update her beliefs about agent 1's deadline:

$$g_1(d_2) = \begin{cases} 0 & \text{if } d_2 > \alpha(\frac{1}{2}) \\ g_0(d_2) \frac{\int_{t}^{d_2} g_0(d_2) dd_1}{\int_{t}^{d_2} g_0(d_2)} & \text{otherwise} \end{cases}$$

$$g_1(d_2) = g_0(d_2) \left[ 1 + \frac{\int_{t}^{d_2} g_0(d_2) dd_1}{\int_{t}^{d_2} g_0(d_2)} \right]$$

$$\geq 2 \text{ because } \alpha(\frac{1}{2}) \leq \text{median}(g_0)$$

Based on these updated beliefs, the support of $m$ can include $a_{L}$ only if

$$a_1 = 1 - \epsilon \geq \int_{t}^{d_1} g_1(d_2) dd_2 \geq 2[G(d_1) - G(t)] = 2G(d_1)$$

In other words, agent 1's type $d_1$ cannot be too high. Specifically, this can be written as $d_1 \leq \beta(\frac{1}{2})$.

In equilibrium, every strategy in the support of $n$ has to give agent 2 at least the same payoff that she would get by going to the waiting game which equals her subjective probability of winning the waiting game. There are two cases. First, if $d_2 > \beta(\frac{1}{2})$, agent 2 knows that she will win the waiting game with probability one, so the split $(x, 1-x)$ could not occur. The second case occurs when $d_2 \leq \beta(\frac{1}{2})$. Agent 2 can use the fact that $d_2 \leq \beta(\frac{1}{2})$ to update her beliefs about agent 1's deadline:

$$f_1(d_1) = \begin{cases} 0 & \text{if } d_1 > \beta(\frac{1}{2}) \\ f_0(d_1) \frac{\int_{t}^{d_1} f_0(d_1) dd_1}{\int_{t}^{d_1} f_0(d_1)} & \text{otherwise} \end{cases}$$

$$f_1(d_1) = f_0(d_1) \left[ 1 + \frac{\int_{t}^{d_1} f_0(d_1) dd_1}{\int_{t}^{d_1} f_0(d_1)} \right]$$

$$\geq 2 \text{ because } \beta(\frac{1}{2}) \leq \text{median}(f_0)$$

Based on these updated beliefs, the support of $n$ can include $a_{L}$ only if

$$a_2 = 1 - \epsilon \geq \int_{t}^{d_2} f_1(d_2) dd_2 \geq 2[G(d_2) - G(t)] = 2G(d_2)$$

In other words, agent 2's type $d_2$ cannot be too high. Specifically, this can be written as $d_2 \leq \beta(\frac{1}{2})$.

$$g_2(d_2) = \begin{cases} 0 & \text{if } d_2 > \beta(\frac{1}{2}) \\ g_0(d_2) \frac{\int_{t}^{d_2} g_0(d_2) dd_1}{\int_{t}^{d_2} g_0(d_2)} & \text{otherwise} \end{cases}$$

$$g_2(d_2) = g_0(d_2) \left[ 1 + \frac{\int_{t}^{d_2} g_0(d_2) dd_1}{\int_{t}^{d_2} g_0(d_2)} \right]$$

$$\geq 2 \text{ because } \alpha(\frac{1}{2}) \leq \text{median}(g_0)$$

Based on these updated beliefs, the support of $m$ can include $a_{L}$ only if

$$a_1 = 1 - \epsilon \geq \int_{t}^{d_1} g_1(d_2) dd_2 \geq 2[G(d_1) - G(t)] = 2G(d_1)$$

In other words, agent 1's type $d_1$ cannot be too high. Specifically, this can be written as $d_1 \leq \beta(\frac{1}{2})$.

In equilibrium, every strategy in the support of $n$ has to give agent 2 at least the same payoff that she would get by going to the waiting game which equals her subjective probability of winning the waiting game. There are two cases. First, if $d_2 > \beta(\frac{1}{2})$, agent 2 knows that she will win the waiting game with probability one, so the split $(x, 1-x)$ could not occur. The second case occurs when $d_2 \leq \beta(\frac{1}{2})$. Agent 2 can use the fact that $d_2 \leq \beta(\frac{1}{2})$ to update her beliefs about agent 1's deadline:

$$f_1(d_1) = \begin{cases} 0 & \text{if } d_1 > \beta(\frac{1}{2}) \\ f_0(d_1) \frac{\int_{t}^{d_1} f_0(d_1) dd_1}{\int_{t}^{d_1} f_0(d_1)} & \text{otherwise} \end{cases}$$

$$f_1(d_1) = f_0(d_1) \left[ 1 + \frac{\int_{t}^{d_1} f_0(d_1) dd_1}{\int_{t}^{d_1} f_0(d_1)} \right]$$

$$\geq 2 \text{ because } \beta(\frac{1}{2}) \leq \text{median}(f_0)$$

Based on these updated beliefs, the support of $n$ can include $a_{L}$ only if

$$a_2 = 1 - \epsilon \geq \int_{t}^{d_2} f_1(d_2) dd_2 \geq 2[G(d_2) - G(t)] = 2G(d_2)$$

In other words, agent 2's type $d_2$ cannot be too high. Specifically, this can be written as $d_2 \leq \beta(\frac{1}{2})$.

$$g_2(d_2) = \begin{cases} 0 & \text{if } d_2 > \beta(\frac{1}{2}) \\ g_0(d_2) \frac{\int_{t}^{d_2} g_0(d_2) dd_1}{\int_{t}^{d_2} g_0(d_2)} & \text{otherwise} \end{cases}$$

$$g_2(d_2) = g_0(d_2) \left[ 1 + \frac{\int_{t}^{d_2} g_0(d_2) dd_1}{\int_{t}^{d_2} g_0(d_2)} \right]$$

$$\geq 2 \text{ because } \alpha(\frac{1}{2}) \leq \text{median}(g_0)$$

Based on these updated beliefs, the support of $m$ can include $a_{L}$ only if

$$a_1 = 1 - \epsilon \geq \int_{t}^{d_1} g_1(d_2) dd_2 \geq 2[G(d_1) - G(t)] = 2G(d_1)$$

In other words, agent 1's type $d_1$ cannot be too high. Specifically, this can be written as $d_1 \leq \beta(\frac{1}{2})$.

5 Incorporating discounting

Time discounting is a standard way of modeling settings where the value of the good decreases over time e.g. due to inflation or due to perishing. In the previous sections we assumed that agents do not discount time. However, we now show that our results are robust to the case where agents do discount time in addition to having firm deadlines. Let $\delta_1$ be the discount factor of agent 1 and $\delta_2$ be the discount factor of agent 2. The utility of agent $i$ from an agreement where he receives a share $x$ at time $t < d_i$ is then $\delta_i x$. We denote by $\Gamma(a, b, \delta_1, \delta_2)$ the bargaining game where $\delta_1 \delta_2 \delta_1 \delta_2$ are common knowledge. We now prove that our previous result for $\Gamma(a, b)$ holds also for a large range of parameters in $\Gamma(a, b, \delta_1, \delta_2)$. So interestingly, the bargaining power of an agent does not change with her discount factor in contrast to the results of most other bargaining games. In other words, the deadline effect completely suppresses the discounting effect. This crisp result is important in its own right for the design of automated negotiating agents and it also motivates the study of deadline-based models as opposed to focusing only on discounting-based ones.
Proposition 5.1 For any $\delta_1, \delta_2, 0 < \delta_1 \leq 1, 0 < \delta_2 \leq 1$, there exists a sequential equilibrium of $\Gamma(a, b, \delta_1, \delta_2)$ where the agent with the latest deadline receives the whole surplus exactly at the earlier deadline.

Proof. The equilibrium strategies and proof of sequential equilibrium are identical to those in the proof of Proposition 3.1 with the difference that the threshold is no longer $d_1$ but $\delta_1 d_1$. Also the posteriors are now defined only until $\delta_1^{*}$ and not 1.

Theorem 5.1 If $\delta_1, \delta_2 > \frac{1}{2}$, there does not exist a Bayes-Nash equilibrium of $\Gamma(a, b, \delta_1, \delta_2)$, (in pure or mixed strategies) where agents agree to a split other than $(1,0)$ or $(0,1)$.

Proof. We prove the case for pure strategy equilibrium. The extension for mixed strategy equilibrium is identical to that in Theorem 4.1. The proof is a variant of the proof in Theorem 3.1 and we keep the notation from there.

Assume for contradiction $\Gamma$ that there exist types $d_1 > 0$ and $d_2 > 0$ and a pure strategy Bayes-Nash equilibrium $(s_1, s_2)$ where the agents agree to a split of the total surplus available at time $t$ according to proportions $(\tau_1, \tau_2) = (x, 1-x)$ where $x \in (0,1)$ at time $t \geq 0$. We assume w.l.o.g. that agent 1 receives at least one half i.e., $x \geq 1/2$. We can therefore write $x = \frac{1}{2} + \varepsilon$ where $\frac{1}{2} > \varepsilon \geq 0$.

In equilibrium agent 2 will accept $1 - x$ only if she does not expect to receive more by unilaterally moving to the waiting game. The expected payoff from the waiting game is now agent 2's subjective probability of winning. Hence agent 2 would accept only if

$$\delta_2 \left(1 - e^{-\delta_2} \right) \geq \int_0^{d_2} f_0(d_1) d_1 \geq \delta_2 \int_0^{d_2} f_0(d_1) d_1$$

Dividing both sides by $\delta_2$ we get:

$$\frac{1}{2} - \varepsilon \geq \frac{1}{\delta_2} - \int_0^{d_2} f_0(d_1) d_1 = \frac{1}{\delta_2} - \int_0^{d_2} f_0(d_1) d_1 = \frac{1}{\delta_2} - \int_0^{d_2} f_0(d_1) d_1$$

In other words: $d_2 \leq \frac{\delta_2}{\delta_2 - \varepsilon}$ (14)

Now agent 1 can use this to update her beliefs about agent 2's deadline in the same way as in equation (4) $\Gamma$ with the difference that now $g_1 \geq 2d_2g_0$. Since $\delta_2 > 0.5$ (by the assumption that $\delta_1 \delta_2 > 0.5$) we know that $g_1 > g_0$ (when $g_1$ is not zero).

Based on these updated beliefs $\Gamma$, agent 1 would accept only if

$$\delta_1 \left(1 - e^{-\delta_1} \right) \geq \int_0^{d_1} f_0(g_1(d_2) d_1 \geq \delta_1 \int_0^{d_1} f_0(g_1(d_2) d_1$$

Dividing both sides by $\delta_1$ and using the updated beliefs $\Gamma$, we can now rule out "high" types of agent 1. Formally $\Gamma$ does not exist if

$$\delta_1 \left(1 - e^{-\delta_1} \right) \geq \int_0^{d_1} f_0(g_1(d_2) d_1 \geq \delta_1 \int_0^{d_1} f_0(g_1(d_2) d_1$$

Once more belief updating by agent 2 (in the same way as in (6)) yields $f_1 \geq 2d_2 f_0$. Since $\delta_1 \delta_2 > 0.5$ we get that $f_1 > \delta_0$ (when it is not zero). Based on these updated beliefs $\Gamma$, we can rule out the following types: $d_2 \leq \alpha(\frac{1}{\delta_2 - \varepsilon}) \leq \alpha(\frac{1}{\delta_2 - \varepsilon})$.

This process of belief update and acceptance threshold resetting continues to alternate between agents. After $r$ rounds of this alternation $\Gamma$ all types have been eliminated except those that satisfy $d_2 \leq \alpha(\frac{1}{\delta_2 - \varepsilon})$ and $d_2 \leq \alpha(\frac{1}{\delta_2 - \varepsilon})$. This process can continue for an unlimited number of steps $\Gamma$. So for practice the effect of deadlines suppresses that of discount factors. Therefore the equilibrium cannot exist if $d_1 < 0$ or $d_2 < 0$. Contradiction.

Example: If the annual interest rate is 10% the discount factor would be $\delta = \frac{1}{1.1^{0.1}} \approx 0.999$ per year. For the conditions of Theorem 5.1 to be violated $\Gamma$ at least one agent's discount factor would have to be $\delta_1 \leq \frac{1}{2}$. This would mean that the unit of time from which its deadline is drawn would have to be no shorter than 7 years because $\frac{1}{2} \leq \frac{1}{1.1^{0.1}}$. Since most deadline bargaining situations will certainly have shorter deadlines than 7 years $\Gamma$ shows that "sit-and-wait" is the only rational strategy. So in practice the effect of deadlines suppresses that of discount factors. This is even more common true in automated negotiation because that is most likely going to be used mainly for fast negotiation at the operative decision making level instead of strategic long-term negotiation.

6 Robustness to risk attitudes

We now generalize our results to agents that are not necessarily risk neutral. Usually in bargaining games the equilibrium split of the surplus depends on the agents' risk attitudes. However we show that this does not happen in our setting. This is surprising at first since a risk averse agent generally prefers a smaller but safe share to the risky option of the waiting game even if she expects to win it with high probability. However we show that the type-elimination effect described in the theorems so far is still present and dominates any concessions which may be consistent with risk aversion.

Let the agents' risk attitudes be captured by utility functions $u_i$ where $i = 1, 2$. Without loss of generality we let $u_1(0) = 0$ and $u_1(1) = 1$ for both agents.

Proposition 6.1 There exists a sequential equilibrium of $\Gamma(a, b, u_1, u_2)$, where the agent with the latest deadline receives the whole surplus exactly at the earlier deadline.

Proof. The equilibrium strategies and proof of sequential equilibrium are identical to those in the proof of Proposition 3.1 with the difference that the threshold is no longer $d_i$ but $u_i(d_i)$. The following definition is used to state our main result for the case of different risk attitudes.
Definition 6.1 The maximum risk aversion of agent $i$ is
\[ \rho_i \equiv \max_x u_i(x) \]  
(17)

We can now show that our impossibility result applies to a large range of risk attitudes of the agents:

Theorem 6.1 If $\rho_1 \rho_2 < 2$, there does not exist a Bayes-Nash equilibrium (pure or mixed) of $\Gamma(a, b, u_1, u_2)$, where agents agree to a split other than $(1, 0)$ or $(0, 1)$.

Proof. We prove the case for pure strategy equilibrium. The extension to mixed strategies is identical to that in Theorem 4.1. Assume for contradiction that there exist types $d_1 > 0$ and $d_2 > 0$ and a pure strategy Bayes-Nash equilibrium $(\pi_1, \pi_2)$ where the agents agree to a split $(\pi_1, \pi_2) = (x, 1 - x)$ where $x \in (0, 1)$ at time $t \geq 0$. Assume without loss of generality that agent 1 receives at least half i.e. $x \geq \frac{1}{2}$. Thus we can write $x = \frac{1}{2} + c'$. Then the expected payoff from the waiting game is agent 2's subjective probability that $d_2 < d_1$. So agent 2 would accept only if

\[ \rho_2 (\frac{1}{2} - c) \geq u_2 (\frac{1}{2} - c) \geq \int_{d_2}^{d_1} f_0 (d_1) dd_1 = P(d_2) \]  
(18)

In other words $d_2 < \alpha (\rho_2 (\frac{1}{2} - c))$. Now agent 1 can use this to update her beliefs about agent 2's deadline in the same way as in (4) with the difference that now $g_1 \geq \frac{1}{2} g_0$. Since (by assumption) $\rho_2 < \frac{1}{2}$ then $g_1 > g_0$ (when $g_1$ is not zero). Based on these updated beliefs agent 1 would accept only if

\[ u_1 (\frac{1}{2} - c) \geq \int_{d_1}^{d_1} g_1 (d_2) dd_2 \]  
(19)

Using $\rho_1$ and the updated beliefs $g_1, \Gamma$ we can rule out "high" types of agent 1. Formally $u_1 (\frac{1}{2} - c) \geq \int_{d_1}^{d_1} g_1 (d_2) dd_2$.

Once more belief updating by agent 2 (in the same way as in (4)) yields $f_1 \geq \frac{1}{2} f_0$. Since $\rho_1 \rho_2 < 2$ we get $f_1 > f_0$ (when $f_1$ is not zero). Based on these updated beliefs agent 2 would accept only if

\[ u_2 (\frac{1}{2} - c) \geq \int_{d_1}^{d_1} f_1 (d_1) dd_1 \]  
(20)

Using $\rho_2$ and the updated beliefs $f_1, \Gamma$ we can rule out the following types: $d_2 \leq \alpha (\rho_2 \rho_1 (\frac{1}{2} - c)) < \alpha (\rho_2 (\frac{1}{2} - c))$. This process of belief update and acceptance threshold resetting continues to alternate between agents. After $r$ rounds of this alternation and choosing $\alpha$, all types have been eliminated except those that satisfy $d_1 \leq \beta (\rho_1 (\frac{1}{2} - c))$ and $d_2 \leq \alpha (\rho_2 (\frac{1}{2} - c))$. This process can continue for an unlimited number of steps $r$ i.e. the upper bounds approach zero. Therefore the equilibrium cannot exist if $d_1 > 0$ or $d_2 > 0$. Contradiction.

\[ \square \]

7 Designing bargaining agents

Our motivation for studying bargaining with deadlines stems from our desire to construct software agents that will optimally negotiate on behalf of the real-world parties that they represent. That will put experienced and poor human negotiators on an equal footing and save human negotiation effort.

Deadlines are widely advocated and used in automated electronic commerce to capture time preference. For example when a user delegates priceline.com to find an inexpensive airline flight on the web the user gives it one hour to complete (while priceline.com uses an agent with a deadline the setting is a form of auction not bargaining). Users easily understand deadlines and it is simple to specify a deadline to an agent.

Our results show that in distributive bargaining settings with two agents with deadlines it is not rational for either agent to make or accept offers. But what if a rational software agent receives an offer from the other party? This means that the other party is irrational and could perhaps be exploited. However the type-elimination argument from the proofs above applies here too and it is not rational for the software agent to accept the offer no matter how good it is. To exploit the other party the agent would have to have an opponent model to model the other party's irrationality. While game theory allows us to give precise prescriptions for rational play it is mostly silent about irrationality and how to exploit it.

Another classic motivation for automated negotiation is that computerized agents can negotiate faster. However in distributive bargaining settings where the agents have deadlines this argument does not hold because in such settings rational software agents would sit-and-wait until one of the deadlines is reached. From an implementation perspective this suggests the use of daemons that trigger right before the deadline instead of agents that use computation before the deadline.

Finally our results suggest that a user will be in a much stronger bargaining position by inputting time preferences to her agent in terms of a time discount function instead of a deadline even if the discounting is significant. To facilitate this software agent vendors should provide user interfaces to their agents that allow easy human-understandable specification of time discounting functions instead of inputting a deadline.

8 Designing bargaining protocols

The following mechanism implements in dominant strategies the equilibrium of the deadline bargaining game described above. First agents report their deadline $d_i, \Gamma$ to the protocol—possibly insincerely ($d_i \neq d_i$). The protocol then assigns the whole dollar to the agent with the highest $d_i, \Gamma$ but this only takes place at time $t = \delta_i, \Gamma$ i.e. at the earlier reported deadline. Truth-telling is a dominant strategy in this mechanism. By reporting $d_i < d_i$ agent $i$'s probability of winning is reduced. By reporting $d_i > d_i$ agent $i$ increase its proba-
Conclusions

the computational speed of automated agents can significantly enhance negotiation. Additional efficiency can stem from the fact that computational agents can negotiate with large numbers of other agents quickly and virtually with no negotiation overhead. However, this paper showed that in one-to-one negotiation where the optimal deal in the bargaining set has been identified and evaluated and distributing the profits is the issue, an agent's power does not stem from speed but on the contrary from the ability to wait.

We showed that in one-to-one bargaining with deadlines, the only sequential equilibrium is one where the agents wait until the first deadline is reached. This is in line with some human experiments where adding deadlines introduced significant delays in reaching agreement (Roth & Murnighan & Schonmaier 1988). We also showed that deadline effects almost always completely suppress time discounting effects. Impossibility of an interim agreement also applies to most types of risk attitudes of the agents. The results show that for deadline bargaining settings it is trivial to design the optimal agent: it should simply wait until it reaches its deadline or the other party concedes. On the other hand, a user is better off by giving her agent a time discount function instead of a deadline since a deadline puts her agent in a weak bargaining position. Finally, we discussed mechanism design and presented an effective protocol that implements the outcome of the free-form bargaining game in dominant strategy equilibrium.

References


Mimeo.


