# Simple Randomized Algorithms for Tractable Row and Tree Convex Constraints

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### Abstract

We identify tractable classes of constraints based on the following simple property of a constraint: "At every infeasible point, there exist two directions such that with respect to any other feasible point, moving along at least one of these two directions decreases a certain distance metric to it". We show that connected row convex (CRC) constraints, arc-consistent consecutive tree convex (ACCTC) constraints, etc fit this characterization, and are therefore amenable to extremely simple polynomial-time randomized algorithms—the complexities of which are shown to be much less than that of the corresponding (known) deterministic algorithms and the (generic) lower bounds for establishing path-consistency. On a related note, we also provide a simple polynomial-time deterministic algorithm for finding tree embeddings of variable domains (if they exist) for establishing tree convexity in path-consistent networks.

## Introduction

While the task of solving constraint satisfaction problems (CSPs), in general, is NP-hard, much work has been done on identifying tractable subclasses. Broadly, these subclasses have resulted from restrictions imposed on: (1) the *topology* of the associated constraint network (Dechter 1992), (2) the structure of the *constraints* themselves (see (Van Beek and Dechter 1995), (Jeavons *et al* 1998), (Deville *et al* 1999) and (Bulatov *et al* 2000)), or (3) a combination of both (Dechter and Pearl 1991). While the notions of minimum induced-width and hypergraph acyclicity play a key role in characterizing the complexity of solving a given CSP by looking only at the topology of its associated constraint network (Dechter 1992), the notions of row and tree convexity, among others, have been identified in the context of exploiting the structure of the constraints themselves (see (Van Beek and Dechter 1995) and (Zhang and Freuder 2004)). Row and tree convex constraints generalize other types of constraints like monotone and functional constraints (Van Hentenryck *et al* 1992), and together with relational path-consistency, ensure the global consistency of a constraint network.

More specifically, if a binary constraint network is path-consistent, and all of the binary relations can be made row or tree convex by finding suitable domain orderings for the variables, then the network is globally consistent—enabling us to find a solution in a backtrack-free manner. However, neither row convex constraints nor tree convex constraints by themselves constitute tractable languages since further conditions are necessary to ensure that the additional constraints resulting from enforcing path-consistency also remain row or tree convex respectively. Connected row convexity (CRC) is a further restriction on row convexity that ensures the closure over composition, intersection and transposition—the basic operations of path-consistency algorithms—hence making the language of CRC constraints tractable (Deville *et al* 1999). Similarly, tractable classes of tree convex constraints satisfying the properties of local chain convexity and union closure are identified in (Zhang and Freuder 2004). An algebraic theory of tractable constraint languages is developed in (Jeavons *et al* 1998), etc.

In this paper, we identify tractable classes of constraints based on the following simple property of a constraint: "At every infeasible point, there exist two directions such that with respect to any other feasible point, moving along at least one of these two directions decreases a certain distance metric to it". We show that CRC constraints, arc-consistent consecutive tree convex (ACCTC) constraints, etc fit this characterization, and are therefore amenable to extremely simple polynomial-time randomized algorithms—the complexities of which are shown to be much less than that of the corresponding (known) deterministic algorithms and the (generic) lower bounds for establishing path-consistency. Further, to the best of our knowledge, ACCTC constraints were not previously identified to be tractable. Finally, on a related note, we also provide a simple polynomial-time deterministic algorithm for finding tree embeddings of variable domains (if they exist) for establishing tree convexity in path-consistent networks.

## Preliminaries and Definitions

A CSP is defined by a triplet $\langle \mathcal{X}, \mathcal{D}, \mathcal{C} \rangle$, where $\mathcal{X} = \{X_1, X_2 \dots X_N\}$ is a set of *variables*, and $C = \{C_1, C_2 \dots C_M\}$ is a set of *constraints* between subsets of them. Each variable $X_i$ is associated with a discrete-valued *domain* $D_i \in \mathcal{D}$, and each constraint $C_i$ is a pair $\langle S_i, R_i \rangle$ defined on a subset of variables $S_i \subseteq \mathcal{X}$, called the *scope* of $C_i$. $R_i \subseteq D_{S_i}$ ($D_{S_i} = \times_{X_j \in S_i} D_j$) denotes all compatible tuples of $D_{S_i}$ allowed by the constraint. A *solution* to a