Towards Synthesizing Optimal Coordination Modules for Distributed Agents

Manh Tung Pham and Kiam Tian Seow
Division of Computing Systems
School of Computer Engineering
Nanyang Technological University
Republic of Singapore 639798
{pham0028,asktseow}@ntu.edu.sg

Abstract

In a discrete-event framework, we define the concept of a coordinate language and show that it is the necessary and sufficient existence condition of coordination modules for distributed agents to achieve conformance to a given agent constraint language. We also present a synthesis algorithm to compute near optimal coordination modules.

Background & Notation

We use the following notations from language and automata theory (Cassandras and Lafortune 1999):

- For an event set \( \Sigma \), \( \Sigma^* \) denotes the set of all finite strings over \( \Sigma \), including the empty string \( \varepsilon \); and for two event sets \( \Sigma^1 \subseteq \Sigma^2 \), \( P_{\Sigma^2, \Sigma^1} \) denotes the natural projection from \( (\Sigma^2)^* \) to \( (\Sigma^1)^* \), erasing all \( \sigma \in \Sigma^2 - \Sigma^1 \) in \( s \in (\Sigma^2)^* \).
- For a language \( L \) over an event set \( \Sigma \), i.e., \( L \subseteq \Sigma^* \), \( \bar{L} \) denotes the set of all prefixes of its strings. \( L \) is said to be prefix-closed if \( \bar{L} = L \).
- For an automaton \( A = (X^A, \Sigma^A, \delta^A, x_0^A, X_m^A) \), \( L(A) \) and \( L_m(A) \) denote its generated prefix-closed and marked languages, respectively; and if \( x \in X^A \), \( s \in (\Sigma^A)^* \), \( \delta^A(s, x) \) denotes that \( \delta^A(s, x) \) is defined; \( Trim(A) \) denotes the automaton which computes and returns a nonblocking automaton which generates the same marked language as \( A \); and \( A = A_1 \parallel A_2 \) denotes that automaton \( A \) is the synchronous product of the two automata \( A_1 \) and \( A_2 \).
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Consider a system modeled by an automaton \( A \), with the event set \( \Sigma^A \) partitioned into (i) \( \Sigma^A = \Sigma^c \cup \Sigma^uc \) and (ii) \( \Sigma^A = \Sigma^c \cup \Sigma^uc \cup \Sigma^uc \), where \( \Sigma^c \), \( \Sigma^uc \), \( \Sigma^a \) and \( \Sigma^ia \) denote the sets of controllable, uncontrollable, observable and unobservable events of \( A \), respectively. Let \( K \) be a sublanguage of \( L_m(A) \), i.e., \( K \subseteq L_m(A) \). The statements ‘\( K \) is controllable w.r.t \( A \), \( \Sigma^c \) and \( \Sigma^a \)’ refer, respectively, to the concepts of controllability (Ramadge and Wonham 1987) and observability (Lin and Wonham 1988) of a language \( K \) in supervisory control theory. Finally, for an automaton \( C \), the \( Supcon(C, A, \Sigma^A) \) procedure (Wonham and Ramadge 1987) computes a nonblocking automaton \( S \) such that \( L_m(S) \) is the supremal controllable sublanguage (Ramadge and Wonham 1987) of \( L_m(C) \cap L_m(A) \) w.r.t \( A \) and \( \Sigma^A \).

Discrete-Event Agents & Coordination

Consider a system of two agents modeled by the respective automata \( A_1 = (X^{A_1}, \Sigma^{A_1}, \delta^{A_1}, x^{A_1}_0, X^{A_1}_m) \) \( i \in \{1, 2\} \), where \( \Sigma^{A_1} \cap \Sigma^{A_2} = \emptyset \). The event set \( \Sigma^{A_i} \) of agent \( A_i \) is partitioned into the controllable set \( \Sigma^{A_i c} \) and the uncontrollable set \( \Sigma^{A_i uc} \). In enabling distributed agents to coordinate, each agent \( A_i \) is equipped with a coordination module (CM) modeled by an automaton \( S_i \) with the following properties:

1. \( \Sigma^{S_i} = \Sigma^{A_i} \cup ComSet(S_i, A_j) \), where \( ComSet(S_i, A_j) \subseteq \Sigma^{A_j} \) \( (i, j \in \{1, 2\}, i \neq j) \). \( \Sigma^{S_i} \) is called the coordination event set for agent \( A_i \), and \( ComSet(S_i, A_j) \) is the set of events that agent \( A_j \) needs to communicate to \( A_i \) to synchronize \( S_i \).
2. \( S_i \) is \( \Sigma^{A_i uc} \)-enabling, namely, \( (\exists s \in (\Sigma^{S_i})^*)(\forall \sigma \in \Sigma^{A_i}) \left((s \in L(S_i \parallel A_i) \text{ and } P_{\Sigma^{S_i}, \Sigma^{A_i}}(s, \sigma) \in L(A_j)) \Rightarrow (s \sigma \in L(S_i \parallel A_i))\right) \).
3. \( S_i \) and \( S_j \) are cooperative, namely, \( (\forall s \in (\Sigma^{A_i})^*)(\forall \sigma \in \Sigma^{A_j}) \left((s \in L(S_i) \text{ and } P_{\Sigma^{S_i}, \Sigma^{A_i}}(s, \sigma) \in L(A_j)) \Rightarrow (P_{\Sigma^{A_i}, \Sigma^{A_j}}(s, \sigma) \in L(S_j))\right) \).

Let \( A = A_1 \parallel A_2 \) and \( S_{12} = (S_1, S_2) \) denote the CM pair of \( S_1 \) and \( S_2 \). Write \( S_{12} / A \) for the system of two agents \( A_1 \) and \( A_2 \) coordinating through their respective CM’s.

Definition 1. Coordinated Behaviors

1. Prefix-closed coordinated behavior \( L(S_{12}/A) \)
   
   (a) \( c \in L(S_{12}/A) \).
   
   (b) \( (\forall s \in L(S_{12}/A)) (\forall \sigma \in \Sigma^{A_2}) (s \sigma \in L(S_{12}/A) ⇔ (s \in L(A) \text{ and } P_{\Sigma^{A_2}, \Sigma^{S_1}}(s, \sigma) \in L(S_j))) \).

2. Marked coordinated behavior \( L_m(S_{12}/A) \)

   \( L_m(S_{12}/A) = L(S_{12}/A) \cap L_m(A) \cap L_m(S_1) \cap L_m(S_2) \).

CM pair \( S_{12} \) is nonblocking if \( L_m(S_{12}/A) = L(S_{12}/A) \).

Definition 2. Coordination Language: Let \( \Sigma^{com} \subseteq \Sigma^A \). A language \( K \subseteq L_m(A) \) is coordinate w.r.t \( A \) and \( \Sigma^{com} \) if

1. \( K \) is controllable w.r.t \( A \) and \( \Sigma^c = \Sigma^{A c} \cup \Sigma^{A uc} \); and
2. \( K \) is observable w.r.t \( A \) and \( P_{\Sigma^c, \Sigma^{A c} \cup \Sigma^{com}}(s) \in (i \in \{1, 2\}) \).
Theorem 1. Let $\emptyset \neq K \subseteq L_m(A)$ and $\Sigma_{com} \subseteq \Sigma^A$. Then, there exists a nonblocking CM pair $S_{12} = (S_1, S_2)$, with CM $S_i$ for $A_i$, such that $L_m(S_{12}/A) = K$ and $\Sigma_{com} = ComSet(S_1, A_2) \cup ComSet(S_2, A_1)$, if and only if $K$ is coordinable w.r.t $A$ and $\Sigma_{com}$.

Problem Statement and Solution Properties

Problem. Multiagent Coordination Problem (MCP): Given an inter-agent constraint automaton $C$ over $\Sigma^A$, construct a nonblocking CM pair $S_{12} = (S_1, S_2)$ such that $L_m(S_{12}/A) \subseteq L_m(A) \cap L_m(C)$.

When solving MCP, it is desirable to synthesize optimal CM’s, i.e., CM’s with the following properties: 1) Minimal Intervention - the coordination does not unnecessarily displace controllable events; 2) Minimal Communication - the number of events to be communicated between the agents is minimal; and 3) Efficient Implementation - each CM is of minimal state size (among all CM’s satisfying the first two properties).

Let $S = Supcon(C, A, \Sigma^C)$, where $\Sigma^C = \Sigma^A_1 \cup \Sigma^A_2$. Then minimal intervention can be guaranteed by synthesizing CM’s $S_1$ and $S_2$ such that $L_m(S_{12}/A) = L_m(S)$. Procedure $CM$ below computes CM’s $S_1$ given $S$, system event set $\Sigma^A$, and event set $\Sigma^{CM}_1 \subseteq \Sigma^A$ for $\Sigma^{CM}_1 = \Sigma^{CM}_2$. By the constructive proof of Theorem 1 presented elsewhere, it can be shown that if $L_m(S)$ is coordinable w.r.t $A$ and $\Sigma^{CM}_1 \cup \Sigma^{CM}_2$ and $S_i = CM(S, A, \Sigma^{CM}_i)$ ($i \in \{1, 2\}$), the CM pair $(S_1, S_2)$ is nonblocking and $L_m(S_{12}/A) = L_m(S)$.

Algorithm: Coordination Module Synthesis

Input: Agents $A_1$, $A_2$ and constraint $C$ where $\Sigma^A_1 = \Sigma^A_2 = \emptyset$

Output: A near optimal nonblocking CM pair $S_{12} = (S_1, S_2)$ such that $L_m(S_{12}/A) \subseteq L_m(A) \cap L_m(C)$

begin
1. Compute automaton $S'_i = (\Sigma^{CM}_i, X_p, \delta^{S'_i}, x_0^{S'_i}, X_m^{S'_i})$:
   - $x_0^{S'_i} \in X_p$ with $\pi(x_0^{S'_i}) = \{\delta^S(s, x_0^S) | P_{\Sigma^A, \Sigma^{CM}_i}(s) = \epsilon\}$;
   - $X_m^{S'_i} = \{x_p \in X_p | (\exists s \in L_m(S)) \delta^S(s, x_0^S) \in \pi(x_p)\}$;
   - $\forall \sigma \in \Sigma^{CM}_i \exists x_p \in X_p \text{ (if only if}) \exists s \in L_m(S) \delta^S(s, x_0^S) \in \pi(x_p)$;
2. When defined, $\delta^{S'_i}(\sigma, x_p)$ with $x'_p = \{\delta^{S'_i}(s', x) | x \in \pi(x_p), P_{\Sigma^A, \Sigma^{CM}_i}(s') = \sigma\}$;
3. Return $S_i = Trim(S'_i)$;
end

Conclusion

The contributions of this paper to discrete-event multiagent coordination include the existence condition (Theorem 1) of CM’s and a synthesis algorithm for near optimal CM design.

References


