Decision Making in Qualitative Influence Diagrams

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Abstract

The increasing number of knowledge-based systems that build on a *Bayesian belief network* or *influence diagram* acknowledge the usefulness of these frameworks for addressing complex real-life problems. The usually large number of probabilities and utilities required for their application, however, is often considered a major obstacle. The use of *qualitative abstractions* may to some extent remove this obstacle. *Qualitative belief networks* and associated algorithms have been developed before. In this paper, we address *qualitative influence diagrams* and outline an efficient algorithm for qualitative decision making.

Introduction

In the late 1980s, the framework of *Bayesian belief networks* was introduced for *reasoning with uncertainty* (Pearl 1988). The framework provides a formalism for encoding a joint probability distribution on a set of statistical variables and offers algorithms for probabilistic inference. In practice, reasoning with uncertainty is often performed to support a decision maker in solving complex real-life problems. The belief-network framework in itself does not provide for *decision making under uncertainty*, as decision making involves not only knowledge of the uncertainties in a problem under study, but also knowledge of the decisions that are at a decision maker's disposal and of the desirability of their uncertain consequences. The framework of *influence diagrams* is tailored to decision making (Howard and Matheson 1981). It provides a formalism for capturing the various types of knowledge involved in a decision problem and offers algorithms for computing preferred decisions. The framework is closely related to the belief-network framework; in fact, influence diagrams may be looked upon as enhanced belief networks.

The belief-network and influence-diagram frameworks have demonstrated their practicability in a wide range of problem domains. Experience shows, however, that the usually large number of probabilities and utilities required poses a major obstacle to their application (Druzdzel and Van der Gaag 1995). Motivated by this experience, the framework of *qualitative belief networks* was introduced in the early 1990s by M.P. Wellman (1990). A qualitative belief network abstracts from numerical probabilities by encoding *qualitative probabilistic relationships* among its variables. For reasoning with a qualitative belief network, an elegant algorithm is available from M.J. Druzdzel and M. Henrion (1993). As belief networks may be extended to influence diagrams, qualitative belief networks may be enhanced to *qualitative influence diagrams* (Wellman 1990). A qualitative influence diagram abstracts from the numerical quantities involved in a decision problem under study by encoding *qualitative probabilistic* and *preferential relationships* among its variables.

Since their introduction, research has focused mainly on qualitative belief networks, with less attention for qualitative influence diagrams. As we consider decision making a valuable addition to reasoning with uncertainty, we re-introduce qualitative influence diagrams and outline a new algorithm for efficient *qualitative decision making*, that builds on Druzdzel and Henrion's algorithm for qualitative reasoning with uncertainty.

The paper is organised as follows. In Section 2 we review the belief-network and influence-diagram frameworks. In Section 3 qualitative belief networks are presented. In Section 4 we introduce qualitative influence diagrams; in addition, we outline our algorithm for qualitative decision making. In Section 5 we give some conclusions and directions for further research.

Belief networks and influence diagrams

The framework of *Bayesian belief networks* for reasoning with uncertainty is rooted in probability theory (Pearl 1988). It offers a formalism for encoding a joint probability distribution on a set of statistical variables, in which information about independences is explicitly separated from numerical quantities.

A *belief network* consists of a qualitative part and an associated quantitative part. The qualitative part is a graphical representation of the independences holding among the variables in the encoded probability distribution. It takes the form of an *acyclic directed graph* G. Each node A in G represents a statistical variable that takes one of a finite set of values. We assume all

procedure Preferred-Decisions(*from*,*message*):

  Propagate-Sign$_{influence}$(*from*,*from*,*message*);
  Propagate-Sign$_{utility}$(V,V,'+');
  for each decision node D
  do if sign[*utility*,D] = '?' and $\alpha(D)$ causes the ambiguity
    then sign[*utility*,D] $\leftarrow \oplus_i$ (sign[*influence*,A_i] $\otimes \delta_i$),
      where $A_i \in \alpha(D)$ and δ_i is determined
      from $Y_U^{\delta_i}(\{D, A_i\})$

So, if a '+' reaches D, the preferred decision is d; if a '−' reaches it, $\bar{d}$ is the preferred decision. If D receives a '0', then both decision alternatives are equally preferred. If D, however, receives an ambiguous sign, the preferred decision cannot be determined from the influence on utility of the node by itself. In fact, the ambiguity may indicate that the represented decision problem involves a true *trade-off*. By exploiting the signs of influence of the nodes that model the trade-off and their additive synergies on utility with node D, the ambiguity may be resolved; we illustrate the basic idea by means of our running example. Further details of our algorithm and a formal proof of its correctness will be provided in a forthcoming technical paper.

Example 5 Consider once more the qualitative *Sore Throat* influence diagram from Figure 4. Suppose that, after having observed a sore throat, we observe tonsillitis in a child. To reflect the new observation, a '+' is entered for node T. T updates its own sign to '+' and sends a '−' to nodes R and V; node R subsequently updates its sign of influence to '−'. Our algorithm now proceeds by sending a '+' from the value node V to the decision node E. Because of its ambiguous qualitative influence on utility, E receives a '?' and the preferred decision cannot yet be determined. From $U^?(E)$, we conclude, however, that either

$$u(te) > u(t\bar{e}) \text{ and } u(\bar{t}e) < u(\bar{t}\bar{e}), \text{ or}$$
$$u(te) < u(t\bar{e}) \text{ and } u(\bar{t}e) > u(\bar{t}\bar{e})$$

must hold. The first set of inequalities would correspond with a positive additive synergy on utility of nodes E and T, as it induces

$$u(te) + u(\bar{t}\bar{e}) \geq u(t\bar{e}) + u(\bar{t}e)$$

The second set of inequalities would correspond with a negative additive synergy on utility. Since the diagram specifies a positive additive synergy on utility of T and E, we know that the first set of inequalities holds. The preferred decision can now be determined: from the synergy, we have that in case of a positive sign of influence for T, the preferred decision is e, and in the case of a negative sign, the decision $\bar{e}$ is preferred. Since tonsillitis has actually been observed in the child under consideration, the algorithm yields the decision to perform a tonsillectomy as the preferred decision. □

Conclusions and further research

Qualitative abstractions of belief networks and influence diagrams have been introduced to remove the obstacle of acquiring a large number of probabilities and utilities. Research so far has focused mainly on qualitative belief networks. Since we consider decision making a valuable addition to reasoning with uncertainty, we have re-introduced the framework of *qualitative influence diagrams*. We have proposed a new algorithm for qualitative decision making under uncertainty, that builds on a similar algorithm for qualitative probabilistic reasoning. In developing our algorithm, we have assumed that a qualitative influence diagram under study includes binary variables only. Our algorithm is readily extended, however, to apply to more general diagrams.

One of the major drawbacks of qualitative abstractions is their coarse level of detail. Although for some problem domains this level will suffice, there are decision problems for which a finer level of detail is required. We would like to test our algorithm for qualitative decision making on various real-life applications to gain insight as to the level of detail generally required.

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