A Bayesian Approach to Operational Decisions in Transportation Businesses

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Abstract
The transportation business sector, along with many others, has not, to a large extent, evaluated the possibility of gaining advantages through the use of more elaborated decision tools for operational decision making. This article presents a case study where an interval based decision tool was used for transportation business applications. The investigation was made in collaboration with a well-established shipping company as well as a well-established haulage contractor. The tool used in this study utilizes a generalization of the principle of maximizing the expected utility, and is particularly suitable when a decision maker does not possess precise information of future scenarios and expected outcomes.

Introduction
By operational decisions, we usually mean decisions made by personnel in the operating core of the company. Typically, such decisions have to be made rapidly. One common feature of decisions of this kind, as well as many others, is the lack of precise background information.

In the case study presented herein, it is demonstrated how operational decisions in the transportation sector can be handled, even when the background information is numerically imprecise. For this purpose, an implementation of the decision theory of (Danielson and Ekenberg, 2001) was used. This theory is a suitable approach for the particular problem herein, when both imprecise probability and value statements are involved. Other candidates could have been considered as well. However, with very few exceptions, these are primarily focused on probability estimates, and to a less extent with imprecise value statements, and evaluations of these with respect to decision rules. Furthermore, there is usually an artificial distinction between the modeling of quantitative and qualitative probabilities, cf. (Ekenberg, 2001), (Ekenberg, et al., 2001).

The next section provides a background to decision making in the transportation sector. Section 3 gives an overview of a decision theory. Section 4 presents the case study and the use of an interval decision tool and, finally, some conclusions are provided.

Transportation
In general, shipping companies are often referred to as intermediaries. Some of these are specialized in organizing formalities for international transports, and others in organizing routes within a nation. The carriers, e.g., haulage contractors and railway companies, are responsible for the physical transportation, and the shipping company handles the required administration.

The most common type of national shipping companies are the consolidators. These purchase a whole capacity from one or a number of carriers at a particular line. This capacity is thereafter sold to companies in need of shipping small parties of goods. The price per weight-unit for the service then decreases when more goods are sent. The difference between the price paid to the carrier and the price customers are willing to pay are therefore of high concern for the consolidation shipping companies.

Senders of goods usually outsource the transportation to external companies, i.e., there is a form of third party logistics involved. The companies, offering transportation services, like haulage contractors, are in this case marketing the third component in such a system for third party logistics. When a customer in the value chain is buying transportation services from more than one supplier, the customer makes the choice of supplier based on offered services and their respective prices. For the customer, a better service means a possibility for a decrease of inventory as well as manageable time schedules (Ballou, 1999, p 188).

For instance, the shipping company in the case study operates between the purchaser of transport services, the carriers and the authorities. Furthermore, the company is responsible for the entire transportation even if the goods are transported by a number of different carriers. The identified decisions, made by the personnel responsible for the operational information flow, concern price-setting and delegation of goods to the carriers as well as selecting the best solution of problems with overload and underload. In the second and third case, the applicability of probability theory is limited, which makes a bayesian viewpoint less interesting. However, in the first case, it is important to find the optimal (or at least a reasonable) discount. This is mainly because the customers’ statements of future transportation frequencies often are unreliable. Consequently, these estimates are necessarily vague and classical decision theory, based on precise data, is
not entirely appropriate for many such decisions.

A Decision Tool

Interval approaches model decision situations, where numerically imprecise statements occur. In (Danielsson and Ekenberg, 1998), imprecise probabilities and utilities are represented by numerical intervals. Interval sentences are of the form: “The probability of $c_{ij}$ lies between the numbers $a_k$ and $b_k$" and are translated into $p_{ij} \in [a_k, b_k]$. Comparative sentences are of the form: “The probability of $c_{ij}$ is greater than the probability of $c_{ij'}$”. Such a sentence is translated into an inequality $p_{ij} \geq p_{ij'}$. The conjunction of constraints of the types above, together with $\sum_j p_{ij} = 1$ for each alternative $A_j$ involved, is called the probability base ($P$). The value base ($V$) consists of similar translations of vague and numerically imprecise value estimates. The collection of probability and value statements constitutes the information frame. In a frame, an alternative $A_j$ is represented by its consequence set $C_i = \{c_{i1}, \ldots, c_{ih_i}\}$.

Relative Strength

The alternatives can be evaluated according to various decision rules. Herein, we restrict the analysis to the relative strength of the alternatives.

**Definition:** Given an information frame $<C_1, \ldots, C_m, P, V>$ the strength of $C_i$ compared to $C_j$ is $\frac{1}{2} \max(\delta_{ij})$, where $\delta_{ij} = E(C_i) - E(C_j)$ and $E(C_i)$ denotes the expected value of $C_i$.

To analyze the strength of the alternatives, $\max(\delta_{ij})$ is calculated. This means that we choose the feasible solutions to the constraints in $P$ and $V$ that are most favorable to $E(C_i)$ and least favorable to $E(C_j)$.

The strength might sometimes be a too rough measure and often an investigation of the relative strength of the alternatives can provide more balanced information.

**Definition:** Given an information frame $<C_1, \ldots, C_m, P, V>$ the relative strength $\Delta_{ij}$ of $C_i$ compared to $C_j$ is $(\max(\delta_{ij}) - \max(\delta_{ij'})/2$, where $\delta_{ij} = E(C_i) - E(C_j)$.

For the pair wise evaluation of alternatives, studying their relative strengths, (Danielsson and Ekenberg, 1998) suggests various algorithms.

Contractions

A problem with evaluating interval statements is that the results usually overlaps, i.e., an alternative might not be dominating for all instances of the feasible values in the probability and value bases. A suggested solution to this is to further investigate in which regions of the bases the respective alternatives are dominating. For this purpose, contractions are introduced in the framework. Contractions can be seen as generalized sensitivity analyses to be carried out to determine the stability of the relation between the consequence sets under consideration. A natural way to investigate this is to consider values near the boundaries of the intervals as being less reliable than more central values. The idea behind this is therefore to investigate how much the different intervals can be decreased before an expression such as $\Delta_{ij} > 0$ ceases to be consistent.

**Definition:** Let $X$ be a probability or utility base with the variables $x_1, \ldots, x_n$. Let $p \in [0, 1]$ be a real number and let $\{p_i \in [0, 1]: i = 1, \ldots, n\}$ be a set of real numbers. Assume that $[a_i, b_i]$ denotes the interval corresponding to the variable $x_i$ in the solution set of the base, and that $\bar{k} = (k_1, \ldots, k_n)$ is a consistent point in $X$. A $p$-contraction of $X$ is to add the interval statements $\{x_i \in [a_i + p \cdot p_k \cdot (k_i - a_i), b_i - p \cdot p_k \cdot (b_i - k_i)] : i = 1, \ldots, n\}$ to the base $X$. $\bar{k}$ is called the contraction point.

The contraction avoids the complexity in combinatorial analyses, but still offers possibilities to study the stability of a result, by providing a mean for investigating the importance of interval boundary points. By co-varying the contractions of an arbitrary set of intervals, the influence of the structure of the information frame on the solutions can be investigated. Contrary to volume estimates, contractions are not measures of the sizes of the solution sets but rather of the strength of statements when the original solution sets are modified in controlled ways. According to the definition above, both the set of intervals under investigation and the scale of individual contractions can be controlled.

As a special case, for a 100 % contraction, the volume of each base is reduced to the contraction point. The results from the algorithms for comparing alternatives then coincide with the ordinary expected value for this point.

Security levels

In some cases, the classical utility theory cannot sufficiently handle different risk behaviors. For instance, in many situations, there is a need for excluding particularly high-risk alternatives, e.g., when there is too great a probability that an alternative will result in unacceptable outcomes. It is therefore often important to investigate when an alternative exposes a decision maker of a risk that cannot be afforded, even when the expected utility is reasonably high. The use of Security levels in this sense is suggested in (Ekenberg 2001).

The JML Application

The JML Application is a decision software developed according to the theory above. The software calculates (among other things) the strength and the relative strength between two consequence sets. It also uses the concept of contraction for refinements of the analyses.

Business Decisions

Customer Negotiations and Contract Formulation

An offer is based on a gross tariff with high prices, and the result of the negotiations with a customer is usually an agreement on given discounts in a net tariff. The sizes of these discounts are based on assumptions of the expected customer loyalty. Usually, a contract ranges over one year and the prices in the net tariff might be as much as 50 percent
lower compared to the gross tariff. The offered price might be modified, but if this is too low, it will not be profitable for the haulage contractor to accept it. A further complicating factor is the uncertainty about the customer performance according to the given discount, i.e., the customer might not pay for the service as often as it initially was stated.

The background information accessible in such a scenario is usually the following:

- The type of goods to be transported. This is of importance for determining whether to base the price on weight or size.
- The customer’s forecasted transportation frequency during the validity of the contract.
- The destination, time and mode of transportation of the goods.
- The quality of the bought service, e.g., punctual delivery.

Convincing the customer to sign a contract promising loyalty to the shipping company is of high concern, i.e., the customer is not supposed to hire another shipping company for this kind of transports. However, there are several alternatives regarding the size of the offered discount.

Simulation with the JML Application

The potential customer provides the following information:

- The goods consist of car parts.
- The goods weigh 400 kg and fits nicely on two palls.
- The goods should be transported between the Swedish cities Sundsvall and Västerås.

Furthermore, this is the customers first contact with the shipping company.

There are at least three feasible alternatives:

- **A1**: The offered price has a discount of 0 – 10% off the gross price.
- **A2**: The offered price has a discount of 10 – 30% off the gross price.
- **A3**: The offered price has a discount of 30 – 45% off the gross price.

Given these alternatives, we have the following consequence sets. The interval $[a_k, b_k]$ represents how many times the customer will pay for the service if $A_k$ is chosen.

A transportation frequency is being approximated from the customer’s information, which in our example will serve as the utility interval. $x$ monetary units is the profit of the shipping company, when the customer is paying the gross price. If a contract is signed by both parties, the total price is the number of paid transports multiplied with the paid price.\(^1\)

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\(^1\)We assume that the different utilities are proportional to earned money units for the shipping company.

### Alternative A1

$c_{11}$: The offer is not accepted, ($v_{11} = 0$).

c_{12}$: The offer is accepted but no long term agreement is accomplished, ($v_{12} = [0.9x, x]$).

c_{13}$: The offer is accepted and an agreement over one year is accomplished, ($v_{13} = x \cdot n_1$, where $n_1 \in [a_1, b_1]$ and $a_1, b_1 \in \mathbb{Z}^+$. The utility of this consequence depends on how many times the customer will pay for the service.

### Alternative A2

$c_{21}$: The offer is not accepted, ($v_{21} = 0$).

c_{22}$: The offer is accepted, but no long term agreement is accomplished, ($v_{22} = [0.7x, 0.9x]$).

c_{23}$: The offer is accepted and an agreement over one year is accomplished, ($v_{23} = y_2 \cdot n_2$, where $y_2$ is the money earned by the shipping company for each transport during the agreed period). Because of the discount, $y_2 \leq x$. Furthermore, $n_2 \in [a_2, b_2]$ and $a_2, b_2 \in \mathbb{Z}^+$.

### Alternative A3

$c_{31}$: The offer is not accepted, ($v_{31} = 0$).

c_{32}$: The offer is accepted but no long term agreement is accomplished, ($v_{32} = [0.55x, 0.7x]$).

c_{33}$: The offer is accepted and an agreement over one year is accomplished, ($v_{33} = y_3 \cdot n_3$, where $y_3$ is the money earned by the shipping company for each transport during the agreed period). We know that $y_3 \leq y_2$. Furthermore, $n_3 \in [a_3, b_3]$ and $a_3, b_3 \in \mathbb{Z}^+$.

The decision problem can be modeled by the matrix below, where $s_i$ and $c_{ij}$ represent the possible states and consequences, respectively.

<table>
<thead>
<tr>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$c_{11} = 0$</td>
<td>$c_{12}, v_{12} = [0.9x, x]$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$c_{21}, v_{21} = 0$</td>
<td>$c_{22}, v_{22} = [0.7x, 0.9x]$</td>
</tr>
<tr>
<td>$A_3$</td>
<td>$c_{31}, v_{31} = 0$</td>
<td>$c_{32}, v_{32} = [0.55x, 0.7x]$</td>
</tr>
</tbody>
</table>

### Probability and Utility Estimates

#### Probabilities

According to the information available to the shipping company, the following estimates are used.

- $p_{11} \in [0.20, 0.40]$ – with such a high price, there is quite a low probability for a signed one year contract.
- $p_{12} \in [0.40, 0.70]$ – it is reasonably probable that the customer accepts the price for one transport, in particular if he is in a hurry.
- $p_{13} \leq 0.05$ – there is a very low probability of a signed contract with this high price.

- $p_{21} \in [0.05, 0.15]$ – the price is reduced, but not dramatically.
- $p_{22} \in [0.15, 0.40]$ – it is quite improbable that the customer makes an agreement for one transport only.
- $p_{23} \in [0.40, 0.70]$ – there is a reasonable chance that the customer will sign a one year contract and promise to be loyal.
\( p_{31} \leq 0.01 \) – with such a low price it is highly improbable that the customer will choose a competitor instead.

\( p_{32} \in [0.10, 0.30] \) – it is quite unlikely that the customer chooses to buy a single transport, but not sign a one year contract.

\( p_{33} \in [0.60, 0.80] \) – it is very probable that the customer will sign a one year contract and promise to be loyal.

**Utilities**

The gross price for one transport is SEK 2500, whereof 15\% results in \( x = \text{SEK 375} \). Furthermore, the higher the discount is, the more transports are bought. Because of this, \( \alpha_3 \geq \alpha_2 \geq \alpha_1 \) and \( b_3 \geq b_2 \geq b_1 \). The following values are then easily calculated.

- \( v_{11} = 0 \) – no agreement accomplished.
- \( v_{12} \in [338, 375] \).
- \( v_{13} \in [16875, 37500] \) – this interval is based on a discount of 0–10\% and a transportation frequency of 50–100 shippings during the year.

- \( v_{21} = 0 \) – no agreement accomplished.
- \( v_{22} \in [250, 338] \).
- \( v_{23} \in [13125, 50025] \) – this interval is based on a discount of 10–30\% and a transportation frequency of 50–150 shippings during the year.

- \( v_{31} = 0 \) – no agreement accomplished.
- \( v_{32} \in [206, 250] \).
- \( v_{33} \in [20625, 52500] \) – this interval is based on a discount of 10–30\% and a transportation frequency of 100–200 shippings during the year.

![Figure 1: The decision problem illustrated as a decision tree in the JML Application.](image1)

The expected utilities of the alternatives can now be calculated. It follows that \( A_3 \) is the best alternative. Alternative \( A_1 \) is, by far, the worst alternative.

The shrinking area in the figure depicts the value under different degrees of contraction. The plotted nearly horizontal line represents the relative strength, \( \Delta_2 \), over all contraction points. As can be seen, \( \Delta_2 \) is below zero, independent of the degree of contraction. Furthermore, after about 70\% contraction, there is no possibility that \( a_2 \) would be better than \( a_3 \). This strongly indicates that \( a_3 \) should be a better alternative than \( a_2 \).

However, we might be considerably less optimistic concerning the transportation frequency approximated by the customer, e.g., let \( b_2 = b_3 = 150 \) and \( a_3 = 75 \). This means that \( v_{33} \in [15409, 39375] \). We also raise the probability of \( o_{23} \) to the same level as \( c_{23} \) (i.e., we assume that the customer is not particularly price sensitive). Nevertheless, it still seems unreasonable to assume that \( p_{32} \) is greater than \( p_{33} \), because \( c_{23} \) is still a better offer for the customer. Therefore, the probability base is restricted by \( p_{33} > p_{32} \).

![Figure 2: The figure shows the resulting graph of the pair wise comparison of \( A_2 \) and \( A_3 \).](image2)

The figure shows the same decision problem with modified values according to the above.

![Figure 3: The figure shows the same decision problem with modified values according to the above.](image3)
As can be seen from the figure, $\Delta_{23}$ is now positive (constantly above zero) and consequently, $A_2$ is a marginally better alternative. By modifying the parameters and studying the contractions we can find the most important factors in the decision. In this case, considerable modifications of the values were necessary for changing the preference ordering $A_3 \succ A_2 \succ A_1$.

Consequently, the preference ordering over the alternatives is very stable to changes.

Security levels
Furthermore, an outcome being less than SEK 20 000 is considered to be very low. Therefore, we also investigate whether there is a probability greater than 0.5 for an outcome being less than this amount, considering the contraction levels 0%, 20%, 40%, 60%, 80% and 100%. The result of this investigation is shown in Figure 4.

![Figure 4: A security analysis.](image)

$A_1$ violates the security levels for all contractions. This is represented by dark gray in the figure. For $A_2$, there is at least one point even at a 20% contraction level resulting in a probability of 1.0 that the security level will not be reached. Alternative $A_3$ does not violate the security levels in any region (represented by light gray), and seems therefore to be the best option also for a very risk averse decision maker.

Conclusions

In this article, we have shown how operational decisions in the transportation sector can be handled, even when the background information is vague or numerically imprecise. A case study was discussed, when both imprecise probability and value statements were involved. We also demonstrated a decision tool for these kinds of problems.

However, the problems discussed above are not unique for the transportation business. Aspects on vague input data on the customers’ loyalties can be found in most newborn business-to-business relationships. Typical situations include forecasted future estimations, where it is more adequate to use qualitative or interval approaches rather than classical theories. For instance, when there is a high enough discount for satisfying a customer and low enough discount for being profitable, decision analysis could be of great importance when beforehand analyzing candidates for offering reasonably large discounts. Furthermore, it is also possible to investigate how sensitive the decision is to changes in the input parameters, i.e., analyze the stability of the result. In this way the risk involved can be estimated. If the relative strengths of the alternatives not are stable to moderate changes of utility and probability estimates, the decision should be further analyzed.

References


