Fusion of Possibilistic Knowledge Bases from a Postulate Point of View

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Abstract
This paper proposes a postulate-based analysis of the fusion of possibilistic logic bases which are made of pieces of information expressing knowledge associated with certainty degrees. We propose two main sets of postulates: one focuses only on plausible conclusions, while the other considers both plausible conclusions and the certainty degrees associated with them. For each rational postulate, the class of operators that satisfies it is identified. The existence of weights associated with the pieces of information considerably enriches the postulates-based analysis, and leads to a refined classification of combination operators.

Introduction
The fusion of pieces of information originated from different sources can be considered in various representations frameworks. Ranked propositional logic bases offers a reasonably expressive framework for representing pieces of information with their levels of reliability. Possibilistic logic [4] is a logic of weighted formulas, equipped with a semantics, where the weights can be understood as lower bounds of necessity degrees in the sense of possibility theory. Possibilistic logic inference associates the smallest weights of necessity degrees in the sense of possibility theory. Possibilistic logic postulates. It is in spirit of previous works on postulates for possibilistic case. However, this adaptation is not fully satisfactory. Indeed, from the classification of operators that they proposed, there is no operator which satisfies all postulates. The reason is that some postulates which are natural in propositional logic cannot be directly used in the ranked models framework.

Background
Let $\mathcal{L}$ be a finite propositional language. $\vdash$ denotes the classical consequence relation. A possibilistic knowledge base is a set of weighted formulas $B = \{ (\phi_i, a_i) : i = 1, n \}$ where $\phi_i$ is a propositional formula and $a_i$ belongs to the interval $[0,1]$ and represents the level of certainty or priority attached to $\phi_i$.

Definition 1 Let $B$ be a possibilistic base, and $a \in [0,1]$. We call the $a$-cut (resp. strict $a$-cut) of $B$, denoted by $B_{\geq a}$ (resp. $B_{>a}$), the set of propositional formulas in $B$ having a certainty degree at least equal to $a$ (resp. strictly greater than $a$).

The expression $Inc(B) = \max \{ a_i : B_{\geq a_i} \text{ is inconsistent} \}$ denotes the inconsistency degree of $B$. When $B$ is consistent, we have $Inc(B) = 0$.

Definition 2 $B$ and $B'$ are said to be equivalent, denoted by $B \equiv B'$, iff $\forall a \in [0,1], B_{\geq a} \equiv B'_{\geq a}$, where $\equiv$ is the classical logic equivalence.

There are two possible definitions of the inference process in possibilistic logic framework depending if we take into account the weights associated with conclusions or not:

Definition 3 • A formula $\phi$ is said to be a plausible consequence of $B$, denoted by $B \vdash p \phi$, iff $B_{Inc(B)} \vdash \phi$.

• A possibilistic formula $(\phi, a)$ is said to be a possibilistic consequence of $B$, denoted by $B \vdash \pi (\phi, a)$, iff

1) $B_{\geq a}$ is consistent,
2) $B_{\geq a} \vdash \phi$, and
3) $\forall b > a, B_{\geq b} \vdash \phi$.

We define the plausible (resp. possibilistic) closure of $B$, denoted by $\mathcal{C}n_{p}(B)$ (resp. $\mathcal{C}n_{\pi}(B)$), as follows:

$\mathcal{C}n_{p}(B) = \{ \phi_i : B \vdash p \phi_i \}$

(resp. $\mathcal{C}n_{\pi}(B) = \{ (\phi_i, a_i) : B \vdash \pi (\phi_i, a_i) \}$).
Note that if we ignore the weights in $Cn_\pi(B)$ we simply get $Cn_P(B)$.

### Merging possibilistic knowledge bases

A possibilistic merging operator, denoted by $\oplus$, is a function from $[0,1]^m$ to $[0,1]$. Intuitively, $\oplus$ will be used to merge the certainty degrees associated with pieces of information provided by different experts. More precisely, let $B = \{B_1, \ldots, B_n\}$ be a set of $n$ consistent possibilistic bases. Our aim is to merge the bases of $B$ into a new one, denoted by $B_\oplus$, using the merging operator $\oplus$.

$B_\oplus$ is a set of weighted formulas $(\phi_i, a_i)$ such that $\phi_i$ is a propositional formula, and $a_i$ is the result of combining the certainty degrees associated with $\phi_i$ in each base of $B$. More formally, (see also Figure 1)

$$B_\oplus = \{ (\phi_i, a_i) : \phi \in L, B_i \vdash_\pi \phi \}.$$ 

Two natural properties for $\oplus$ are:

1. If $\forall i = 1, \ldots, n, a_i > b_i$, then $\oplus(a_1, \ldots, a_n) > \oplus(b_1, \ldots, b_n)$.

The first property says that if an explicit conclusion of any base exists, then it should not be an explicit conclusion of the merged result. The second property is simply the monotonicity property which means that if all the experts say that a formula $\phi$ is more plausible than another formula $\psi$, then the result of merging should confirm this preference.

When we only have two bases $B_1 = \{ (\phi_i, a_i) : i \in I \}$ and $B_2 = \{ (\psi_j, b_j) : j \in J \}$, then $B_\oplus$ is equivalent to:

$$\{ (\phi_i \oplus (b_1, \ldots, b_n)) : i \in I \} \cup \{ (\psi_j \oplus (0, \ldots, 0)) : j \in J \} \cup \{ (\phi_i \vee \psi_j, \oplus(a_i, b_j)) : i \in I \text{ and } j \in J \}.$$ 

**Example 1** Let $B_1 = \{ (m_1 \vee m_2, 9), (\neg m_1 \vee \neg m_2, 9), (c_1 \vee c_2, 9), (\neg c_1 \vee \neg c_2, 9), (c_1 \vee c_2, 7), (f, 5) \}$ and $B_2 = \{ (\neg f \vee \neg c_1 \vee m_1, 8), (\neg f \vee c_2 \vee m_2, 8), (\neg c_2, 2) \}$. Let $\oplus$ be a merging operator defined by $\oplus(a, b) = a + b - ab$. Then, $B_\oplus = B_1 \cup B_2 \cup \{ (\neg f \vee \neg c_1 \vee m_1 \vee m_2, 98), (\neg f \vee \neg c_2 \vee m_1 \vee m_2, 98), (\neg f \vee \neg c_1 \vee m_1, 98), (\neg f \vee \neg c_2 \vee m_2, 98), (\neg f \vee c_1 \vee c_2 \vee m_2, 94), (\neg f \vee c_1 \vee m_2 \vee m_2, 92), (\neg f \vee c_2 \vee m_1 \vee m_2, 92), (\neg c_1 \vee \neg c_2, 76) \}$.

In the rest of this paper, $\oplus$ is also supposed commutative and associative. We now define some classes of operators $\oplus$ useful for the rest of the paper:

**Definition 4** $\oplus$ is said to be:

- **excessively optimistic** if $\exists (a_1, \ldots, a_n)$ s.t.
  $$\forall i = 1, \ldots, n, a_i \neq 0 \text{ and } \oplus(a_1, \ldots, a_n) = 0.$$
  Excessively optimistic operators are such that merging not completely certain formulas can lead to totally certain beliefs. Excessively pessimistic formulas mean that merging some what certain information can lead to a completely uncertain formula.

- **regular** if it is neither excessively optimistic nor excessively pessimistic.

- **strictly monotone** if $\forall (a_1, \ldots, a_n), \forall (b_1, \ldots, b_n)$, with $a_i \neq 1, b_j \neq 1$:
  $$\text{if } \forall i, a_i > b_i \text{ and } \exists j, a_j < b_j, \text{ then } \oplus(a_1, \ldots, a_n) > \oplus(b_1, \ldots, b_n).$$

### Postulates for propositional merging

We briefly give main postulates of merging propositional bases [5]. Let $E = \{ K_1, \ldots, K_n \}$ ($n \geq 1$) be a multi-set of $n$ propositional bases to be merged. $\oplus$ is called an information set. $\bigwedge E$ (resp. $\bigvee E$) denotes the conjunction (resp. disjunction) of the propositional bases of $E$. The symbol $\bigcup$ denotes the union on multi-sets.

For the sake of simplicity, if $K$ and $K'$ are propositional bases and $E$ is an information set we simply write $E \sqcup K$ and $K \sqcup K'$ instead of $E \sqcup \{ K \}$ and $\{ K \} \sqcup \{ K' \}$ respectively. We will denote $K^n$ the multi-set $\{ K, \ldots, K \}$ of size $n$. A propositional merging operator $\Delta$ is a function applied on $E$ and which returns a propositional base, denoted by $\Delta(E)$.

In [5], the authors have proposed a set of basic properties that a merging operator has to satisfy:

- **A1**: Consistency $\Delta(E)$ is consistent,
- **A2**: Information complementarity If $E$ is consistent, then $\Delta(E) = \bigwedge E$,
- **A3**: Syntax independence If $E_1 \leftrightarrow E_2$, then $\vdash \Delta(E_1) \equiv \Delta(E_2)$, where $E_1 \leftrightarrow E_2$ means that there exists a bijection $f$ from $E_1 = \{ K_1, \ldots, K_n \}$ to $E_2 = \{ K'_1, \ldots, K'_n \}$ such that $\forall K \in E_1, \exists K' \in E_2, f(K') \equiv K'$.
- **A4**: Cautiousness If $K \not\sqcup K'$ is inconsistent, then $\Delta(K \sqcup K') \not\sqcup K$.

In the case of $n$ bases, $A_4$ can be generalized as follows: $\bigwedge E$ is inconsistent then $\Delta(E) \not\sqcup K_i, \forall K_i \in E$.

- **A5**: Conjunction primacy $\Delta(E_1) \land \Delta(E_2) \sqcup \Delta(E_1 \sqcup E_2)$,
- **A6**: Recovering conjunction If $\Delta(E_1) \land \Delta(E_2)$ is consistent, then $\Delta(E_1 \sqcup E_2) \equiv \Delta(E_1) \land \Delta(E_2)$.

Two classes of merging operators have been particularly analyzed in the literature: majority and arbitration operators [5] defined respectively by:

**Maj**: Majority $\forall K, \exists n, \Delta(E \sqcup K^n) \sqcup K$, and

** Arb**: Arbitration $\forall K, \forall n, \Delta(E \sqcup K^n) \equiv \Delta(E \sqcup K)$.
Plausible inference point of view

The first adaptation focuses only on the set of plausible conclusions without taking care of degrees of inferences. The counterpart of the propositional postulates $A_i$ will be denoted by $P_i$ ($P$ for plausible).

Consistency

Possibilistic logic, contrary to propositional logic, does not entail everything in presence of inconsistency. Therefore, rather than to require that the resulting base is consistent we require that the conclusions obtained from the fused bases are consistent, namely the adaptation of $A_1$ is:

$$P_1: CnP(B_1^*) \text{ is consistent.}$$

Hence, we have the following trivial result:

**Proposition 1** All operators satisfy $P_1$.

Information complementarity

$P_2$ If $B_1 \cup \cdots \cup B_n$ is consistent, then $CnP(B_1^*) \equiv CnP(B_1^* \cup \cdots \cup B_n^*)$.

$P_2$ says that the result of merging should recover all the information provided by the sources if they do not conflict. Such requirement can be satisfied if the merging operator guarantees that each formula which is entailed from at least one base, should also be explicitly present in the result of the merging. This can be captured by conjunctive operators defined by:

**Definition 5** $\otimes$ is called a conjunctive operator if $\otimes(a_1, \cdots, a_n) > 0$ when for some $i$, $a_i > 0$.

**Proposition 2** A merging operator $\otimes$ satisfies $P_2$ iff $\otimes$ is a conjunctive operator.

Syntax independence

$A_3$ has an immediate counterpart in the possibilistic logic setting, namely:

$P_3$ If $B \Leftrightarrow B'$ then $CnP(B) \equiv CnP(B')$, where $B \Leftrightarrow B'$ is defined as follows:

$$\forall B_i \in B, \exists B'_i \in B' \text{ such that } B_i \equiv B'_i$$

**Proposition 3** All operators satisfy $P_3$.

Cautiousness

The idea in the propositional postulate $A_4$ is that when two bases are conflicting, the result of merging should not give preference to any base. This requirement is natural in possibilistic logic since formulas are flat and have the same reliability level. However, this cannot be right away in possibilistic logic framework. Indeed,

**Proposition 4** There is no strictly monotone operator which satisfies the immediate adaption of $A_4$ (Namely, if $B_1$ and $B_2$ is inconsistent, then $CnP(B_1^*) \not\sqsupseteq CnP(B_1)$ and $CnP(B_2^*) \not\sqsubseteq CnP(B_2)$).

Note that strictly monotone operators are necessarily conjunctive. The converse is false. For example the max operator defined by $\mathcal{E}(a_1, \cdots, a_n) = \max(a_1, \cdots, a_n)$ is conjunctive but not strictly monotone.

The following counter-example illustrates Proposition 4:

**Counter-example 1** Let $B = \{B_1, B_2\}$ be s.t. $B_1 = \{(a, \phi)\}$ and $B_2 = \{(-\phi, b)\}$ where $a > b$ and $b > 0$. Let $\oplus$ be a strictly monotone operator. By construction, we have $B_1 = \{(\phi, \oplus(a, 0))\} \cup \{(-\phi, \oplus(0, b))\} \cup \{\top, \oplus(a, b)\}$. We have $\oplus(a, 0) > 0$ and $\oplus(0, b) > 0$ since $\oplus$ is strictly monotone and $\oplus(0, 0) = 0$. We also have $\oplus(a, 0) > \oplus(b, 0)$ since $a > b$ and $\oplus$ is strictly monotone. Hence, $CnP(B_1^*) = \{\phi\}$ from which $CnP(B_1)$ is entailed.

There is a class of operators which satisfies the immediate adaptation of $A_4$ (however, fails to satisfy $P_2$). They are called disjunctive operators defined as follows:

**Definition 6** $\oplus$ is called a disjunctive operator if $\oplus(a_1, \cdots, a_n) = 0$ when $\exists j$ s.t. $a_j = 0$.

Note that disjunctive operators cannot be strictly monotone. Since formulas are weighted in possibilistic logic framework, a natural adaptation of $A_4$ in this setting is to say that if $B_1$ and $B_2$ are conflicting and if they are equally prioritized, then the result of fusion should neither infer $B_1$ nor $B_2$. The question now is "how to express in possibilistic logic that two bases are equally prioritized?". Let us first introduce the notion of a certainty degree of a subbase:

**Definition 7** Let $A$ be a subbase of $B$. We define its certainty degree, denoted by $Deg(A \cap B)$, by $Deg(A \cap B) = \min\{a : (\phi, a) \in A \cap B\}$.

This definition means that $Deg(A \cap B)$ is equal to the degree of the least certain formulas in $B$ which belong to $A$. Now, we define the priority between two bases as follows:

**Definition 8** $B_1$ is said to be more prioritized than $B_2$ if for each conflict $C$ in $B_1 \cup B_2$, we have $Deg(C \cap B_1) > Deg(C \cap B_2)$.

Namely, $B_1$ is more prioritized than $B_2$ if the least formulas in each conflict $C$ between $B_1$ and $B_2$ are in $B_1$. $B_1$ and $B_2$ are said to be equally prioritized if for each conflict $C$ in $B_1 \cup B_2$, the least prioritized formulas in $C$ belong to both $B_1$ and $B_2$.

**Example 2** Let $B_1$ and $B_2$ be the two following possibilistic bases defined as follows:

$$B_1 = \{\phi \lor \psi \lor \xi, \theta; (\neg \psi, \xi); (\neg \theta, \xi)\} \text{ and } B_2 = \{(\neg \phi, \psi); (\neg \xi, \theta); (\xi \lor \theta, \psi); (\varphi \lor \psi, A)\}.$$

There are two conflicts in $B_1 \cup B_2$: $C_1 = \{\phi \lor \psi \lor \xi, \neg \theta, \neg \psi\}$ and $C_2 = \{\neg \psi, \xi \lor \phi, \theta\}.$

We have $Deg(B_1 \cap C_1) = Deg(B_2 \cap C_1) = \gamma$ and $Deg(B_1 \cap C_2) = Deg(B_2 \cap C_2) = \delta$.

Then, $B_1$ and $B_2$ are equally prioritized. However if $B_2 = \{(\neg \phi, \psi); (\neg \xi, \theta); (\xi \lor \theta, \psi); (\varphi \lor \psi, A)\}$ then $B_1$ is prioritized than $B_2$ since $Deg(C_1 \cap B_1) > Deg(C_1 \cap B_2)$ and $Deg(C_2 \cap B_1) > Deg(C_2 \cap B_2)$.

We now give a suitable adaptation of $A_4$ in possibilistic logic framework:

$P_4$ If $B_1$ and $B_2$ is inconsistent and, $B_1$ and $B_2$ are equally prioritized, then $CnP(B_1^*) \not\sqsubseteq CnP(B_1)$ and $CnP(B_2^*) \not\sqsupseteq CnP(B_2)$.

Then we have:

**Proposition 5** All operators satisfy $P_4$.

The remaining postulates have immediate adaptations: let $B = B' \cup B''$.
If a reinforcement operator which does not conflict completely sure formulas of

Proposition 6 All operators satisfy \( P_5 \). \( \oplus \) satisfies \( P_6 \) iff \( \oplus \) is a strictly monotone operator.

Majority

\[ P_{\text{Maj}} \quad \forall B', \exists n, Cn_p((B \cup B')_\oplus) \vdash Cn_p(B'). \]

Intuitively, majority is related to the idea of reinforcement, namely if a same formula is believed to a degree \( a \) by two agents, it should be believed with a larger weight in the result of merging. Reinforcement operators are defined as follows:

Definition 9 \( \oplus \) is said to be a reinforcement operator if

\[ \oplus(a_1, \ldots, a_n) > \max(a_1, \ldots, a_n) \text{ when } \exists i, a_i \neq 0. \quad \text{And,} \]

\[ \oplus(a_1, \ldots, a_n) = 1 \text{ if for some } i, a_i = 1. \]

Note that reinforcement operators are also conjunctive. However, reinforcement and strictly monotone operators are unrelated. For example \( \oplus = \frac{a+b}{2} \) is strictly monotone but not a reinforcement operator.

Note that reinforcement property is not enough to satisfy \( P_{\text{Maj}} \). An example of a reinforcement operator which does not satisfy \( P_{\text{Maj}} \) is \( \oplus \) satisfying \( P_{\text{Maj}} \). To satisfy \( P_{\text{Maj}} \), \( \oplus \) should also be strictly monotone.

Proposition 7 \( \oplus \) satisfies \( P_{\text{Maj}} \) if \( \oplus \) is a strictly monotone and a reinforcement operator.

Arbitration

\[ P_{\text{Arb}} \quad \forall B', \forall n, Cn_p((B \cup B')_\oplus) \equiv Cn_p(B \cup B'). \]

Arbitration postulate means that \( \oplus \) ignores the redundancy of information. Formally, we should have:

\[ \oplus(a, 0, \ldots, 0) = \oplus(a, a, 0, \ldots, 0) = \cdots = \oplus(a, \ldots, a, 0, \ldots, 0). \]

This requirement can be obtained by operators called idempotent operators and defined as follows:

Definition 10 \( \oplus \) is called an idempotent operator if

\[ \forall a_i, \oplus(a_i, \ldots, a_i) = a. \]

Proposition 8 \( \oplus \) satisfies \( P_{\text{Arb}} \) iff \( \oplus \) is an idempotent operator.

To summarize the above results, we have:

1. \( \oplus \) satisfies \( \{P_1, P_2, P_3, P_4, P_5, P_6\} \) if and only if \( \oplus \) is a strictly monotone operator. An example of such operators is the probabilistic sum, i.e. \( \oplus(a, b) = a + b - ab \).

2. \( \oplus \) satisfies \( \{P_1, P_2, P_3, P_4, P_5, P_6, P_{\text{Maj}}\} \) if and only if \( \oplus \) is a strictly monotone and reinforcement operator. An example of such operators is the probabilistic sum.

3. If \( \oplus \) is an idempotent operator then it satisfies \( \{P_1, P_2, P_3, P_4, P_5, P_{\text{Arb}}\} \). Examples of such operators are \( \min \) and \( \max \).

\[ \text{Possibilistic inference point of view} \]

In this section, we briefly give the adaptation of propositional postulates, denoted by \( W_2 \) (W for weighted) in possibilistic logic framework when we keep track of the weights attached to conclusions. Therefore we are interested in analysing the possibilistic closure \( Cn_\pi(B_\oplus) \). We recall that this set is defined by:

\[ Cn_\pi(B_\oplus) = \{(\phi_i, a_i) : B_\oplus \vdash (\phi_i, a_i)\}. \]

Postulates \( W_1, W_2 \) and \( W_4 \) are the direct counterparts of \( P_1, P_3 \) and \( P_4 \). Namely,

\( W_1 Cn_\pi(B_\oplus) \) is consistent.

\( W_3 \) If \( B_1 \Rightarrow B_2 \), then \( Cn_\pi(B_1) \Rightarrow Cn_\pi(B_2) \).

\( W_4 \) If \( B_1 \) and \( B_2 \) is inconsistent and, \( B_1 \) and \( B_2 \) are equally prioritized, then

\[ Cn_\pi(B_\oplus) \not\vdash B_1 \text{ and } Cn_\pi(B_\oplus) \not\vdash B_2. \]

Then, we have the following result:

Proposition 9 All operators satisfy \( W_1, W_3 \) and \( W_4 \).

Information complementarity

The idea in \( A_2 \) is that the result of merging should recover classical conjunction when the bases are agreeing. When the weights attached to formulas are considered, recovering the complementarity between the bases means that, if a formula \( (\varphi, a) \) is entailed from at least one base \( B_i \) then it should also be entailed from \( Cn_\pi(B_\oplus) \), with a weight at least equal to \( a \). Hence, we have the following adaptation of \( A_2 \):

\( W_2 \) If \( B_1 \cup \cdot \cdot \cdot \cup B_n \) is consistent, then

\[ \forall \phi_i, \text{ if } \exists i, B_i \vdash (\varphi, a) \text{ then } B_\oplus \vdash (\varphi, b), \quad \text{ with } b \geq a. \]

We have shown in the first adaptation (when only considering plausible conclusions) that such requirement is obtained by operators called conjunctive operators and satisfying the following property: \( \oplus(a_1, \ldots, a_n) > 0 \) if \( \exists a_i > 0 \). However, it may exist conjunctive operators which do not satisfy \( A_2 \).

Example 3 Let \( B_1 \) and \( B_2 \) be two possibilistic bases such that \( B_1 = \{\{\varphi, 9\}\} \) and \( B_2 = \{\{\varphi \lor \psi, 7\}, \{\psi, 5\}\} \) which are together consistent. Let \( \oplus \) be the mean operator defined by:

\[ \oplus(a, b) = \frac{a+b}{2}. \]

Then, \( B_\oplus = \{\{\varphi, 4.5\}, \{\varphi \lor \psi, 25\}, \{\psi, 25\}, \{\varphi \lor \psi, 7\}\} \) which does not entail neither formulas entailed from \( B_1 \) nor those entailed from \( B_2 \).

Indeed, \( W_2 \) excludes some conjunctive operators. Note that \( W_2 \) is stronger than \( P_2 \).

To satisfy the requirement expressed by \( W_2 \), we define a sub-class of conjunctive operators called strongly conjunctive, defined by:

Definition 11 A merging operator \( \oplus \) is called strongly conjunctive if

\[ \oplus(a_1, \ldots, a_n) \geq \max(a_1, \ldots, a_n). \]

Hence, we have the following result:

Proposition 10 \( \oplus \) is a strongly conjunctive operator iff \( \oplus \) satisfies \( W_2 \).

Conjunction primacy

The adaptation of \( A_5 \) needs a discussion. A possible adaptation would be the following: let \( B = B' \cup B'' \).

\[ Cn_\pi(B_\oplus) \cup Cn_\pi(B' \oplus) \vdash Cn_\pi(B_\oplus). \]

This adaptation does not reflect faithfully the idea behind \( A_5 \), when \( Cn_\pi(B_\oplus) \cup Cn_\pi(B' \oplus) \) is inconsistent. Indeed,
in the propositional postulate $A_5$ when $\Delta(E_1) \land \Delta(E_2)$ is inconsistent, the postulate is trivially satisfied. But the above adaptation is not trivially satisfied since possibilistic inference does not entail everything in presence of inconsistency. Hence, it is more natural to restrict to the case of consistency.

Another point is how to fix the definition of conjunction in the sense of Definition 9 used between $Cn_\tau(B'_{\parallel})$ and $Cn_\tau(B''_{\parallel})$. A stronger reinforcement operator, defined below, is appropriate. Hence, we have:

\[ W_5 \] if $Cn_\tau(B'_{\parallel}) \cup Cn_\tau(B''_{\parallel})$ is consistent, then $Cn_\tau(B'_{\parallel} \circ_s B''_{\parallel}) \vdash \tau Cn_\tau(B_{\parallel})$, where $\circ_s$ is defined as follows:

Let $\alpha_1 = Inc(B'_{\parallel})$ and $\alpha_2 = Inc(B''_{\parallel})$. Then,

$$\circ_s(a, b) = \begin{cases} 1 & \text{if } a > \alpha_1 \text{ or } b > \alpha_2 \\ \max(a, b) & \text{otherwise} \end{cases}$$

The use of a parametrized operator means that combining two formulas whose certainty is above the inconsistency level, get the maximal weight.

**Proposition 11** All operators satisfy $W_5$.

**Conjunction recovering**

Contrary to $A_5$, $A_6$ refers to a decomposition of one group into two subgroups. Then, an idempotent conjunction (which leads to the union of knowledge bases) is enough to adapt $A_6$:

\[ W_6 \] if $Cn_\tau(B'_{\parallel}) \cup Cn_\tau(B''_{\parallel})$ is consistent, then $Cn_\tau(B_0_{\parallel}) \vdash \tau Cn_\tau(B'_{\parallel}) \cup Cn_\tau(B''_{\parallel})$.

We have shown in the first adaptation that the operator should be strictly monotone in order to recover the conjunction. However when we consider the weights attached to formulas, this is not enough. Indeed:

**Proposition 12** $\oplus$ satisfies $W_6$ iff $\oplus$ is a strongly conjunctive and strictly monotone operator.

As a summary of this adaptation, we get the following result:

$\oplus$ satisfies $\{W_1, W_2, W_3, W_4, W_5, W_6\}$ if and only if $\oplus$ is a strongly conjunctive and a strictly monotone operator. Note that this adaptation is stronger than the plausible adaptation, which only requires $\oplus$ to be strictly monotone to satify $P_1, \ldots, P_5$.

**Conclusion**

In this paper two adaptations of propositional fusion postulates to the possibilistic framework has been given. We have pointed out the existence of several classes of merging operators. In particular different kinds of conjunctive operators have been defined, due to the information complementarity postulates. Among them associative strictly monotone (and also strongly conjunctive in the second adaptation) operators are particularly attractive, for knowledge fusion because they indeed satisfy all the basic postulates. A futur work will be to investigate the integration of integrity constraints and see how postulates given in [6] can be extended to the possibilistic logic framework. Integrity constraints are requirements that the resulting base should satisfy. In possibilistic logic, such constraints are represented by means of fully certain formulas (with weight 1). Another futur work is to see to what extent postulates of social choice [7] can be used for merging knowledge bases.

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**References**


