Object Determination Logic Quantification - A System for Natural Language Processing

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Abstract

In this paper we describe a new quantification theory based on the Object Determination Logic(ODL). This theory extends the system of classical quantifiers as a system including the classical and linguistic ones. It takes into account typicality, as a dimension of cognition. This theory represents the background of a computational model in the semantic analysis of natural languages.

This paper presents the basic elements of this quantification system (QSODL) and the possibilities of implementing it in a computational system.

Keywords : more or less determined object, concept, typicality, quantifier.

Introduction

Let us start with some examples :

1. *Alsatians are beer drinkers.*
1'. *Les alsaciens sont des buveurs de bière.*
2. *All Frenchmen know "La Marseillaise".*
2'. *Tout français connaît la Marseillaise.*

Example 1.seems to say that :
All typical Alsatians are beer drinkers.
Example 2. seems to say that :
All typical Frenchmen know "La la Marseillaise".

Their formalization in classical logic is :
$(\forall x)$ [(Alsatian x) $\supset$ (beer drinker x)]
and :
$(\forall x)$ [(Frenchman x) $\supset$ (knows "la Marseillaise" x)]

The illative version (Curry and Feys 1958) is :
Π_2 (to be Alsatian) (to be beer drinker)
Π_2 (to be Frenchman) (to know "La Marseillaise")

Words like "all", "the" mean "all typical ....". The operators $\forall$ and Π_2 do not encode the typicality in their semantics.

Quantification operations were introduced by logic in the form of the well-known quantification operators, denoted by $\forall$ and $\exists$. On the other hand, the operations of "qualitative determination" were not taken into account by mathematical logic. This logic essentially stressed the deductive effects stated by a part of the sense of propositional connectors (and, or, if...than,..), by predication operations and by quantification operators.

Otherwise, natural languages use a large number of quantifiers (all, some, a, most of, at least..., more... than...,...) and other operators as classifiers in some languages such as Chinese and Vietnamiese.

- How can we define precisely these operations of "qualitative determination"?
- How can we articulate them with quantifiers ?
- Is the operation of determination specifically a cognitive operation ?
- Are "classical" quantifiers sufficient for the analysis of quantification in natural languages ?

The answer to these questions is a new approach to quantification theory based on the qualitative determination operations imposed by linguistic operations analysis.

To take *typicality* into account is a phenomenon proper to cognition but completely blotted out by the formal categorizations of mathematical logic. The proof is the fact that traces of typicality can be found in some words which express quantification. Words such as : *all, every, there is a, some* , which have the value of quantifiers, receive, in some contexts, an additional value – that of typicality. In our previous paper (Pascu and Carpentier 2002) we presented a new logic named Object Determination Logic (ODL) based on the notions of concept and object. Concepts are functions defined on objects with the truth values "true" or "false". ODL defines more or less determined objects. Starting from a concept it constructs classes of more or less determined objects related to a totally indetermined object – the totally indetermined object τf.

In this paper we propose a quantification theory taking this aspect into account. This theory is constructed starting with Curry's (Curry and Feys 1958) quantifiers and using the natural deduction rules. It is named the Quantification System of Object Determination Logic (QSODL).

Curry's quantification operators

Curry's quantification theory (the illative theory) (Curry and Feys, 1958) is represented by the Π_2 and Σ_2 operators.

These operators are defined by the following rules of introduction and elimination:

For Π_2 :

$$\begin{array}{|l} \begin{array}{|l} (f\ x) \\ \vdots \\ (g\ x) \end{array} \\ \Pi_2\ f\ g \end{array} \qquad [\text{ i - }\Pi_2\ ]$$

$$\frac{\Pi_2\ f\ g,\ (f\ x)}{(g\ x)} \qquad [\text{ e - }\Pi_2\ ]$$

For Σ_2 :

$$\frac{(f\ a) \wedge (g\ a)}{\Sigma_2\ f\ g} \qquad [\text{i - }\Sigma_2\ ]$$

$$\frac{\Sigma_2\ f\ g,\ (f\ a) \vdash B,\ (g\ a) \vdash B}{B} \qquad [\text{ e - }\Sigma_2\ ]$$

with their semantics :

$\Pi_2\ f\ g$: Ext$f \subset$ Ext$g$ says that : **all the objects falling under f fall under g as well** . The extension of the concept f, Ext f is the class of all totally determined objects falling under f.

$\Sigma_2\ f\ g$: Ext$f \cap$ Ext$g \neq \emptyset$ says that : **there is at least an object falling simultaneously under f and g** .

A new semantics

In the paper (Pascu and Carpentier 2002) we defined classes Etendue(f) and Etendue(τf). The first one is the class of all objects (more or less determined) falling under f; the second one represents all more or less determined objects which can be constructed starting from τf. The object τf is the totally indetermined object associated to the concept f.

If we make the following hypothesis (very strong) that "everything that is constructible is also deductible" and conversely, that is :

Etendue$(\tau f) =$ Etendue$f$

Then the following relations hold :

1. Etendue $f \subset$ Etendue $g \Longrightarrow$ Ext $f \subset$ Ext g
2. Ext $f \bigcap$ Ext $g \neq \emptyset \Longrightarrow$ Etendue $f \bigcap$ Etendue $g \neq \emptyset$

These relations show that the following statements are not equivalent :

" All totally determined objects of f are totally determined objects of g."

and

"All objects of f are objects of g. "

This idea determines the definition of two Π_2 operators : a strong universal quantification operator denoted by (Π_2^{strong}) and a weak universal quantification operator denoted by (Π_2^{weak}) :

$\Pi_2^{strong} fg$: Etendue $f \subset$ Etendue g

$\Pi_2^{weak} fg$: Ext $f \subset$ Ext g

For existential quantification :

$\Sigma_2^{strong} fg$: Etendue $f \cap$ Etendue $g \neq \emptyset$

$\Sigma_2^{weak} fg$: Ext$f \cap$ Ext $g \neq \emptyset$

Note :

$\Pi_2^{weak} \equiv \Pi_2$ and $\Sigma_2^{weak} \equiv \Sigma_2$

and

$\Pi_2^{strong} fg \Longrightarrow \Pi_2^{weak} fg$

$\Sigma_2^{weak} fg \Longrightarrow \Sigma_2^{strong} fg$

¿From the cognitive point of view the interpretation of two operators can vary between a lower limit (given by Ext) and an upper limit (given by Etendue). There are no epistemical means of proving the cognitive adequation of one or of the another.

A new quantification theory

$\Pi^\star$ and $\Sigma^\star$ operators

A cognitive dimension to interprete quantification operators is typicality. A typical object is an object x obtained from τf by applying only determinations "typically compatible with f"(Pascu and Carpentier 2002).

This new theory of quantification is represented by a system of quantifiers larger than the traditional one.

There are two problems when the Π_2 operator is defined:

- This operator must be applied to the nominal phrase and not to the entire clause. This constraint is issued from the Subject-Predicate linguistic structure of the clause.
- This operator must take typicality into account.

We consider the predicate $TYPIQUE(f)(x)$ which means : x is a typical object of f. We introduce the typical universal quantification operator.

- The introduction-elimination rules for the $\Pi^\star$ operator (typical universal quantification operator) are :

$$\frac{TYPIQUE(f)(x), (fx) \vdash (gx)}{g(\Pi^\star f)}$$

or

$$\begin{array}{|l} TYPIQUE(f)\ (x) \\ \qquad \begin{array}{|l} (f\ \ x) \\ \vdots \\ (g\ x) \end{array} \\ g(\Pi^\star f) \end{array} \qquad [\text{ i- }\Pi^\star\ ]$$

If x is a typical object generated from τf and if from $(f\ x)$ one can deduce $(g\ x)$, then $g(\Pi^\star f)$.

$$\frac{g(\Pi^\star f), TYPIQUE\ (f)\ (x), (f\ x)}{(g\ \ x)} \ [\text{ e - }\Pi^\star\ ]$$

If $g(\Pi^\star\ f)$, $(f\ x)$ and x is a typical object generated from τf then $(g\ x)$.

$g(\Pi^\star\ f)$ is read : **every typical object of f is an object of g**, abbreviated **every typical f is g.**

For existential quantification we introduce $\Sigma^\star$ and we change the semantics of this quantifier as follows.

• The introduction-elimination rules for the existential quantification operator $\Sigma^\star$ are :

$$\frac{(f\ \ a) \wedge (g\ a)}{g(\Sigma^\star f)} \qquad [\text{ i - } \Sigma^\star\ ]$$

$$\frac{g(\Sigma^\star f), (f\ a) \vdash B, (g\ a) \vdash B}{B} \qquad [\text{ e - } \Sigma^\star\ ]$$

• The introduction-elimination rules for the typical existential quantification operator Σ_2 are :

$$\frac{TYPIQUE\ (f)\ (a), (f\ \ a) \wedge (g\ a)}{\Sigma_2 f\ g} \qquad [\text{ i - } \Sigma_2\ ]$$

$$\frac{TYPIQUE\ (f)\ (a), \Sigma_2 f\ g, (f\ a) \vdash B, (g\ a) \vdash B}{B} \qquad [\text{e - } \Sigma_2\ ]$$

$g(\Sigma^\star f)$ is read : **there is a f which is g.**

$\Sigma_2 fg$ is read : **there is a typical f which is g.**

A first asymmetry between Π and Σ is due to the asymmetry of Π in f and g and to the symmetry of Σ in f and g:

$(\forall x)((fx) \supset (gx))$

$(\exists x)((fx) \wedge (gx))$ or

$(\exists x)((gx) \wedge (fx))$

This asymmetry generates the second asymmetry between operators Π et Σ as regards typicality : what is the operator which includes typicality between Π_2 and $\Pi^\star$, and between Σ_2 and $\Sigma^\star$?

The change of semantic features as regards typicality between Σ_2 and $\Sigma^\star$ is a hypothesis derived from the fact that one proves the existence of an object by giving an object construction. But construction generally leads to typical objects and not to atypical ones. Applying $\Sigma^\star$ to f, one states the existence of an object falling under f (not necessarily typical). The proof of $g(\Sigma^\star f)$ is the construction of an object (typical or not) falling under f and verifying g. The proof of $\Sigma_2 fg$ is a typical object of f verifying g.

The system of these four quantifiers gives a cube on the model of Aristotle's square (fig.1).

Examples

There are traces of typicality in natural languages. These traces are expressed explicitly or not in the syntax, but there is always a degree of semantic ambiguity which cannot be solved either at clause level or at context level. We present some examples which can be formalized by quantifiers in our system.

Π_2 operator

The Π_2 operator is a quantifier which allows us to formalize mathematical propositions universally quantified and general statements from common language. It is expressed in English by "all", "the", "a", but also by "every". It can be expressed by locutions as "all without exception", "absolutely all"....

1. Every man is mortal.

Π_2 (to be a man)(to be mortal)

2. All men are mortal.

Π_2 (to be a man)(to be mortal)

Example 1 can be interpreted by :

(to be mortal)(τ (to be a man))

Example 2, because of the plural has rather the interpretation of a Π_2.

3. Man is mortal.

can have two interpretations :

Π_2 (to be a man)(to be mortal)

(to be mortal)(τ (to be a man))

4. All Frenchmen drink wine.

Π_2 (to be Frenchman)(to be a wine drinker)

In example 4 the pronoun "all" gives the sense of Π_2 rather than $\Pi^\star$.

$\Pi^\star$ operator

$\Pi^\star$ operator is the quantifier which allows the formalization of sentences from common language with predications of typical objects. It is encoded in natural language in the same way as Π_2 (without giving the typicality). Sometimes, it is expressed by expressions as : "with some exceptions", "generally".

5. Every Alsatian drinks beer. (G. Kleiber)

(to be beer drinker) ($\Pi^\star$ (to be Alsatian))

6. An Alsatian drinks beer.

(to be beer drinker) ($\Pi^\star$ (to be Alsatian))

or

(to be beer drinker) (τ (to be Alsatian))

7. Frenchmen drink wine.

(to be wine drinker)($\Pi^\star$ (to be French))

$\Sigma^\star$ operator

This operator allows the formalization of a sentence containing an existential quantifier. It is encoded by "a, (an)", "there is a, (an)", "there are", "it exists a".

8. A thief stole my bag. (Un voleur m'a dérobé mon sac)

(to steal the bag of...)($\Sigma^\star$ (to be a thief))

A particular totally undetermined human being stole my bag. The word "thief" expresses here a totally undetermined object.

9. There are students who arrive late for my class.

(to be late in the class of...)($\Sigma^\star$ (to be student))

There is at least one student (typical or not) who arrives late for my class.

10. There is a prime number which is even

(to be even)($\Sigma^\star$ (to be a prime number))

Σ_2 operator

This quantifier allows the formalization of clauses from common language containing an existential quantification of typical objects.

11. There are birds which glide well in the wind.

Σ_2 (to be a bird)(to glide well in the wind)

12. There is a continuous function which is not derivable

Σ_2 (to be a continuous function)(to be non-derivable)

There is at least one continuous function which is non-derivable and this function is a typical one versus these two properties.

A proposition for a new generalized theory

There are other cognitive processes like counting or numerical estimation which are often considered as quantification in linguistics. The relation between quantification in logic and that in linguistics can generate a unification of the two approaches. We will try to sketch it.

We start from the following question :

- Is it legitimate to consider all determiners as quantifiers as Keenan say (Keenan,1997) ?

First of all, it can be noted that qualification and quantification as cognitive operations are characterized by :

- to construct classes of objects starting from object properties — for the first one
- to construct classes of objects starting from other classes by comparing their dimensions — for the second one.

They are different and their operators must be different.

Moreover a generalized quantifier in Keenan's sense is not a quantifier in the sense of logic.

We propose the following distinctions :

- Qualification applies a determination or a string of determinations to an object (more or less determined). It results in another object more determined than the initial one :

  $y = (\Delta\ x)$

  It is applied to objects.
- Quantification applies a quantification operator (Π_2, Σ_2, $\Pi^\star$, $\Sigma^\star$,..) to a concept f. It results in the statement of a relation between Etendue(τf) and another class of objects.
- Other "counting" operations expressed by applying a "counting operator" to a concept f. It always results in the statement of a relation between classes of objects. "Counting" operators are very particular. Most of the determiners described by Keenan are "counting operators".

Conclusions

Quantification is a cognitive process which allows the determination of the denotative scope of a concept.

- Are there objects falling under a concept ? If so, is there one or more ?
- Do all objects from the universe of discourse fall under this concept or not?

The quantification of classical logic is not sufficient to express typicality. A larger system of quantifiers is defined. The features of it are :

- It is based on the categorization system of ODL (Pascu 2001)
- This system takes typicality into account.

We are working with a system which can find the typicality of objects expressed in a text.

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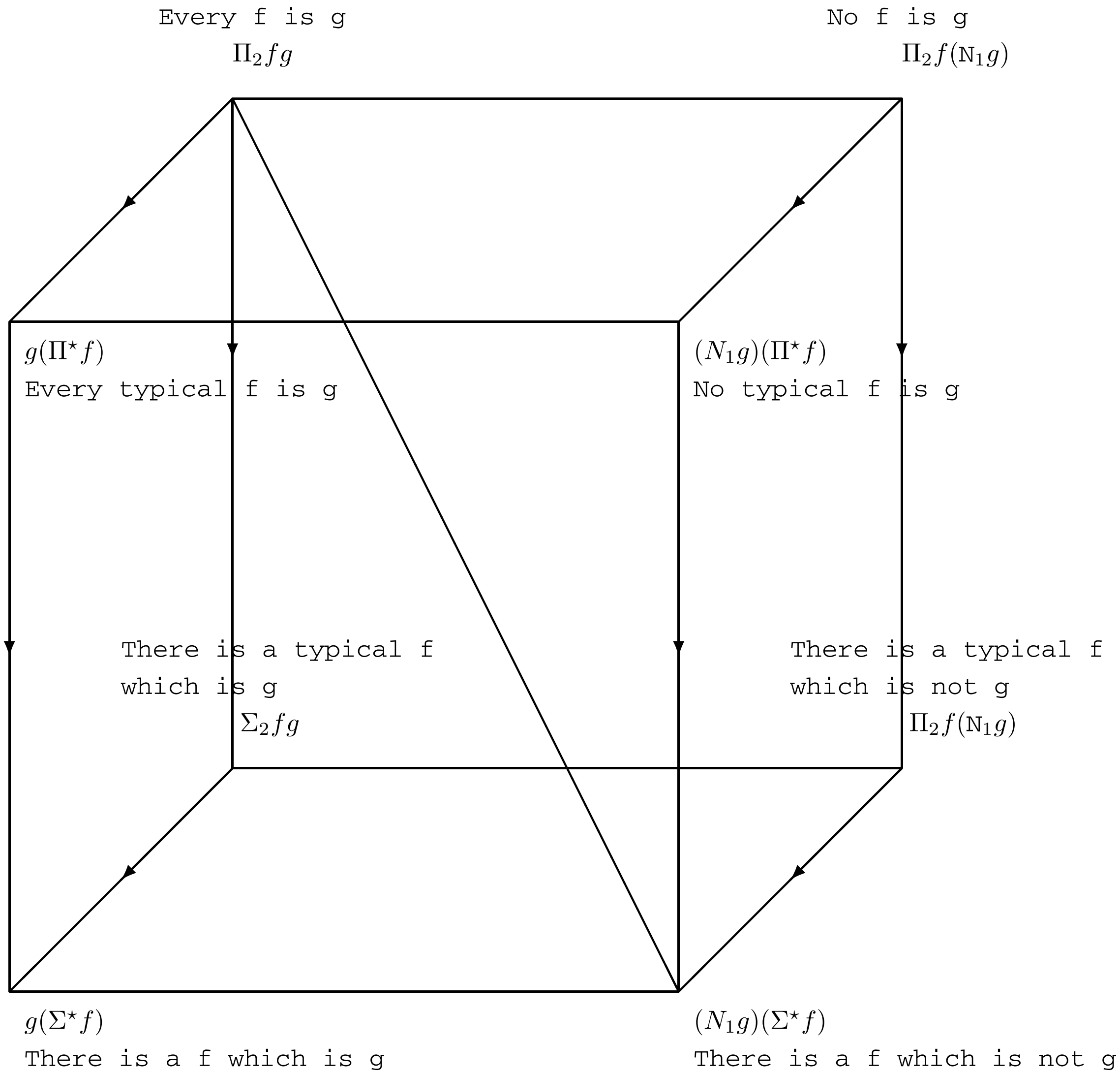

Figure 1: Quantifier's cube