Foundations of Similarity and Utility

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Abstract

In this paper we discuss a rigorous foundation of similarity reasoning based on the concept of utility. If utility is formulated in mathematical terms it can serve as a formal specification for a similarity system. However, utility can also be formulated in informal ways. We consider subjective versions, competing ones and those that change dynamically over time. These are illustrated by examples from risk analysis, speech recognition, and urban planning. From the examples we derive a number of challenges.

1. Introduction

In the huge literature on similarity researchers often discuss what "similarity really is". There is no unique answer presently available. A major reason for this is that the discussion is mostly application driven.

In (Richter and Aamodt 2005) the foundations and roots of Case-Based Reasoning have been discussed. This was not to be simple because CBR is at the intersection and the interest of different disciplines of a rather heterogeneous nature, including cognitive science, knowledge representation and reasoning, machine learning, and mathematics. Each discipline has its own roots and its own foundations. An overview over many kinds of similarity measures is given in (Richter 2007).

Similarity measures depend not only on the object class, but also on the task and the context. For example, there are different measures for technical devices when they are constructed, sold, monitored, or repaired. Other areas include risk analysis, e-commerce tasks, image interpretation, speech recognition, and explanation.
The presence of a wide variety of such measures raises the question of what the meaning i.e., the semantics of a measure is and whether there is a unified approach to semantics.

These considerations often create the impression that similarity reasoning is more an art than a scientific discipline and definitely not an engineering discipline. Engineering disciplines usually depend on clearly and precisely stated foundations. This requires also a formally stated similarity.

Here we take a special view on similarity reasoning, namely as a computational device. There are at least two reasons why similarity systems should have a formal basis. Firstly, one can prove properties about the systems in a mathematical way; that means such properties can be guaranteed. Secondly, one can build engineering systems on a solid foundation. This has turned out to successful for programming languages. Of course, a formal basis is not intended to solve practical problems. For instance, the theory of programming does not tell us how to write clever Java programs. Here, the analogy of writing a program is similarity assessment. Such a foundation, however, should also be flexible enough to be useful in situations that are not described in precise mathematical terms.

We will base the semantics of similarity reasoning on *utility*. This aspect has been around for several years, see (Bergmann et al. 2001); here we will investigate this more systematically. The utility can be a classification with minimal error or error cost, a most useful diagnosis, or intended document that gives best answers to a query.

Utility in the mathematical sense is an axiomatic theory. On the other hand, the concept is broad enough to allow extensions beyond pure mathematics that are needed in applications.

A formal basis has two parts, syntax and semantics. We will first describe these. Then we will investigate utilities that are not formally defined. Some of them are of a very problematic nature and we investigate their consequences to similarity measures. Here we use the foundations of CBR discussed in (Richter and Aamodt 2005). Finally, we illustrate this by examples and present some challenges.

2. Syntax and Semantics of Similarity Reasoning

2.1 Terminology

We introduce some notation for well-known concepts. Similarity measures sim are defined on ordered pairs:

$$\text{sim}: U \times U \rightarrow (0, 1) \text{ or sim}: U \times V \rightarrow (0, 1)$$

for sets U and V. For simplicity we assume that all objects from U and V are attribute-value vectors.

The role of *a* and *b* in sim(*a*,*b*) is not symmetric; usually *a* is a given object (called the *query object*) and *b* is a possible *answer object*.

In the classical situation cases are (problem, solution) pairs. In electronic commerce (where we restrict ourselves to the situation where the customer sends a demand to the shop) one has the query objects as the customer demanded products and the answer objects are the available products; hence we have $V \subseteq U$. In information retrieval U and V are different. In more general situations like document search U and V may be disjoint.

We denote the nearest *neighbour relation* by NN(a,b), stating that b is a nearest neighbour of a.

For a fixed object a we define $\geq_a, >_a, \sim_a$ by:

$b \geq_a c \Leftrightarrow \text{sim}(a,b) \geq \text{sim}(a,c)$

$b >_a c \Leftrightarrow (b \geq_a c) \wedge \neg (c \geq_a b)$

$b \sim_a c \Leftrightarrow (b \geq_a c) \wedge (c \geq_a b)$.

We refer to systems performing the nearest neighbour search as *similarity systems* here. A similarity system performs two types of computations:

- Computing the similarity value between objects a and b.
- Searching for the nearest neighbour(s).

2.2 Syntax

The syntax in almost all formal systems is defined bottom up, starting with atomic elements. This can be expressed using the local-global principle:

Each (language) object A is *(globally)* described by some effectively computable construction operator C from local (atomic) elements A_i, where I is an arbitrary index set:

$$A = C(A_i \mid i \in I).$$

Instantiating this to similarity reasoning leads to:

There are *local measures* sim_i on the attributes A_i and there is some constructor function F such that for a and b,

$a = (a_i \mid i \in I)$, $b = (b_i \mid i \in I)$:

$$\text{sim}(a, b) = F(\text{sim}_i(a_i , b_i) \mid i \in I).$$

A popular example is (n being the number of attributes):

$$\text{sim}(a,b) = \sum (w_i \cdot \text{sim}_i(a_i , b_i) \mid 1 \leq i \leq n).$$

Because in principle arbitrary construction operators are allowed the syntax of similarity systems has more expressive power than the syntax of predicate logic and programming languages.

The definition of the measure provides the syntax for similarity systems. The syntax of the search algorithms is irrelevant here.

2.3 Semantics

Describing the semantics of similarity measures is more involved. In analogy to programming languages we distinguish two forms of semantics, namely operational and denotational semantics. The *operational semantics* describes how computations are performed. In programming theory they are defined by the step-by-step computation of a program. In similarity reasoning the operational semantics is defined by the way the similarity values are computed and the nearest neighbours are stepwise obtained. Hence we have the following steps:

(1)Enter candidates for the nearest neighbours and compute the similarity to the query object.

(2) Create the next set of nearest neighbour candidates.

Here the second step is the most essential one. It is given by the definition of the mathematical symbols used by the measure for computing sim(a,b) for given a and b.

The *denotational semantics* for similarity systems is more involved and essentially not developed. For programming languages one has to specify which function has to be computed; this is often done in terms of a fixed point semantics. This allows formulating a specification and is the basis of correctness proofs. In similarity systems it should describe the intended meaning of the measure and is related to questions like:

- Q1: What does sim(a,b) = 0.85 tell me and what does sim(a,b) > sim(a, c) mean?
- Q2: Why should I be interested in a nearest neighbour rather than in the 5^{th}-nearest neighbour?

The last question is crucial. In order to discuss it we consider the partial orderings introduced above. The observation is that $\geq_a$ is a preference relation. One way to express this in natural language is "b is better than c with respect to a $\Leftrightarrow$ $b >_a c$".

Preference relations are concepts in utility theory mwhich states that preference relation can be induced by a utility function u

$$u : U \rightarrow \Re.$$

The induced preference is:

$$\text{b is preferred over c} \Leftrightarrow u(b) > u(c).$$

Utility functions are also constructed according to the *local-global principle:*

$$u(a) = G(u_i(a_i) \mid i \in I)$$

where the u_i are local utilities and G is some constructor function.

Each similarity measure sim and each object $a \in U$ induce a utility function $u_{sim,a}(.)$ defined on the elements of V by setting

$$u_{sim,a}(b) := \text{sim}(a,b)$$

Now we take utility functions as the format for specifications of a measure. More concretely, the utility functions play the role of the specification for the nearest neighbour(s).

The situation for similarity systems is the same as in programming languages: For correctness of the similarity measure one has to prove that the program result meets the specification. One way to formulate this is "the operational semantics computes the fixed point specification". For similarity systems that means, if u describes the specification, then the correctness proof has to ensure

$$u_{sim,a}(.) = u(.) \text{ for each } a \in U.$$

A weaker correctness is if the induced relations coincide. (For the soundness of similarity measures see (Bergmann 2002).) It is sufficient to ensure that the nearest neighbours are in fact the "best" objects in V.

This answers the questions Q1and Q2 :

- sim(a,b) = 0.85 says that the utility of choosing b for given a is 0.85 in terms of utility units and sim(a,b) > sim(a, c) says that the utility of b is bigger than the one of c, for the object a.
- The nearest neighbour principle is a consequence of the *Maximum Expected Utility* principle (there may be only a probability distribution on the utilities).

A related question is: Given a language for measures (for instance, linear sums), which measures can be realized within the language?

Fixed points also occur here. For a given query object a we call it a fixed point if NN(a,a) holds; this is always the case if the similarity measure is symmetric. Of course, fixed points can only exist for $V \subseteq U$ because otherwise the answer object cannot be a query object again. Then we obtain:

If NN(a,b) holds then b is a fixed point.

In electronic commerce this means that if b is a best available product for a query product a, and b is used as a new query, then b is returned again. In fact, the nearest neighbour search is a stepwise approximation of the fixed point(s). However, the computed fixed point may not meet the specification.

The formal correctness relies on a formal specification. In this context, it means that a formal specification for the utility is given. In order to perform correctness proofs one needs a deductive calculus like the standard *Hoare calculus*. The principal way is to use the local-global principle that is valid for similarity measures as well as for utilities.

Challenge:

Develop a calculus that decides correctness in the context of similarity systems, at least in principle.

As in programming languages it is often unclear or difficult to check whether a formal specification reflects truly what the user wants. We will discuss this next. A guideline to write correct programs is that both similarity measures and utilities are built using the local-global principle. That allows performing similarity assessment in a close relation to the utility. This means, the constructor functions for the measures should be parallel to the constructor functions of utility functions.

3. Similarities, Types of Utilities and their Problems

The types we consider are ordered by the level of exactness in which they are formulated. We start with precise mathematical concepts that can be the basis for a formal specification and then turn to weaker notions. The reduction of the correctness problem to looking at utilities allows for model-based utilities a formal treatment, at least in principle. For informal utilities this is no longer possible in the strict sense. But this view, due mainly to a systematic use of the local-global principle, allows also to establish some kind of weak correctness that is satisfactory in many applications.

For this purpose we distinguish four basic types of utilities and similarities:

- Model-based
- Subjective
- Mixed
- Mixed and dynamic, i.e.wicked problems

The examples presented in Section 5 will illustrate the problems connected with these types.

3.1 Model-Based

The simplest situation is the deterministic one where all influence factors are completely known and the utility can be exactly computed. Unfortunately, this ideal situation occurs rarely. The first assumption given up is the deterministic view which is replaced by a probabilistic approach. This is mathematically described in the *von Neumann-Morgenstern theory*. It uses expected utilities and has a clear axiomatic foundation in (von Neumann and Morgenstern 1944).

A difficulty occurring regularly in practice is that the utilities and their associated probabilities are partially unknown.

3.2 Subjective

In the von Neumann-Morgenstern theory probabilities were assumed to be "objective". In this respect, they followed the "classical" view that randomness and probabilities, in a sense, "exist" inherently in nature. There are great deficiencies in the classical approach and it has been questioned in various directions. As a consequence, subjective probabilities and utilities have been created. It has to be observed here that "subjective" should not be confused with "irrational".

A subjective probability describes an individual's personal judgement about how likely a particular event is to occur. It is not based on any precise computation, but is often a reasonable assessment by a knowledgeable person.

Axioms for subjective expected utility have been given in (Savage 1954), which was a highly influential book. The rational behaviour of humans is expressed in the equation

$$\text{subjective value} = \text{subjective probability} \times \text{subjective utility},$$

where the subjective value has to be maximized. There is a huge literature about the subject; see, for instance, (Anscomb, Aumann 1963). The axiomatization given there is somewhat in between the ones of v.Neumann-Morgenstern and Savage. This means, it has more mathematical elements than Savage has but is not purely mathematical.

Subjectivity is, however, not restricted to the likelihood of events. Utilities as a whole, preferences, interpretation of images and music, and even the interpretation of concepts may be subjective. An early attempt concerning subjective similarity of impression by groups of humans was described in (Künnapas and Künnapas 1974).

3.3 Mixed

Mixed problems occur if the constructor function has to combine two or more local utilities of a heterogeneous nature. This is in particular difficult if one has to integrate model-based as well as subjective components. Often the data structures are not even compatible. To make things worse, utilities may compete and change dynamically.

When combining subjective preferences and similarities the constructor function has also a subjective character.

3.4 Wicked Problems

In mixed problems we often encounter problems, where the utilities come from different stakeholders and can in addition be changed, i.e., they are dynamic. They occur in wicked problems. Although wicked problems don't have a precise definition, they have several characteristics (Rittel, Webber 1984). Some major ones are:

- The problems are of a mixed nature and contain model-based, subjective, and context-dependent elements. One has different participants (called stakeholders) with different preferences and this makes it problematic to judge the quality of the solution.
- The influence factors for the utility and hence for the similarity measure are difficult to determine and the quality of a solution can often be judged only a posteriori.
- The stakeholders may change their preferences, i.e. they are dynamic.
- The combination of different subjective utilities has in addition the problem that it is not clear what the amalgamated utility should look like. The optimal utility depends on a compromise that becomes only clear a posteriori.

That means we encounter situations where the task of dealing with mixed problems is taken to an extreme.

4. Exactness and Approximation

It is well known that similarity reasoning is not exact. But what does this precisely mean?

The operational semantics is always exact. What is inexact is the specification; this often informal. CBR has developed techniques to deal with such situations. The utility, representing the specification, is primary, and the similarity is thought as an a priori estimate. If the utility is completely unknown then the problem of finding the nearest neighbour is unsolvable. A partially known utility can be regarded as an approximation of an unknown utility which results in an approximation.

A crucial function of the similarity measure is, to control the error. This is closely related to questions Q1 and Q2 in Section 2.3.

5. Application Examples

The application examples follow the formal-informal types for utilities outlined in Section 3 and indicate how these abstract aspects are reflected in reality. For each application we will name challenges and relate them to Section 3. These name problems, where one has no or only partial and unsatisfactory methods for defining similarities and applying CBR.

5.1 Risk Analysis

We start with a mixed problem. In the context of risk analysis for investment planning one wants to avoid or minimize the situation in which several assets of a portfolio go down drastically at the same time, i.e., they should not be concordant with respect to going down. The essential part of the analysis is to compare two shares of a portfolio, and we will concentrate on that.

One knowledge source is implicit and hidden in the numerical data. This is a problem of statistics. On the other hand, there are several symbolic data of importance where no statistical data may exist. Several experts have emphasized that qualitative, i.e., symbolic attributes are also important for describing risks. Such data may be the type of the company, political stability or type of the products sold.

We first define the concordance notion. Let (x^T, y^T) and $(\underline{x}^T, \underline{y}^T)$ be two observations of the continuous random variables (X^T, Y^T).

Def.: (x^T, y^T) and $(\underline{x}^T, \underline{y}^T)$ are called

- *concordant* if $(x - \underline{x})(y - \underline{y}) > 0$
- *discordant* if $(x - \underline{x})(y - \underline{y}) < 0$.

This has a probabilistic version (see (Embrechts et al. 2003)), where one considers independent vectors (X^T, Y^T) and $(\underline{X}^T, \underline{Y}^T)$ of continuous random variables. Between these two vectors the probability of concordance is

$$\mathrm{Prob}((X - \underline{X})(Y - \underline{Y}) > 0)$$

and the probability of discordance is

$$\mathrm{Prob}((X - \underline{X})(Y - \underline{Y}) < 0).$$

The order difference is $Q = \mathrm{Prob}((X - \underline{X})(Y - \underline{Y}) > 0) - \mathrm{Prob}((X - \underline{X})(Y - \underline{Y}) < 0)$.

This give rise to a similarity measure called the *measure of* *concordance* between random variables X and Y.

In statistics a distribution combining marginal distributions that respects dependencies can always (under some very weak assumptions) be modeled with constructors of a special form called *copulas*.

A simple example is the measure called *Kendall`s tau* (the order difference from above); see (Embrechts et al 2003). Let X and Y be standard exponential variables and suppose ($\underline{X}$,$\underline{Y}$) is an independent copy of (X,Y). Let C(u,v) and $\underline{C}(u,v)$ be the copulas of (X,Y) and ($\underline{X}$,$\underline{Y}$), resp. This means, if F and G are the common margins of X and $\underline{X}$ and Y and $\underline{Y}$ resp., then their joint distributions are H(x,y)) = C(F(x), G(y)) and $\underline{H}$(x,y) = $\underline{C}$(F(x), G(y)), resp.

We then define the τ – measure as τ (X, Y) : Q :=

$$\mathrm{Prob}((X - \underline{X})(Y - \underline{Y}) > 0) - \mathrm{Prob}((X - \underline{X})(Y - \underline{Y}) < 0).$$

Hence τ measures the difference of the probabilities for concordance and disconcordance, which is intuitively clear for risk analysis. We remark that t as concordance measures in general are copulas. The measure can be computed according to the following formula:

$$\tau(X, Y) = Q(C, \underline{C}) = 4 \times \iint \underline{C}(u,v)\,dC(u,v) - 1$$

where the integral is taken over $(0, 1)^2$. The factor 4 is due to the fact that one considers all four quadrants because the range of the measure is (-1, 1) instead of (0,1).

For the risk, concordance should be avoided. The risk in this sense often also depends on non-statistical, symbolic attributes for which no statistical evidence is available but that also contribute to concordance.

Example attributes are:

- Company type; range = {steel, military, energy, tourism}. For the local similarity sim_{CT} with respect to concordance it is reasonable to assume sim_{CT} (steel, tourism) < sim_{CT} (steel, military).
- Economical structure of the country. If, for instance, one company is likely to improve on certain events, a company in another country may go down because of the same event.

These attributes have to be associated with weights. The local measures and the weights reflect subjective views on domain knowledge, while in the same way a distribution functions represent objective statistical knowledge.

Challenges: To model dependencies is always a problem. The situation here is particularly difficult: The dependencies in the symbolic part can be handled with standard CBR techniques and the dependencies in the statistical part are handled by the concordance measure. But how should dependencies between some symbolic attributes and some distributions be handled? The latter are not visible anymore; they are encapsulated in the measure. Thus, this is an example of a mixed problem that has two parts. One part is encapsulated in a measure and one has no access to its components. This seems to be a general problem for mixed problems.

5.2 Speech Recognition

This is again a mixed problem. It occurs in an ongoing project in assisted living ((Spracherkennung 2006)) where the task is to support handicapped persons who have problems with moving and operating tools in the room in which they live. Wishes are expressed by spoken commands and the ultimate goal is that the intended command is carried out. This is cost sensitive because some errors do create a danger for the person. In addition, dialects and the enunciation of older persons have to be respected. Although there are not many commands, they can be formulated in many different ways that cannot all be foreseen in detail. In contrast to other approaches we will apply not only Fourier transforms but also wavelets (Simon 2006). In principle we will encounter two similarity measures:

- The phonetic distance
- The symbolic distance.

Both measures contain subjective and model-based aspects. The phonetic distance relates spoken words and stored words. Here words are considered as spoken units that are not segmented. An overview over phonetic distances is given in (Heeringa 2004).

A crucial technical term is *formant***.** It is a peak in an acoustic frequency spectrum that results from the resonant frequencies of some acoustical system. Formants are the distinguishing frequency components of speech, in particular to identify vowels. They move about in a range of approximately 1000 Hz. The formants f_i are ordered by their frequencies; usually 3 formants suffice for a vowel.

For digitalization *linear predictive coding* (LPC) is presently used, see (Park 2006). It ultimately generates a *frequency* response as seen in Figure 1. This is then approximated by cubic splines, leading to some characteristic curves.

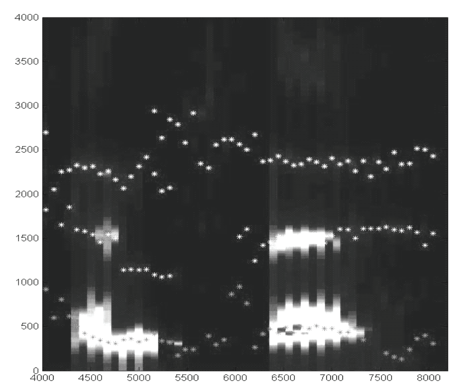

Figure 1. Spectrogram of hi-lf-e

As an example we consider the word *hilfe* (i.e., *help*). It has the first three formants for hi-lf-e and the spectrogram (that visualizes the formants).

The approximation with cubic splines for the first three formants is shown in figure 2.

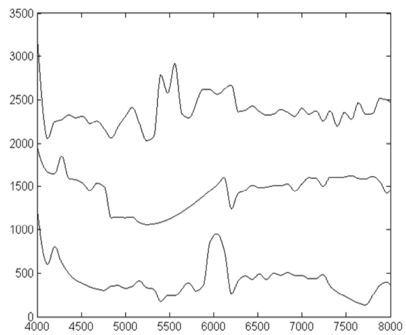

Figure 2. Three formants

These curves are compared with the reference curves.

Although one can identify some information about the human mouth from the curve, the arguments of the phonetic distances are “meaningless”, i.e., they are not interpreted with respect to the content of the commands. In contrast to that, the main purpose of the symbolic measures is just to respect the content of the command. In particular, sometimes the command cannot or should not be executed. There are two reasons for that:

1) It is for some reason impossible
2) The execution seems to be too dangerous.

For this possibility a new command is created, a question back to the user. Abstractly this means the nearest neighbour search is interrupted by a user interaction.

Challenges: In addition to the problems of the previous example we face the fact that the phonetic part encapsulates the influences. Therefore the dependencies are again hidden. Furthermore, the encapsulated part has no knowledge about second part that concerns costs. Thus, any kind of interaction between the two phases is impossible The two measures are not only of heterogeneous character, there is also some kind of temporal ordering involved: One first looks at the phonetic and then at the symbolic distance. How to treat that systematically? Is the integration into one measure better?

This shows that mixed problems have not only concern relation of model-based and subjective aspects but also that there is some kind of ordering between the components of the measure.

5.3 Urban Planning

Urban planning belongs to the earliest types of wicked planning and made this term popular (see (Rittel, Webber 1984)). In urban planning very many stakeholders are participating who have different preferences, goals, and knowledge. There are three major types:

- Planners (usually architects, technicians, and civil engineers).
- Local authorities, railway companies, churches, and business organizations. They have in addition subjective preferences.
- Citizens. They have mostly subjective preferences and goals.

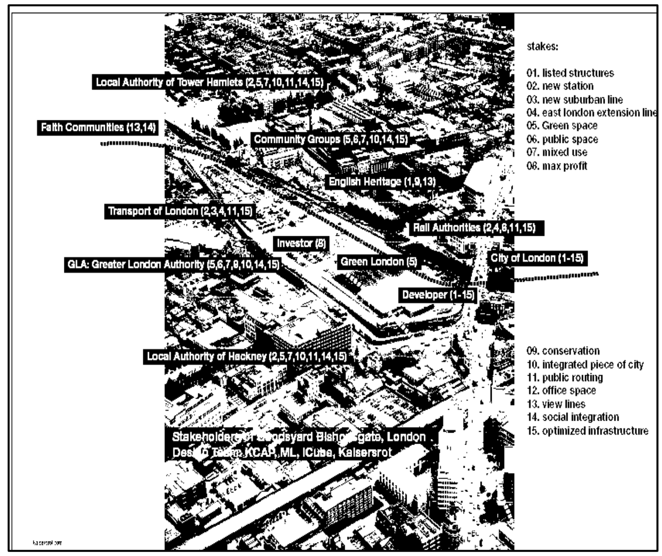

Figure 3. Some stakeholders in an urban planning project

Figure 3 shows a number of stakeholders in a very large project that is concerned with bridging two parts of the city of London (Bishopsgate) (see (Christiansen et al. 2006)).

This is a mixed problem of a very difficult nature. In addition to the partially unknown, dynamic, and conflicting involved utilities it is heavily dependent on the context. Suppose, for example, that there are two identical planning problems in different cities, where the structures of the cities are identical and stakeholders are of the same type. Then it may nevertheless be difficult to transfer an experience from one city to the other one because of different contexts.

Presently it seems to be impossible to make use of previous city plans as a whole. The chances increase if the problem is decomposed into smaller parts. As examples we mention specific constructions that have a static character, search for documents, and answers to objections. Here CBR can provide direct solutions to problems. Some first steps have been performed in (Müller et al 2006).

If two utilities are competing then this looks at the first glance as a multi-criteria decision problem. This view, however, neglects that the utilities are dynamic, i.e., the stakeholders may change their subjective utilities. Consequently, we do not face a decision problem but rather a problem of finding a compromise. The principal idea to employ similarity measures for this purpose has been described in (Du et al. 2006). It starts with the observation that similarity measures have some explanatory character. This is contained in the different weights that stakeholders assign to attributes. If one compares the measures sim^1 and sim^2 of two stakeholders, then one can extract those attributes, where the difference $(w^1_i \cdot sim^1_i(a_i , b_i) - (w^2_i \cdot sim^2_i(a_i , b_i)$ is large. This indicates that stakeholders' opinions differ substantially. These attributes can be the starting point of a focused discussion in order to achieve a compromise.

Challenges: As indicated in Section 3, wicked problems often do not allow a systematic treatment; this applies to the use of CBR as well. The example shows that the presence of heterogeneous stakeholders creates more than problem that has to be solved. The dynamic character of the preferences creates the necessity of a discussion for finding a compromise. How this can be supported by using experiences remains largely unclear.

6. Conclusion

We presented a foundation of similarity reasoning on the basis of utility theory. We considered different types of utilities, ranging from formal mathematical theories to various degrees of informal formulations. These covered subjective expected utilities, competing, and utilities that change dynamically.

A basic difference between model-based and subjective constraints is that the former are unavoidable while the latter ones can be revised, at least in principle. Hence a general challenge remains: Under what conditions is it advisable to separate model-based and subjective aspects of the measure? Another general challenge is to cope with dynamic and interactive systems. How can we make systematic use of experiences?

Let us leave with a final **challenge**: Develop for each utility at least a "standard recipe" that would allow to consider similarity assessment not as a piece of art but an engineering task.

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