Multi-Prototype Concept and Object Typicality in Ontology

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Abstract
Concept representation in ontology is a basic problem on the Semantic Web. In human cognitive process, object typicality plays an important role in concept representation. Traditional ontologies cannot reflect the typicality of objects in concepts. In this paper, we present a cognitive model of ontology with multi-prototype concept and object typicality. It can better approximate to human’s cognition than other models known to us.

Introduction
The development of the Semantic Web requires describing and structuring the web information using ontologies which can be understood by machines and human. An ontology is generally defined as an explicit specification of conceptualization and can be used to provide semantics to resources on the Semantic Web (Staab & Studer 2004).

Traditional ontologies represent concepts as crisp sets of objects (Staab & Studer 2004). Objects are considered either to belong to or not to belong to a concept. There is no ‘better’ or ‘worse’ instance of a concept in traditional ontologies. However, cognitive psychologists find that some objects are more ‘easier’ to be considered as exemplars (representative examples) of a concept in our daily life. For example, people generally consider a sparrow as a better example of the concept ‘bird’ than a penguin. The concept representation in current ontology lacks a formal method to handle the degrees of goodness of objects as exemplars of concepts.

In the study of cognitive psychology, typicality (Galotti. 2004) refers to the goodness degree of objects as exemplars in concepts. People would consider some objects to be more typical than others in a concept. When asked to give examples of a concept, people are more likely to give typical instances than atypical ones as examples (Murphy 2002). For example, at most times, while a user retrieves the information about clothing, the information of pants and shirts is considered to be more relevant than that of belts and gloves. That is because pants and shirts are more typical than belts and gloves in people’s mind (Barsalou 1985). Measure of typicality provides a mechanism to rank the individuals in a way that is closer to human thinking and their psychological belief (Au Yeung & Leung 2006). For the rifeness of object typicality in concepts, it is desirable to represent the object typicality in concepts in ontologies so that ontologies can indicate different degree of goodness and importance of different objects.

In this paper, we propose a new formal cognitive model of ontology with multi-prototype concept and object typicality. The model is based on the theories in cognitive psychology and extends current ontologies to reflect the object typicality in concepts. It can outperform our previous models, and make the object typicality and concept representation be modeled more accurately and appropriately. We argue that our model approximates human’s judging on object typicality well because it is derived from the theories in cognitive psychology.

Background and Related Work
Views of Concepts and Typicality in Cognitive Psychology

In cognitive psychology, the classical view of concepts posits that each concept is defined by a set of properties which are individually necessary and collectively sufficient (Murphy 2002). It was the dominant view in psychology until Rosch found that people judged that different members of a concept vary in ‘goodness’ of objects as exemplars in the concept (Murphy 2002). Cognitive psychologists consider typicality to be a measure of the goodness degree of objects as exemplars in a concept. Object typicality depends on properties shared by most of the objects of the concept, which usually include non-necessary properties of the concept (Galotti. 2004).

Barsalou (Barsalou 1985) measures two factors named central tendency and frequency of instantiation which affect the object typicality in a concept, and he also provides evidence for them. Central tendency is considered as the degree of how close an object is to a prototype. The affectation of central tendency on typicality is that the objects which are more similar to the prototypes in the concept are more typical. Frequency of instantiation of a cluster of similar objects is an estimate of how often one has experienced objects in the cluster as members of a particular category (concept) or how often one has considered objects in the cluster as mem-
numbers of a particular category. Objects of a cluster with higher frequency of instantiation in a concept are more familiar to people, and thus be considered more typical in the concept. Different people with different ‘experience’ with objects in a concept, and they would give different weights to frequency of instantiation of prototypes (Murphy 2002).

The studies of typicality have motivated the development of other views (Galotti, 2004). In the prototype view (Medin & Smith 1984), a concept is represented by a best prototype or a property list (i.e. feature list) which has all the salient properties of the objects that are classified to this concept. An instance is considered more typical in a concept while it is more similar to the prototype of the concept. In the exemplar view (Smith & Medin 1981), a concept is represented by a set of exemplars (some particular objects in people’s memory), and an object is more typical if it is more similar to as many exemplars of the concept.

Vanpaemel et al. (Vanpaemel, Storms, & Ons 2005) proposes a model combining both the prototype view and the exemplar view. They consider a concept that can be represented by some abstractions deduced from exemplars of the concept. Each abstraction can be considered as an abstract prototype of the concept. For an object, it is considered to be an instantiation of an abstraction that is most similar to it. Object typicality is affected by the matching of properties of the object and properties of the abstraction that is most similar to it. Vanpaemel et al. show that both prototype model and exemplar model are the special cases of the model they proposed, and such a combining model outperforms prototype model and exemplar model.

Previous Work

Rifqi (Rifqi 1996) proposes a method to calculate object typicality in large databases, which is later extended by Lesot (Lesot, Mouillet, & Bouchon-Meunier 2005). Their methods are in line with the exemplar view in cognitive psychology. In their works, the object typicality of a member for a category that it belongs to depends on its resemblance to the other members of the category, and its dissimilarity to members of other categories. Besides, Jean-Pierre Descles and Anca Pascu (Descles & Pascu 2006) introduce object typicality in yielding new quantifiers for natural language processing and natural inferences common reasoning.

Recently, Au Yeung and Leung (Au Yeung & Leung 2006) have formalized object typicality by constructing several vectors in ontologies. Their work is based on single prototype model in cognitive psychology. They consider that a concept $r$ can be defined by a characteristic vector $\vec{c}_{r}$ of $r$ which consists all the necessary properties in $r$. An object $a$ can be represented by a property vector $\vec{p}_{a}$, and each element in $\vec{p}_{a}$ corresponds to the degree to which the object possesses a property. While calculating typicality of objects in $r$, they consider a concept $r$ to be represented by an unique prototype vector $\vec{T}_{r}$ consisting of all the necessary properties of sub-concepts of $r$. The typicality of an object in a concept is the degree of similarity matching between the object property vector and the prototype vector of the concept.

<table>
<thead>
<tr>
<th>A</th>
<th>travel-in-the-sky</th>
<th>B</th>
<th>travel-on-sea</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>have-engine</td>
<td>D</td>
<td>travel-on-the-rail</td>
</tr>
<tr>
<td>E</td>
<td>have-wings</td>
<td>F</td>
<td>have-four-wheels</td>
</tr>
<tr>
<td>G</td>
<td>travel-on-the-road</td>
<td>H</td>
<td>have-helm</td>
</tr>
<tr>
<td>I</td>
<td>have-locomotive</td>
<td>J</td>
<td>have-undercarriage</td>
</tr>
<tr>
<td>K</td>
<td>look-like-plane</td>
<td>L</td>
<td>look-like-train</td>
</tr>
<tr>
<td>M</td>
<td>look-like-car</td>
<td>N</td>
<td>look-like-boat</td>
</tr>
</tbody>
</table>

Table 1: Salient properties of the concept $r$

Limitations of Previous Models

A Motivating Example

Suppose that Bob plans to go to New York from San Francisco in next week for a conference. He wants to find some useful information about the typical transportation for getting from San Francisco to New York. Here, we denote the concept ‘Transportation for getting From San Francisco to New York’ by $r$.

We assume that the ‘experience’ of Bob about transportation is stored in a data set. In the concept $r$ of such a data set, forty percent objects are airplanes, thirty percent are cars, twenty percent are boats, eight percent are trains and two percent are amphibics. The order of frequency of instantiation of all kinds of transportation in the data set is as the same order as in the cognitive psychology experiment in (Barsalou 1985). Salient properties of concept $r$ are shown in table 1.1

Limitations of Au Yeung-Leung Model

In the Au Yeung-Leung model, concept $r$ is represented by a single prototype vector while calculating typicality of objects in $r$. The prototype vector of $r$ is constructed from the all necessary properties of sub-concepts of $r$, and properties $K$, $L$, $M$ and $N$ are not included in $\vec{T}_{r}$ of $r$.

$\vec{T}_{r} = (0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25)$

The prototype vector $\vec{T}_{r}$ is a properties list. The value of each property in $\vec{T}_{r}$ is the degree of the prototype possessing the property.

We consider three objects, which are a boat, a car and an amphibic. For an amphibic, it possesses the properties of both cars and boats. We denote an amphibic by $a'$, a boat by $b'$ and a car by $c'$. Let the property vectors of $a'$, $b'$, and $c'$ be:

$$a' : [B]_1 [C]_1 [F]_1 [G]_1 [H]_1 [M]_1$$
$$b' : [B]_1 [C]_1 [G]_1 [H]_1 [N]_1$$
$$c' : [C]_1 [F]_1 [G]_1 [M]_1$$

$$\vec{T}_{a'} = (1, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0)$$
$$\vec{T}_{b'} = (0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0)$$
$$\vec{T}_{c'} = (0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0)$$

1This example about concept $r$ is only for illustration, and we only consider the salient properties and omit some properties of instances of $r$ for interest of space.

2Their values are zero because $K$, $L$, $M$ and $N$ are not necessary properties of any sub-concepts of $r$. 


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The value of each property in object property vector indicates the degree to which the object possessing the property. According to the Au Yeung-Leung model, each object’s typicality depends on the similarity match between prototype vector of concept \( r \) and property vector of the object. We can obtain the following results by the axioms and equations in the Au Yeung-Leung model.

\[ \tau_r(a') = 0.6154, \tau_r(b') = 0.4615, \tau_r(c') = 0.4615 \]

Such a result is unreasonable and not consonant with most people’s intuition. Intuitively, most people would consider that an amphicar is less typical than a car and a boat.

From the motivating example, we can find that the Au Yeung-Leung model has several inadequacies.

Firstly, the frequency of instantiation are not taken into account in the Au Yeung-Leung model well. In the process of calculating the typicality of an amphicar, the frequency of instantiation of amphicars is not taken into account, but only the number of properties the amphicar shared with the prototype is taken into account in the Au Yeung-Leung model. Consequently, an amphicar is considered more typical than a car and a boat because it possesses more properties in the prototype than the other two. If we take frequency of instantiation into account, the typicality of amphicars will be decreased a lot because there is few amphicars in the world and it is not often for people coming across an amphicar.

Secondly, using a single prototype to represent a concept in Au Yeung-Leung model treats the relations among properties of the concept as conjunction only in the single prototype. Such a representation only can represent the conjunctive relations (the relations in which the relations among properties are conjunctive (Murphy 2002)). It has a limitation in representing the disjunction of properties in concepts, and lacks the building blocks to compute typicality for disjunctive concepts (the concepts in which the relations among properties are disjunctive (Murphy 2002)). Besides, only conjunctive relationship among all properties of a concept cannot keep the co-occurrence relations among properties of the concept (Murphy 2002). For example, properties ‘travel-on-sea’, ‘have-engine’, ‘have-helm’ and ‘look-like-boat’ should have a co-occurrence effect on boats, while properties ‘travel-on-the road’, ‘have-engine’, ‘have-four-wheels’ and ‘look-like-car’ should have a co-occurrence effect on cars and so forth.

Thirdly, in the Au Yeung-Leung model, the prototype vector of a concept is constructed by characteristic vectors of its sub-concepts. The single prototype of a concept may miss some salient properties (properties shared by most instances of a concept) since the characteristic vectors of sub-concepts only include necessary properties. For example, ‘look-like-cars’ is not a necessary property in any sub-concepts of concept ‘car’ but it is a salient property of ‘car’ obviously.

Limitations of Lesot’s Model

Since Lesot’s work mainly focuses on an algorithm for calculating object typicality in database instead of a formal model for object typicality in ontology on which we focus, so we discuss its limitations briefly.

For objects \( a' \), \( b' \), and \( c' \) in section 3.2, their typicality values depend on their resemblance to the other members of \( r \), and their dissimilarity to members of other categories in Lesot’s model. For object \( b' \), it shares properties \( B, C, H \), and \( N \) with boats and there are twenty percent objects belong to boats in the data set. For object \( c' \), it shares properties \( C, F, G, M \) with cars and there are thirty percent objects belong to cars. For object \( a' \), it shares properties \( B, C, H \) with all boat objects, and \( N, F, G, M \) with all car objects, and all these six properties with all amphicar objects. Therefore, we can obtain the following result from Lesot’s model.

\[ \tau_r(a') > \tau_r(c') > \tau_r(b') \]

Such a result obtained from Lesot’s model is also not intuitive and appropriate. Lesot’s model also do not take the frequency of instantiation into account well. Besides, it cannot keep the co-occurrence relations mentioned before. The calculation of one object’s typicality need to compare the object to all other objects in the domain. Therefore, the calculation complexity will be higher than prototype-based model. Furthermore, Lesot’s work lacks a formal model for object typicality in ontology.

A New Formal Cognitive Model for Ontology

In this section, we propose a novel formal model using multiple prototypes instead of a single prototype to represent a concept based on the study of cognitive psychology.

Representation of Concepts and Objects in Ontologies

We consider an ontology \( O \) in a domain \( \Delta \) as follows:

\[ O_\Delta = (C, P, I) \]

where \( C \) is a set of concepts, \( P \) is a set of independent properties of concepts, and \( I \) is a set of objects. For a concept \( r \) in \( O \), it possesses a set of properties denoted by \( P_r \); and there is a set of objects denoted by \( I_r \) belonging to \( r \). The instances set \( I_r \) of concept \( r \) can be clustered into several groups (clusters) of similar instances (Barsalou 1985), i.e.,

\[ I_r = (I_{r,1}, I_{r,2}, \ldots, I_{r,n}) \]

where \( n \) is the number of groups of similar instances, and \( I_{r,i} (1 \leq i \leq n) \) is \( i \)th group of similar instances of \( r \).

Inspired by the work of cognitive psychology, we consider that a prototype of a concept is an abstract representative object of a group of similar instances and can be represented by a property vector of the prototype, i.e. prototype \( j \) is an abstract representative object of \( I_{r,j} \), which is extracted from instances of \( j \)th group of similar instances. In the process of calculating object typicality in concept \( r \), \( r \) is represented by a prototype list which consists of a set of

\[ \{a', b', c'\} \]

Please refer to (Xu & Wunsch 2005) for more details about clustering, and we will not discuss it more because it is not the focus of this paper.

Generally, the representative object of a cluster is considered as the mean or the median or the mode of all objects in the cluster (Lesot, Mouillet, & Bouchon-Meunier 2005) (Murphy 2002).
property vectors of prototypes. An object is represented by a property vector of object and the vector consists of properties which the object possesses. The formal definitions of object property vector, prototype property vector and prototype list are as follows.

**Definition 1.** The object property vector \( \overrightarrow{p}_a \) of an object \( a \) is a vector of property:value pairs.

\[
\overrightarrow{p}_a = (p_{a,1} : v_{a,1}, p_{a,2} : v_{a,2}, \cdots, p_{a,k} : v_{a,k})
\]

where \( k \) is the number of properties of the object in the domain, and \( v_{a,i} \) is a real number between 0 and 1 which indicates the fuzzy degree to which object \( a \) possesses the property \( p_{a,i} \).

**Definition 2.** Each prototype property vector \( \overrightarrow{t}_{c,j} \) of a prototype \( j \) in concept \( c \) is a vector of property:value pairs.

\[
\overrightarrow{t}_{c,j} = (p_{c,j,1}^c : v_{c,j,1}^c, p_{c,j,2}^c : v_{c,j,2}^c, \cdots, p_{c,j,m}^c : v_{c,j,m}^c)
\]

where \( m \) is the number of properties of the prototype \( j \) of concept \( c \), and \( v_{c,j,i}^c \) is a real number between 0 and 1 which indicates the fuzzy degree to which the prototype \( j \) of concept \( c \) possesses the property \( p_{c,j,i}^c \).

**Definition 3.** The prototype list \( \overrightarrow{T}_c \) of concept \( c \) consists of a set of prototype property vectors.

\[
\overrightarrow{T}_c = (\overrightarrow{T}_{c,1}, \overrightarrow{T}_{c,2}, \cdots, \overrightarrow{T}_{c,n})
\]

where \( n \) is the number of the prototypes of concept \( c \). Each element \( \overrightarrow{T}_{c,i} \) in \( \overrightarrow{T}_c \) is a property vector of prototype of concept \( c \).

**Similarity and Dissimilarity Measurement Between Objects and Prototypes**

In our model, the similarity between a prototype of a concept \( c \) and an object \( a \) is calculated by a function:

\[
sim : P \times T \rightarrow [0, 1]
\]

where \( T \) is the set of all prototype vectors of a concept \( c \), and \( P \) is the set of all object property vectors. For the dissimilarity between a prototype \( i \) of a concept \( c \) and an object \( a \) in our model, we define it as the complement of similarity.

\[
\text{dissimilar}(\overrightarrow{p}_a, \overrightarrow{T}_{c,i}) = 1 - \sim(p_a, T_{c,i})
\]

The output of the \( \sim \) function is a real number in the range of \([0,1]\). It means that the object \( a \) is identical with the prototype \( i \) of concept \( c \) when \( \sim(\overrightarrow{T}_{c,i}, \overrightarrow{p}_a) = 1 \). If \( \sim(\overrightarrow{T}_{c,i}, \overrightarrow{p}_a) = 0 \), then it means that the object \( a \) is not similar to the prototype \( i \) of concept \( c \) at all.

Similarity of two objects is derived from their degree of distance from them through decreasing functions (Santini & Jain 1995). Distance between an object and a prototype depends on matching of corresponding properties of them in our model. The choices of similarity functions and distance functions depend on applications. There have been some methods proposed to calculate the similarity between two objects in previous works. More details are discussed in (Santini & Jain 1995).

Here, we present an possible similarity function to measure the similarity between an object and a prototype of a concept.

\[
sim(T_{r,j}, \overrightarrow{p}_a) = \{ \exp(-\sqrt{\sum_{i=1}^{n} (v_{j,i}^r - v_{a,i})^2}) \text{ for } \forall i, \exists v_{a,i} = 0 \}
\]

Where \( n \) is the number of properties of the prototype \( T_{r,j} \) and \( \overrightarrow{p}_a \), and \( \theta \) is a positive parameter which can adjust the rate of decay of the exponential function to make the function fit to different applications. \( \sqrt{\sum_{i=1}^{n} (v_{j,i}^r - v_{a,i})^2} \) is the Euclidean distance between \( \overrightarrow{p}_a \) and \( T_{r,j} \). The exponential function has the effect that objects are extremely close to the prototype have a large effect of typicality, and the effect falls off very quickly as they become moderately and less similar (Murphy 2002).

**Modeling Influencing Factors of Typicality**

**Central Tendency**  In our model, among prototypes of a concept \( c \), the one which is the most similar to an object \( a \) is named the most similar prototype of \( a \) for \( c \).

**Definition 4.** The most similar prototype \( s \) of object \( a \) for concept \( c \) is a prototype in the prototype list of \( c \) which satisfies the following condition:

\[
sim(\overrightarrow{T}_{c,s}, \overrightarrow{p}_a) = \max\{\sim(\overrightarrow{T}_{c,1}, \overrightarrow{p}_a), \ldots, \sim(\overrightarrow{T}_{c,n}, \overrightarrow{p}_a)\}
\]

where \( \overrightarrow{T}_{c,s} \) is the property vector of prototype \( s \).

According to (Barsalou 1985), the central tendency of an object to a concept is affected by degree of internal similarity and external dissimilarity. Internal similarity is the similarity of the object and its most similar prototype \( s \) for the concept. The internal similarity of object \( a \) for concept \( c \) denoted by \( \beta(\overrightarrow{p}_a, \overrightarrow{T}_c) \) is determined as following:

\[
\beta(\overrightarrow{p}_a, \overrightarrow{T}_c) = \sim(\overrightarrow{p}_a, \overrightarrow{T}_{c,s})
\]

In our model, the external dissimilarity \( \delta(\overrightarrow{p}_a, \overrightarrow{T}_c) \) is considered as the average of dissimilarities of the object and other concepts excluding \( c \).

\[
\delta(\overrightarrow{p}_a, \overrightarrow{T}_c) = \sum_{s \neq c} \text{dissimilar}(\overrightarrow{p}_a, \overrightarrow{T}_{c,s}) / N_{\Delta} - 1
\]

where \( s \) is the most similar prototype of object \( a \) for concept \( c \). \( N_{\Delta} \) is the number of concepts in domain \( \Delta \).

The central tendency of an object \( a \) to concept \( c \) denoted by \( \alpha(\overrightarrow{p}_a, \overrightarrow{T}_c) \) is defined as an aggregation of internal similarity and external dissimilarity of \( a \) for \( c \).

\[
\alpha(\overrightarrow{p}_a, \overrightarrow{T}_c) = \gamma(\beta(\overrightarrow{p}_a, \overrightarrow{T}_c), \delta(\overrightarrow{p}_a, \overrightarrow{T}_c))
\]

where \( \gamma \) is an aggregation function used to combing the effects of internal similarity and external dissimilarity.

**Axiom 1.** For a concept \( c \) and an object \( a \), if \( \beta(\overrightarrow{p}_a, \overrightarrow{T}_c) = 0 \), then \( \alpha(\overrightarrow{p}_a, \overrightarrow{T}_c) = 0 \).

**Axiom 2.** For a concept \( c \) and an object \( a \), if \( \beta(\overrightarrow{p}_a, \overrightarrow{T}_c) = 1 \) and \( \delta(\overrightarrow{p}_a, \overrightarrow{T}_c) = 1 \), then \( \alpha(\overrightarrow{p}_a, \overrightarrow{T}_c) = 1 \).

**Axiom 3.** For a concept \( c \), two objects \( a \) and \( b \),

- If \( \delta(\overrightarrow{p}_a, \overrightarrow{T}_c) = \delta(\overrightarrow{p}_b, \overrightarrow{T}_c) \) and \( \beta(\overrightarrow{p}_a, \overrightarrow{T}_c) > \beta(\overrightarrow{p}_b, \overrightarrow{T}_c) \), then \( \alpha(\overrightarrow{p}_a, \overrightarrow{T}_c) > \alpha(\overrightarrow{p}_b, \overrightarrow{T}_c) \).
If the frequency of instantiation of similar instances denoted by \(c\) is equal to 0, it means that the object \(a\) is not similar to \(c\) at all, the central tendency of \(a\) to \(c\) should be zero no matter how dissimilarities between \(a\) and other concepts in the domain are. When the internal similarity and external dissimilarity of object \(a\) to concept \(c\) both are equal to 1, it means that object \(a\) is similar to its most similar prototype for concept \(c\) only and the central tendency of \(a\) to \(c\) should be highest.

Axioms 1 and 4 specify the boundary cases of the degrees of central tendency. When the internal similarity of object \(a\) to concept \(c\) is equal to 0, it means that the object \(a\) is not similar to \(c\) at all, the central tendency of \(a\) to \(c\) should be zero no matter how dissimilarities between \(a\) and other concepts in the domain are.

The aggregation function \(\gamma\) is application-based (Lesot, Mouillet, & Bouchon-Meunier 2005). The choice of aggregation function \(\gamma\) in a specific application is not in the scope of this paper.\(^5\)

Here we present a possible function to aggregate the internal similarity and external dissimilarity for central tendency as an example.\(^6\)

\[
\alpha(p_a, t_c) = \beta(p_a, t_c) \cdot \delta(p_a, t_c)
\]

**Frequency of Instantiation** If the \(j\)th group of similar instances denoted by \(I_{r,j}\) of a concept has more instances, people will be more familiar with the prototype abstracted from \(I_{r,j}\) and consider the prototype to be more salient than others (Barsalou 1985). We define a prototype salience vector to indicate the frequency of instantiation of each group of similar instances.

**Definition 5.** The prototype salience vector \(\overrightarrow{w}_c\) of concept \(c\) is a vector of real numbers,

\[
\overrightarrow{w}_c = \{w_{c,1}, w_{c,2}, \ldots, w_{c,n}\}, 0 < w_{c,i} \leq 1
\]

where \(w_{c,i} = \frac{N_i}{\sum_{k=1}^{n} N_k} \) and \(\sum_{i=1}^{n} w_{c,i} = 1\)

Each element in the vector corresponds to the frequency of instantiation of the corresponding cluster in concept \(c\) and indicates the relative salience of the corresponding prototype. The value of each element will not be zero because each cluster should include at least one instance belonging to the cluster.

\(^5\) Please refer to (Yager 1988) for more detail.

\(^6\) It is easy to verify that this aggregation function is in line with the axioms 1 to 4. For the interest of space, we omit verifications of all functions in the whole paper.

**Measuring Typicality**

In our model, the typicality \(\tau_c(a)\) of an object \(a\) in concept \(c\) is given by:

\[
\tau_c(a) = \phi(w_{c,s}, \alpha(p_a, t_c))
\]

\(w_{c,s}\) is the element corresponding to the most similar prototype \(s\) of \(a\) for concept \(c\) in the prototype salience vector \(\overrightarrow{w}_c\), and it indicates the affection of the frequency of instantiation. \(\alpha(p_a, t_c)\) indicates the affection of central tendency. \(\phi\) is an aggregation to combine the central tendency and frequency of instantiation. \(\tau_c(a)\) is in the range of \([0,1]\). The higher value of \(\tau_c(a)\), the more typical the object \(a\) in concept \(c\) is.

We formulate some axioms to be observed for the calculation of object typicality \(a\) in concept \(c\).

**Axiom 5.** For a concept \(c\), an object \(a\), if \(s\) is the most similar prototype of object \(a\) for concept \(c\), and \(\alpha(p_a, t_c) = 0\), then \(\tau_c(a) = 0\).

**Axiom 6.** For a concept \(c\), an object \(a\), suppose \(s\) is the most similar prototype of object \(a\) for concept \(c\), if \(\alpha(p_a, t_c) = 1\) and \(w_{c,s} = 1\), then \(\tau_c(a) = 1\).

**Axiom 7.** For a concept \(c\), two objects \(a\) and \(b\), if prototype \(x\) is the most similar prototype of object \(a\) for concept \(c\), and prototype \(y\) is the most similar prototype of object \(b\) for concept \(c\), then

- If \(w_{c,x} = w_{c,y}\) and \(\alpha(p_a, t_c) > \alpha(p_b, t_c)\), then \(\tau_c(a) > \tau_c(b)\).
- If \(w_{c,x} > w_{c,y}\) and \(\alpha(p_a, t_c) = \alpha(p_b, t_c)\), then \(\tau_c(a) > \tau_c(b)\).

**Axiom 8.** For two concepts \(c\) and \(d\), an object \(a\), let \(x\) be the most similar prototype of object \(a\) for concept \(c\) and let \(y\) be the most similar prototype of object \(a\) for concept \(d\), then

- If \(w_{c,x} > w_{d,y}\) and \(\alpha(p_a, t_c) = \alpha(p_a, t_d)\), then \(\tau_c(a) > \tau_d(a)\).
- If \(w_{c,x} = w_{d,y}\) and \(\alpha(p_a, t_c) > \alpha(p_a, t_d)\), then \(\tau_c(a) > \tau_d(a)\).

Axioms 5 and 6 specify the boundary cases of the degrees of typicality. If there is only one prototype \(s\) \((w_{c,s} = 1\) in concept \(c\) and the central tendency of \(a\) to \(c\) is the highest, then \(a\) is one of the most typical instances in concept \(c\). If the central tendency of \(a\) to \(c\) is zero, then it means the object is not similar to \(c\) at all and the typicality should be zero.

Axioms 7 and 8 specify the influence of central tendency and frequency of instantiation on typicality. According to (Murphy 2002), an object is considered as more typical if it possesses a higher value of central tendency and frequency of instantiation.

Here, we present a possible aggregation function for typicality which satisfies our axioms defined above.

\[
\tau_c(a) = \begin{cases} 
\frac{x \cdot w_{c,s} + y \cdot \alpha(p_a, t_c)}{x + y} > 0 & \alpha(p_a, t_c) > 0 \\
0 & \alpha(p_a, t_c) = 0
\end{cases}
\]

where \(x\) and \(y\) are defined by user and used to indicate the importance degree of frequency of instantiation and central tendency respectively.
Discussion

Let’s consider the example discussed in section 3 about the concept ‘Transportation for getting From San Francisco to New York’ denoted by r and objects a’, b’ and c’. Visiting the motivating example by our model shows that our model can handle the typicality of objects in concept r more appropriately and intuitively than our previous models.

For concept r, it is easy to use a k-means (k=5) clustering algorithm (Xu & Wunsch 2005) to get five clusters from a set of instance data of concept r. Each prototype is a representative center of a cluster. We can obtain a prototypes list \( T_r \) which including several prototypes in our model as following:

\[
T_{\text{airplane}} = (A : 1, C : 1, E : 1, J : 1, K : 1)
\]
\[
T_{\text{car}} = (C : 1, F : 1, G : 1, M : 1)
\]
\[
T_{\text{boat}} = (B : 1, C : 1, H : 1, N : 1)
\]
\[
T_{\text{train}} = (C : 1, D : 1, I : 1, L : 1)
\]
\[
T_{\text{amphicar}} = (B : 1, C : 1, F : 1, G : 1, H : 1, M : 1)
\]

In the motivating example, we obtain \( w_{\text{airplane}} = 0.4, w_{\text{car}} = 0.3, w_{\text{boat}} = 0.2, w_{\text{train}} = 0.08 \) and \( w_{\text{amphicar}} = 0.02 \) based on the object data distribution.

Then we measure the similarity between the objects and prototypes in concept \( r \) by equation 1 in which we set \( \theta = 0.5 \). In this example, object \( a', b' \) and \( c' \) are as the same as their corresponding the most similar prototypes in \( r \) (i.e., the internal similarities of them to \( r \) all are one), and the external dissimilarities of them to \( r \) all are one because there is only concept here. Besides, we set \( \tau_c = 0.36 \) and \( \tau_b = 0.71 \) in equation 3, \(^7\) then we can get the following result by axioms and equations in our model:

\[ \tau_r(c') > \tau_r(b') > \tau_r(a') \]

This corresponds to the result of phycological experiments (Barsalou 1985) and is more consistent with human’s cognition in our daily life.

We can summarize that our model has some advantages:

- Our model uses multiple prototypes to represent a concept. In our model, the relations among prototypes of a concept are conjunction, and the relations among properties of each prototype are conjunction. Such a representation can handle both the conjunctive and disjunctive concept, and keep the co-occurrence relations of properties of a prototype in a concept.

- In addition, we formalize two influencing factors of object typicality in concepts based on the study of cognitive psychology. These factors are not taken into account well in our previous models. Modeling these factors can make our model more approximate to human’s cognition than our previous models and let our model be in line with the theories in cognitive psychology.

- The formation of prototypes in our model is based on the clusters of similar instances and we take into account the salient properties. The prototype extracted from a cluster can be a representative of all instances in the cluster, and it keeps a majority of information of those instances.

Conclusions

In this paper, we present a cognitive model of ontologies with multi-prototype concept and object typicality which adopts ideas from cognitive psychology. The model extends current ontologies to reflect the object typicality. We show that our model outperforms its previous models.

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References


