A Framework for Merging Qualitative Constraints Networks

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Abstract

Spatial or temporal reasoning is an important task for many applications in Artificial Intelligence, such as space scheduling, navigation of robots, etc. Several qualitative approaches have been proposed to represent spatial and temporal entities and their relations. These approaches consider the qualitative aspects of the space relations only, disregarding any quantitative measurement. In some applications, e. g. multi-agent systems, spatial or temporal information concerning a set of objects may be conflicting. This paper highlights the problem of merging spatial or temporal qualitative constraints networks. We propose a merging operator which, starting from a set of possibly conflicting qualitative constraints networks, returns a consistent set of spatial or temporal information representing the result of merging.

Introduction

Representing and reasoning about time and space is an important task in many domains such as natural language processing, geographic information systems, computer vision, robot navigation. Many qualitative approaches have been proposed to represent the spatial or temporal entities and their relations. The majority of these formalisms however use qualitative constraints networks ($QCNs$) to represent information about a system.

In some application, e. g. multi-agent systems, spatial or temporal information come from different sources, i. e. each source provides a spatial or temporal QCN representing relative positions between objects. The multiplicity of sources providing spatial or temporal information makes that the underlying $QCNs$ are generally conflicting. Indeed it becomes necessary to solve the conflicts and define a set of *consistent* spatial or temporal information representing the result of merging. This is the focus of the present paper.

Merging multiple sources information has attracted much attention in the framework of (weighted) propositional logic (Revesz 1997; Konieczny & Pérez 1998; Cholvy 1998; Benferhat *et al.* 1999; 2002; Konieczny, Lang, & Marquis 2004). In this paper, we take our inspiration from these works (in particular those proposed in the framework of propositional logic) and propose a procedure for merging $QCNs$.

The rest of this paper is organized as follows. We first present a necessary background on qualitative formalisms for representing space or time, in particular qualitative constraints networks. Then we present the problem of dealing with multiple sources providing spatial or temporal information. Before we propose a procedure for merging $QCNs$. Lastly we conclude.

Qualitative formalisms for space and time

It is natural to use a particular spatial or temporal qualitative algebra to describe non-numerical relationships between spatial or temporal entities (Allen 1981; van Beek 1990; Vilain, Kautz, & Van Beek 1990; Nebel & Bürckert 1995; Egenhofer 1991; Randell, Cui, & Cohn 1992; Mitra 2004). A qualitative algebra uses a limited range of relations between spatial or temporal objects. These relations can be topological relations (Randell, Cui, & Cohn 1992) (thus the objects represent areas of points), or based on an ordering relation. When representing temporal information, we often use precedence relations between ponctual entities or interval entities (Allen 1981; Vilain, Kautz, & Van Beek 1990).

Figure 1 illustrates the basic relations of a well known qualitative formalism, the cardinal directions algebra (Ligozat 1998). It represents the set $\mathcal{B}_{card}$ composed of the nine basic relations of the cardinal directions algebra, used to represent relative positions of points of the plan provided with an orthogonal reference mark.

Given an initial point $P = (x_1, y_1)$ (with x_1 the projection of P according to axis x, and y_1 its projection according to axis y), the plan is split into nine areas, each corresponding to a basic relation of $\mathcal{B}_{card}$. For example, the relation se (for *south east*) corresponds to the area of points $Q = (x_2, y_2)$ such that $x_2 > x_1$ and $y_2 < y_1$.

Formally, a qualitative algebra considers a finite set $\mathcal{B}$ of binary relations defined on a domain $\mathcal{D}$. Each of these relations is called a basic relation and represents a particular qualitative situation between two elements of $\mathcal{D}$. These relations are complete and mutually exclusive, namely two elements of $\mathcal{D}$ satisfy one and only one basic relation of $\mathcal{B}$. X B Y with $B \in \mathcal{B}$, $X, Y \in \mathcal{D}$ stands for "(X, Y) satisfies the basic relation B". A particular relation of $\mathcal{B}$ is the identity relation (eq). Each relation B of $\mathcal{B}$ is associated to an

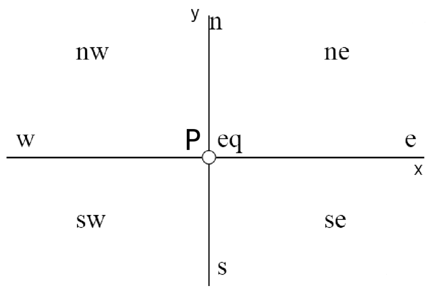


Figure 1: The 9 basic relations of $\mathcal{B}_{card}$.

inverse relation B^{-1} of $\mathcal{B}$ such that $\forall X, Y \in \mathcal{D}$, $X\ B\ Y$ iff $Y\ B^{-1}\ X$. Lastly we denote by $\mathcal{A}$ the set of all subsets of $\mathcal{B}$, i. e. $2^{\mathcal{B}}$.

Let X and Y be two elements of $\mathcal{D}$ and R an element of $\mathcal{A}$, the notation $X\ R\ Y$ stands for (X, Y) satisfies a basic relation of R.

Spatial or temporal information about the relative positions between entities is usually used by a qualitative constraints network (QCN for short). A QCN is characterized by a set of variables representing the spatial or temporal entities and a set of constraints between these variables. A constraint between two variables represents the set of possible basic relations between these variables. Formally, a QCN is defined in the following way :

Definition 1. *A QCN N on $\mathcal{B}$ is a pair (V, C) where V is a finite set of n variables $v_0, \cdots, v_{n-1}$ ($n > 0$), and C is a mapping that associates to each pair (v_i, v_j) of V a subset C_{ij} (or $C(v_i, v_j)$) of basic relations of $\mathcal{B}$, representing the set of possible basic relations between v_i and v_j. For all $v_i, v_j \in V$, C_{ji} is the set of inverse relations of C_{ij}, and $C_{ii} = \{eq\}$.*

Figure 2 gives a QCN on the cardinal directions algebra based on four variables v_0, v_1, v_2 and v_3 which take their values in $\mathbb{R} \times \mathbb{R}$. Let us notice that we do not represent the constraints C_{ii} for all i, since it is the constraint $\{eq\}$. We do not represent either the constraint C_{ji} if C_{ij} is represented, $\forall i, j$. Indeed, C_{ji} is the set of all inverse basic relations of C_{ij}.

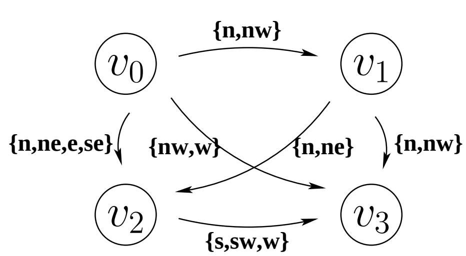


Figure 2: A qualitative constraints network N_1.

Let $N = (V, C)$ be a QCN. An **instanciation** of N on $V' \subseteq V$ is a mapping α from V' to $\mathcal{D}$. An instanciation is **consistent** when $\alpha(v_i)C(v_i, v_j)\alpha(v_j)$, $\forall v_i, v_j \in V'$. A **solution** of N is a consistent instanciation of N on V. N is **consistent** if and only if it admits a solution. A **subnetwork** of N is a QCN $N' = (V, C')$ where $C'_{ij} \subseteq C_{ij}$ $\forall i, j \in \{0, \ldots, n-1\}$. An **atomic network** is a QCN in which each constraint is composed of one and only one basic relation of $\mathcal{B}$. A **(consistent) scenario** of N is an (consistent) atomic sub-network of N. Each solution of N is associated to a consistent scenario of N. N is **minimal** if and only if $\forall i, j \in \{0, \ldots, n-1\}, \forall B \in C_{ij}$, there exists a consistent scenario $\sigma = (V, C')$ of N such that $C'_{ij} = \{B\}$.

Figure 3 gives a consistent scenario of N_1, denoted by σ_1. Indeed, σ_1 is an atomic sub-network of N_1 since it is defined over the same set of variables $\{v_0, v_1, v_2, v_3\}$ and each of its constraints is defined by only one basic relation of $\mathcal{B}_{card}$ belonging to the associated constraint in N_1. Moreover σ_1 is consistent since the instanciation of σ_1 over $\{v_0, v_1, v_2, v_3\}$ given by $\{v_0 = (2, 8), v_1 = (5, 6), v_2 = (2, 1), v_3 = (7, 4)\}$ satisfies all the constraints of σ_1. This instanciation is depicted in Figure 4.

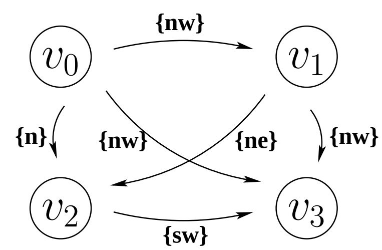


Figure 3: A consistent scenario σ_1 of N_1.

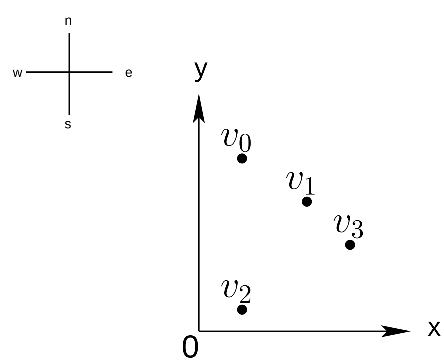


Figure 4: An instanciation of σ_1.

Given a QCN $N = (V, C)$, $[N]$ denotes the set of all consistent scenarios of N. Also, N^V_{ALL} denotes the QCN defined over the set of variables V, and in which each constraint between two distinct variables corresponds to the set $\mathcal{B}$.

Merging $QCNs$: Problem and example

Let $\mathcal{N} = \{N_1, \ldots, N_m\}$ be a set of $QCNs$ defined over the same set of variables $V = \{v_0, \ldots, v_{n-1}\}$ and in which constraints are sets of basic relations of the same set $\mathcal{B}$.

Each QCN N_j $(j \in \{1,\dots,m\})$ represents an agent's knowledge about the relationships between variables of V.

Example. Let us consider three satellites S_1, S_2 and S_3. Information provided by each satellite S_i is described using a QCN $N_i = (V, C_i)$ with $V = \{v_0, v_1, v_2, v_3\}$. Figure 5 gives the three $QCNs$ representing the qualitative information provided by S_1, S_2 and S_3 respectively.

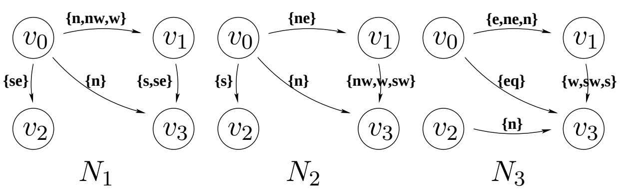


Figure 5: The three $QCNs$ to be merged.

For example, S_1 states that v_1 is in the south or in the south-east of v_3. S_1 has no information about the relationships between v_2 and v_1 and between v_2 and v_3. In this case, the associated constraints are the set $\mathcal{B}$ (i. e. the set of all basic relations). We do not represent them in the graph.

Given the set $\mathcal{N} = \{N_1, \dots, N_m\}$, the problem is to aggregate the different information given by the $QCNs$ $N_1, \dots, N_m$ and to provide a set of information representing the whole set $\mathcal{N}$.

A natural way to deal with $\mathcal{N} = \{N_1, \dots, N_m\}$ is to take the set $\bigcap_{N_i \in \mathcal{N}} [N_i]$ as the result of merging. That is, the set of consistent scenarios that are admitted by *all* N_i $\forall i \in \{1, \dots, m\}$. However this way to define the result of merging is too strong and $\bigcap_{N_i \in \mathcal{N}} [N_i]$ may be empty due to the multiplicity and variety of sources providing information. Let us consider again the above example. Each scenario in $[N_1]$ admits the relation n only between v_0 and v_3. However, each scenario in $[N_3]$ admits the relation eq only between v_0 and v_3. Consequently there is no consistent scenario common to both N_1 and N_3. So $\bigcap_{N_i \in \mathcal{N}} [N_i] = \emptyset$.

A weaker way to aggregate $N_1, \dots, N_m$ would be to take the set $\bigcup_{N_i \in \mathcal{N}} [N_i]$ as the result of merging. Although this way always guarantees a nonempty result of merging (provided that at least one QCN in $\mathcal{N}$ is consistent), it is however very cautious and the number of scenarios in $\bigcup_{N_i \in \mathcal{N}} [N_i]$ may be very large.

Indeed it becomes important to define more parsimonious merging procedures. Merging multiple sources information has been widely studied in literature, in particular when information is represented in the framework of (weighted) propositional logic (Revesz 1993; Lin 1996; Revesz 1997; Konieczny & Pérez 1998; Cholvy 1998; Benferhat *et al.* 1999; 2002). In this paper we take our inspiration from merging procedures proposed in (Lin 1996; Konieczny & Pérez 1998) for merging propositional logic information and define a merging procedure of qualitative constraints networks. We follow three steps :

- The first step consists in computing a local distance between each consistent scenario of $[N^V_{ALL}]$ and each QCN of $\mathcal{N}$.
- The second step consists in aggregating local distances computed in the previous step to compute a global distance for each scenario of $[N^V_{ALL}]$.
- Lastly, the third step defines the result of merging as the set of the "closest" consistent scenarios; those having minimal global distance.

In the following, we will denote by $\sigma(i,j)$, $i,j \in \{0, \dots, n-1\}$ the basic relation between the variables v_i and v_j for the scenario σ.

The merging process

Local distance

We first compute a local distance between each scenario σ of $[N^V_{ALL}]$ and each QCN of $\mathcal{N}$. The distance between a scenario σ and a QCN N is the smallest distance between σ and all consistent scenarios of $[N]$.

$$d(\sigma, N) = \begin{cases} \min\{d(\sigma,\sigma') \mid \sigma' \in [N]\} & \text{if } N \text{ consistent,} \\ 0 & \text{otherwise.} \end{cases}$$

Notice that the local distance $d(\sigma, N)$ is based on the distance between two scenarios. This distance is defined as follows :

Definition 2. *(Distance between scenarios) A distance between scenarios on V is a total function d from $[N^V_{ALL}] \times [N^V_{ALL}]$ to $\mathbb{R}^+$ such that $\forall \sigma, \sigma' \in [N^V_{ALL}]$*

$$\begin{cases} d(\sigma,\sigma') = d(\sigma',\sigma) \\ d(\sigma,\sigma) = 0 \end{cases}$$

Different proposals can be made to define the distance between two scenarios. Due to the lack of space, we recall two distances only.

Definition 3. *(Drastic distance) The drastic distance between two scenarios σ and σ', denoted by $d_D(\sigma,\sigma')$, is defined by :*

$$d_D(\sigma,\sigma') = \begin{cases} 0 & \textit{if } \sigma = \sigma' \\ 1 & \textit{otherwise.} \end{cases}$$

Hamming's distance (Dalal 1988) is however more popular in propositional logic framework.

Definition 4. *(Hamming's distance) The Hamming's distance between σ and σ', denoted $d_H(\sigma,\sigma')$ is defined by*

$$d_H(\sigma,\sigma') = |\{(v_i, v_j) \in V | \sigma(i,j) \neq \sigma'(i,j), i < j\}|,$$

where $|E|$ is the number of elements of the set E.

Hamming's distance between two scenarios σ and σ' is the number of pairs of variables having different constraints in σ and σ'. Notice that this distance is based on an elementary distinction between two basic relations since it has been proposed in propositional logic framework where variables are binary. However the particular context of QCN makes it possible to refine the distance between two basic relations. More precisely, instead of simply stating that two basic relations are different, we define a proximity degree between them. Before we formalize this idea, we give a general definition of the distance between two basic relations.

Definition 5 (Distance between basic relations)**.** *A distance between two relations of $\mathcal{B}$ is a total function d_R from $\mathcal{B} \times \mathcal{B}$ to $\mathbb{R}^+$ such that $\forall r_1, r_2 \in \mathcal{B}$*

$$\begin{cases} d_R(r_1, r_2) = d_R(r_2, r_1) \\ d_R(r_1, r_2) = 0 \textit{ iff } r_1 = r_2 \\ d_R(r_1, r_2) = d_R(r_1^{-1}, r_2^{-1}), \end{cases}$$

where r_1^{-1} (resp. r_2^{-1}) is the inverse relation of r_1 (resp. r_2).

Given a distance d_R between two relations of a qualitative algebra, we define the distance between two scenarios σ and σ' as follows :

$$d(\sigma, \sigma') = \sum_{i<j} d_R(\sigma(i,j), \sigma'(i,j)).$$

Notice that if d_R is defined by $d_R(r_1, r_2) = 1$ if $r_1 \neq r_2$ and $d_R(r_1, r_2) = 0$ otherwise then $d_R(\sigma, \sigma')$ is Hamming's distance. Indeed the new distance we introduced is a generalization of Hamming's distance.

A concept of neighbourhood between basic relations has been introduced by Freksa (Freksa 1992). This concept formally models the notion of proximity between two basic relations. This neighbourhood is often represented by a conceptual neighbourhood graph. A lattice is a richer structure which allows to find this graph. Some lattice's structures have been defined in literature (Ligozat 1991; 1998). Figure 6 presents a lattice of the cardinal directions $(\mathcal{B}_{card}, \leq_{\mathcal{B}})$ defined in (Ligozat 1998). For example, the two relations n and ne will be considered as being neighbours in this context. This is because given two points P and Q of the plane such that $Q\ n\ P$, if we increase the abscissa of the point Q then we directly get the relation $Q\ ne\ P$ without transiting by a third relation. Now the two relations ne and se are not neighbours, since given P and Q such that $Q\ ne\ P$, if we decrease the ordinate of the point Q then we get the relation $Q\ e\ P$ before having the relation $Q\ se\ P$.

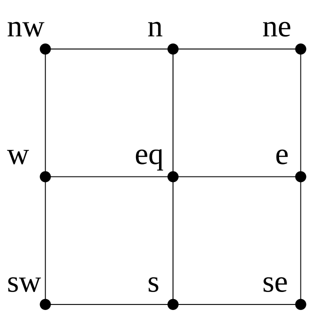


Figure 6: A cardinal relations lattice.

The conceptual neighbourhood graph $G = (\mathcal{B}, \mathcal{C})$ of an algebra corresponds to the Hasse diagram of the relevant lattice. The set of basic relations of the qualitative algebra $\mathcal{B}$ corresponds to the nodes of the graph and $\mathcal{C}$ corresponds to the undirected edges of the graph. Given two basic relations r_1 and $r_2 \in \mathcal{B}$, the edge (r_1, r_2) belongs to $\mathcal{C}$ iff the two relations are considered as being close.

Now that $G = (\mathcal{B}, \mathcal{C})$ is defined, we can define a new local distance between two relations :

Definition 6. *(Neighbourhood distance) The neighbourhood distance between two basic relations r_1 and r_2, denoted $d_V(r_1, r_2)$ is defined as the shortest lenght of the chain in G leading from r_1 to r_2.*

For instance, within the cardinal directions algebra, (nw, n, ne, e) is a shortest chain between the basic relations nw and e. Thus we have $d_V(nw, e) = 3$.

Example (Continued). Let σ be the consistent scenario depicted in Figure 7.a. We compute the local distances $d(\sigma, N_1)$, $d(\sigma, N_2)$ and $d(\sigma, N_3)$ using the neighbourhood distance where N_1, N_2 and N_3 are the $QCNs$ depicted in Figure 5. For instance N_1 admits two consistent scenarios σ_1 and σ_2 (cf. Figure 7.b and Figure 7.c respectively).

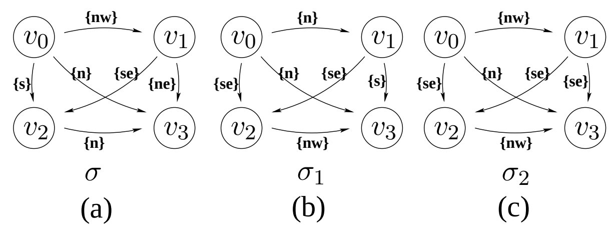


Figure 7: Consistent scenarios σ, σ_1 and σ_2.

We have

$d(\sigma, \sigma_1) = \sum_{i<j} d_V(\sigma(i,j), \sigma_1(i,j))$
$= d_V(nw, n) + d_V(s, se) + d_V(n, n) + d_V(se, se)$
$+ d_V(ne, s) + d_V(n, nw) = 1 + 1 + 0 + 0 + 3 + 1 = 6.$

$d(\sigma, \sigma_2) = \sum_{i<j} d_V(\sigma(i,j), \sigma_2(i,j))$
$= d_V(nw, nw) + d_V(s, se) + d_V(n, n) + d_V(se, se)$
$+ d_V(ne, se) + d_V(n, nw) = 0 + 1 + 0 + 0 + 2 + 1 = 4.$

Thus we have $d(\sigma, N_1) = \min\{d(\sigma, \sigma_1), d(\sigma, \sigma_2)\} = 4$.
Similary we get $d(\sigma, N_2) = 6$ and $d(\sigma, N_3) = 6$.

Computing the global distance

After computing $d(\sigma, N_i)$ between each consistent scenario σ and each $N_i \in \mathcal{N}$, we use an aggregation operator, denoted $\otimes$, in order to compute a global distance $d_{\otimes}(\sigma, \mathcal{N})$.

Definition 7 (Aggregation function)**.** *An aggregation function is a total function $\otimes$ associating a positive real number to every finite tuple of positive real numbers and satisfying the following conditions :*

- *if $x_1 \leq x'_1, \ldots, x_n \leq x'_n$, then $\otimes(x_1, \ldots, x_n) \leq \otimes(x'_1, \ldots, x'_n)$ (monotonicity)*
- *if $x_1 = \ldots = x_n = 0$ then $\otimes(x_1, \ldots, x_n) = 0$ (minimality)*

Several aggregation operators have been studied in literature (Revesz 1993; Lin 1996; Revesz 1997; Konieczny & Pérez 1998). We recall some of them :

- When the sources are independent, an operator supporting the *majority* point of view is appropriate (Lin 1996). It is defined by :

$$d_{\sum}(\sigma, \mathcal{N}) = \sum_{N_i \in \mathcal{N}} d(\sigma, N_i).$$

- An arbitration operator, denoted $\mathcal{MAX}$, has a more consensual behaviour (Revesz 1997) :

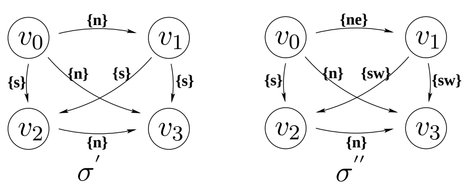
$$d_{\mathcal{MAX}}(\sigma, \mathcal{N}) = \max\{d(\sigma, N_i) \mid N_i \in \mathcal{N}\}.$$

Example (Continued). Let us consider again the consistent scenario σ depicted in Figure 7.a. Recall that $d(\sigma, N_1) = 4$, $d(\sigma, N_2) = 6$ and $d(\sigma, N_3) = 6$.
Thus $d_{\sum}(\sigma, \mathcal{N}) = \sum_{N_i \in \mathcal{N}} d(\sigma, N_i) = 4 + 6 + 6 = 16$, and $d_{\mathcal{MAX}}(\sigma, \mathcal{N}) = \max\{d(\sigma, N_i) \mid N_i \in \mathcal{N}\} = \max\{4, 6, 6\} = 6$.

The result of merging

The result of merging, denoted $\Theta_{\otimes}(\mathcal{N})$, is the set of consistent scenarios that are the "closest" ones w. r. t. $\mathcal{N}$. They are consistent scenarios which minimize the global distance. Formally we have $\Theta_{\otimes}(\mathcal{N}) = \{\sigma \mid \sigma \in [N^V_{ALL}] \ : \ \nexists \sigma' \in [N^V_{ALL}], d_{\otimes}(\sigma', \mathcal{N}) < d_{\otimes}(\sigma, \mathcal{N})\}$.

In our running example, $\Theta_{\mathcal{MAX}}(\mathcal{N})$ is composed of one consistent scenario σ' depicted in Figure 8.a. Its global distance is equal to 3. Indeed we have $d(\sigma', N_1) = 3$, $d(\sigma', N_2) = 3$ and $d(\sigma', N_3) = 1$. Similary $\Theta_{\sum}(\mathcal{N})$ is composed of two consistent scenarios $\sigma^{'}$ and $\sigma^{''}$ depicted in Figure 8. Their global distance is equal to 7. Notice that σ' is common to both $\Theta_{\mathcal{MAX}}(\mathcal{N})$ and $\Theta_{\sum}(\mathcal{N})$.



(a) (b)

Figure 8: The consistent scenarios resulting from $\mathcal{MAX}$ and $\sum$ operators.

In some applications, when the result of merging is composed of several scenarios, we would like to represent it by a QCN. In our example, $\Theta_{\sum}(\mathcal{N})$ is composed of two scenarios $\sigma^{'}$ and $\sigma^{''}$. A way to represent $\sigma^{'}$ and $\sigma^{''}$ by a QCN $N_R = (V, C_R)$ is to define $C_R(i,j)$ as the set $\{\sigma^{'}(i,j), \sigma^{''}(i,j)\}$ for all $i, j \in \{0,1,2,3\}, i < j$ (N_R is depicted on Figure 9).

In the general case, given a set $\Theta_{\otimes}(\mathcal{N})$ of consistent scenarios, we define the associated QCN $N_R = (V, C_R)$ such that $\forall v_i, v_j \in V$,

$$C_R(v_i, v_j) = \bigcup_{\sigma \in \Theta_{\otimes}(\mathcal{N})} \{\sigma(i,j)\}.$$

Each consistent scenario of $\Theta_{\otimes}(\mathcal{N})$ is a consistent scenario of N_R since $\forall \sigma \in \Theta_{\otimes}(\mathcal{N})$, $\forall v_i, v_j \in V$, $\sigma(i,j) \in C_R(v_i, v_j)$. Moreover, N_R is a minimal QCN since $\forall v_i, v_j \in V$, $\forall B \in C_R(v_i, v_j)$, $\exists \sigma \in [N_R]$ consistent ($\sigma \in \Theta_{\otimes}(\mathcal{N})$) such as $\sigma(i,j) = B$.

In our example, N_R (cf Figure 9) admits the scenarios $\sigma^{'}$ and $\sigma^{''}$ only. However generally it is not always possible to build a QCN such that consistent scenarios are exactly the

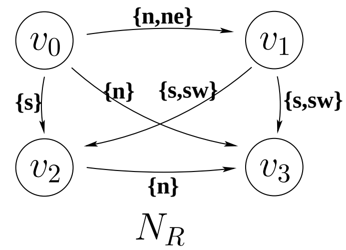


Figure 9: The QCN N_R.

same those appearing in the result of merging. Indeed N_R could admit additionnal non-desired scenarios.

Let us now underline some properties induced by the merging process presented in this section. These properties are an adaptation of those shown by (Lin 1996) in propositional logic framework[1].

Property 1 (Syntactic independence)**.** *Let* $\mathcal{N} = \{N_1, \ldots, N_m\}$ *and* $\mathcal{N}' = \{N'_1, \ldots, N'_m\}$ *be two sets of* $QCNs$ *defined on the same set of variables* V *such that* $\forall i \in \{1, \ldots, m\}$ $[N_i] = [N'_i]$. *Then*

$$\Theta_{\otimes}(\mathcal{N}) = \Theta_{\otimes}(\mathcal{N}').$$

This is because our merging operator is based on the computation of distances between consistent scenarios of $[N^V_{ALL}]$ and $QCNs$. Thus our method does not depend on the syntatic representation $QCNs$ to be merged, but on their respective consistent scenarios.

Definition 8. *The aggregation operator* $\otimes$ *satisfies the irrelevance of the zero element postulate iff for some* $x_i = 0$,

$$\otimes(x_1, \ldots, x_i, \ldots, x_n) = \otimes(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n).$$

Property 2 (Irrelevance of inconsistent $QCNs$)**.** *Let* $\otimes$ *be an aggregation operator which satisfies the irrelevance of the zero element postulate. If* $N_h \in \mathcal{N}$ *is inconsistent, then*

$$\Theta_{\otimes}(\mathcal{N}) = \Theta_{\otimes}(\{N_i \mid i \neq h\}).$$

This property shows that inconsistent sources have no impact on the merging result.

Property 3 (Right hand side inclusion)**.**

$$\bigcap_{N_i \in \mathcal{N}} [N_i] \subseteq \Theta_{\otimes}(\mathcal{N}).$$

Definition 9. *The aggregation operator* $\otimes$ *satisfies the strong minimality postulate iff*

$$\textit{if } \otimes(x_1, \ldots, x_n) = 0 \textit{ then } x_1 = \ldots = x_n = 0.$$

Property 4 (Left hand side inclusion)**.** *Let* $\otimes$ *be an aggregation operator which satisfies the strong minimality postulate. If* $\bigcap_{N_i \in \mathcal{N}} [N_i]$ *is nonempty, then* $\Theta_{\otimes}(\mathcal{N}) \subseteq \bigcap_{N_i \in \mathcal{N}} [N_i]$.

These two properties show that if the sources are not conflicting, the merging process returns the set of the common scenarios of the different sources.

[1] Due to the lack of space, the proofs are omitted

The above properties are satisfied by $\sum$ and $\mathcal{MAX}$. However the aggregation operator $\mathcal{MIN}$ defined as

$$d_{\mathcal{MIN}}(\sigma, \mathcal{N}) = \min\{d(\sigma, N_i) \mid N_i \in \mathcal{N}\}$$

does not satisfy the *strong minimality* postulate, nor *irrelevance of the zero element* $\mathcal{MIN}$ operator does not satisfy in general the properties 2 and 4.

Conclusion and future work

Based on merging methods of propositional information proposed in literature, we proposed a merging procedure which, given a set of possibly conflicting qualitative constraints networks ($QCNs$), returns a consistent set of information representing the result of merging.

The main ingredient in the procedure is a *distance* which computes the closeness of each scenario w. r. t. each QCN to be merged. Then the result of merging is the set of scenarios that are the closest ones w. r. t. *all* $QCNs$.

Going beyond a simple translation of merging procedures proposed in propositional logic framework, we proposed a new distance which is specific to the context of spatial or temporal reasoning and based on the concept of neighbourhood. We have shown that Hamming's distance (used in propositional logic framework) is a special case of the new distance.

Our merging procedure has been implemented into QAT (Qualitative Algebra Toolkit) (Condotta, Ligozat, & Saade 2006), a JAVA constraints programming library allowing to handle $QCNs$. Currently we can compute the result of merging $QCNs$ which are defined on few variables only. Indeed we intend to develop a more efficient syntactic procedure for merging $QCNs$. In particular we will study the properties of a merging procedure which aggregates independently the constraints of each QCN.

The present work can be extended in different directions. As we have restricted our work to the case where the QCNs to be merged are defined on the same set of variables w. r. t. the same qualitative algebra, we will study more general processes allowing to merge $QCNs$ defined on different sets of variables, and in which information is expressed in different formalisms. Another perspective is to merge QCNs modeling both qualitative and quantitative spatial or temporal information.

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