Modelling Decentralised Decision Making

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Abstract
A set of agents (human or computer process) are supposed to decide on an action given a finite set of possible strategies when the background information is vague or numerically imprecise. The agents evaluate the strategies and communicate their opinions about their utilities, their opinions about the credibilities of the reports of the other agents, as well as their opinions of the reliabilities of the other agents. A significant property of the theory is that it admits the representation of imprecise information at all stages. The strategies are evaluated relative to generalisations of the principle of maximising the expected utility. It is also demonstrated how decision trees can be integrated into the framework and we suggest an evaluation rule taking into account all strategies, credibilities, reliabilities, probabilities and utilities involved in the framework. This is done without presupposing the existence of a central coordinating agent that evaluates the different reports of the agents. In a distributed context there are no a priori reasons for such an agent and we show how it can be eliminated in our framework.

Background
In the area of Distributed Artificial Intelligence multi-agent co-ordination is approached in different ways depending on what relations sets of agents have to each other. One aspect of this is to what extent a group of agents is centrally co-ordinated (Ekenberg, Boman, & Danielson 1995), (Ekenberg, Danielson, & Boman 1995), (Katz & Rosenschein 1993), (Rosenschein 1982) and to what extent the control is distributed (Ephrati & Rosenschein 1991), (Zholtkin & Rosenschein 1991). Intermediate forms occur, where agents sub-contract their tasks to other agents (Kraus 1993); also in environments where the agents do not have a common goal and there is no globally consistent knowledge. Within the economic approaches, the Contract Net communication protocol (Smith 1980) has also been appreciated.

The approach in (Ekenberg, Danielson, & Boman 1996a), (Ekenberg, Danielson, & Boman 1996b) treats a centralised scenario where a decision making agent (DMA), which may be a human coordinator as well as another agent process, faces a situation involving a choice between a finite set of strategies \( S_i \) having access to a finite set of autonomous agents \( A_j \) reporting their opinions on the strategies to the DMA, see Figure 1. In such a scenario, the communication model required is restricted to the information flow between the DMA and the respective agents.

In a situation modelled as in Figure 1, some agents may be more reliable than others when evaluating the strategies involved, since different agents may have different capabilities to determine the utilities. The DMA also has access to assessments expressing the credibility of the different agents and is set on choosing the most preferred strategy given the agents' individual reports and the relative credibility of these. A main issue in such a context is how the local utilities of the agents are related to the global utility of the complete system (cf. (Rosenschein & Zholtkin 1994)). A common assumption in this area of the processes involved is that each of the participating agents has an individual preference relation over the set of strategies. Agents bypass negotiation by using a voting mechanism; each agent expresses its preferences and a group choice mechanism is used to select the result. For instance, (Ephrati & Rosenschein 1991), (Zholtkin & Rosenschein 1991) use a refinement of the Clarke tax (Clarke 1971). This is further developed in (Ephrati & Rosenschein 1993).

A feature in, e.g., (Ephrati & Rosenschein 1991) is that the utilities are communicated explicitly and are not dependent on particular local functions in defining a global (social welfare) function. Similar to this, the model in Figure 1 does not necessarily require particular utility functions of the agents, but rather that the utility estimates are expressed by sets of utility functions, determined by different constraints (Ekenberg, Danielson, & Boman 1996a). The decision rule used in agent deliberation has often been the principle of maximising the expected utility (PEMU) which has been equated with the concept of rationality (cf. (Gmytrasiewicz & Durfee...
This equity is inspired by research that grew from efforts of Ramsey, von Neumann, and others (Fishburn 1981). It should be emphasized that even though this paper assumes the underlying use of the PMEU, this assumption can be relaxed (Boman & Ekenberg 1995). (Ekenberg, Danielson, & Boman 1996b).

Another important issue is that for a set of agents to perform adequately, it seems important that the agents are able to evaluate vague background information gathered from different sources. The dynamic adaptation taking place over time as the agents interact with their environment, and with other agents, is affected by the means available to assess and evaluate imprecise information. In this respect, our concern has been how problems modelled by numerically imprecise probabilities and utilities can be evaluated. Much work has been done in this area and the relation between our work and earlier approaches that attempt to incorporate imprecise estimates into probability theory is described in (Ekenberg & Danielson 1994). Our approach attempts to conform to traditional statistical reasoning by using the concept of admissibility (Lehmann 1959). The theory may still be used in vague domains and it avoids restricting assumptions such as common knowledge, precise utility measures, and a dependence on axiomatic characterisations (Rosenschein 1988).

The first step to eliminate the DMA is to distribute the possibility to estimate the credibility of the agents. As in the model in Figure 1, each individual agent may express its respective opinion about the utilities of the strategies under consideration. However, it may still be that different agents may have different capabilities to determine the utilities, and this section suggests how the possibility to make credibility assessments can be distributed.

**Global Utility**

The first step to eliminate the DMA is to distribute the possibility to estimate the credibility of the agents. As in the model in Figure 1, each individual agent may express its respective opinion about the utilities of the strategies under consideration. However, it may still be that different agents may have different possibilities to determine the utilities, and this section suggests how the possibility to make credibility assessments can be distributed.

**Representation**

The agents are asked to assess the competence of the other agents in two respects: (i) they may have opinions on the other agents’ abilities to give adequate estimates on the utilities of different strategies, and (ii) they may have opinions on the reliabilities of the other agents as they in turn assess the abilities of other agents. Each agent has such capabilities of expression since there are no coordinating agents or global credibilities involved. This model is restricted in the sense that there are no good answers to why we should stop here. The agents could also have opinions on the reliability of the other agents when they in turn assess the reliability of the other agents. Furthermore, the agents may have opinions on the abilities of the agents when stating the latter kinds of assessments, etc. Theoretically, this viewpoint implies an infinite regress and makes the problem intractable, but a reasonable, albeit unjustified, assumption is that the assessments converge after a few levels. The description in this section is restricted to a case with three kinds of assessments, but as will be seen later, the model can be extended to take account of an arbitrary number of assessments types.

As an example how this model can be used, consider the following:

**Example:** A set of agents (A₁ and A₂) are to report on their respective assessments concerning the strategies for a risk policy of a company. The objective for the agents is to decide how to allocate resources for preventing potential losses of the company. The prevailing strategies are to prevent disruption of productions and services, to prevent obstruction of research and development, or to distribute the resources over both these objectives. These strategies are labeled S₁, S₂, and S₃ below. Assume that the agents A₁ and A₂ have reported the following utility assessments. The utilities involved could, for example, be monetary values. In that case, they are linearly transformed to real values in the interval [0,1].

For instance, the assessments according to agent A₁ could be the following:

- The utility of strategy S₁ is between 0.20 and 0.50
- The utility of strategy S₂ is between 0.20 and 0.60
- The utility of strategy S₃ is between 0.40 and 0.60
- The utility of strategy S₂ is at least 0.10 better than that of S₁

Agent A₂ can state similar assessments about the utilities of the strategies. Note that the utility estimates are treated only from a global perspective. Below, we will discuss how the agents may use decision trees in imprecise domains to individually perform evaluations of the strategies. (Risk evaluation methods that can be integrated into the framework are discussed in (Ekenberg & Danielson 1995). (Ekenberg, Oberoi, & Orci 1995)).

Moreover, the agents may estimate the credibility of A₁ and A₂ as numbers in the interval [0,1]. The number 0 denotes the lowest credibility and 1 the highest. Thus, the assessments according to agent A₁ could be:

- The credibility of agent A₂ is at least equal to that of A₁
- The credibility of agent A₂ is between 0.30 and 0.70
The assessments according to agent $A_2$ have a similar form. Furthermore, the agents have estimated the reliabilities of $A_1$ and $A_2$ as numbers in the interval $[0, 1]$. The number 0 denotes the lowest reliability and 1 the highest. For instance, the assessments according to agent $A_1$ can be:

- The reliability of agent $A_1$ is between 0.50 and 0.70.
- The reliability of agent $A_2$ is between 0.20 and 0.50.

One further reason for allowing interval as well as comparative assessments is that the agents' information may have different sources. For instance, intervals naturally occur from aggregated quantitative information while qualitative considerations often result in comparisons. Since the sources may be different, the assessments are not necessarily consistent with each other.

Example (cont'd): The reports provided by agent $A_1$ are translated to the following expressions.

\[ u_{11} \in [0.20, 0.50] \]
\[ u_{21} \in [0.20, 0.60] \]
\[ u_{31} \in [0.40, 0.60] \]
\[ u_{21} \geq u_{11} + 0.10 \]

The credibility as well as the reliabilities of $A_1$ and $A_2$ are also represented as numbers in the interval $[0, 1]$. The translation of the credibility and the reliability assessments above results in the following expressions.

\[ c_{12} \in [0.30, 0.70] \]
\[ c_{12} \geq c_{11} \]
\[ r_{11} \in [0.50, 0.70] \]
\[ r_{12} \in [0.20, 0.50] \]

The sets of assessments are transformed into linear systems of equations that are checked for consistency. The credibility assessments, reliability assessments, and utility assessments constitute the credibility base $C$, the reliability base $R$, and the strategy base $S$ respectively. A credibility base with $k$ agents is expressed in the credibility variables $\{c_{11}, \ldots, c_{1k}, \ldots, c_{k1}, \ldots, c_{kk}\}$ stating the relative credibility of the different agents when considering the strategies. The term $c_{ij}$ denotes the opinion of agent $A_i$ about the credibility of agent $A_j$'s utility estimates. A reliability base with $k$ agents is expressed in the reliability variables $\{r_{11}, \ldots, r_{1k}, \ldots, r_{k1}, \ldots, r_{kk}\}$ stating the relative reliability of the different agents. The term $r_{ij}$ denotes the reliability of agent $A_i$ according to agent $A_j$. The credibility and reliability bases are also restricted by normalisation constraints

\[ \sum_{i=1}^{k} c_{ij} = 1 \quad \text{and} \quad \sum_{j=1}^{k} r_{ij} = 1, \]

for each agent $A_i$ involved. A strategy base with $k$ agents and $m$ strategies is expressed in strategy variables $\{u_{11}, \ldots, u_{1k}, \ldots, u_{m1}, \ldots, u_{mk}\}$ stating the utility of the strategies according to the different agents. The term $u_{ij}$ denotes the utility of strategy $S_i$ in the opinion of agent $A_j$. The three bases together constitute an information frame. It is assumed that the variables' respective ranges are real numbers in the interval $[0, 1]$. Below, we will refer to an information frame as a structure $(S, C, R)$.

We will now provide a more detailed description of how problems, arising when eliminating the central position of a coordinating agent, are handled. We begin by discussing the representation and properties of assessments of agent credibility, agent reliability, and utilities of strategies according to the different agents.

Evaluation

In eliminating the DMA we obtain three different bases to consider in the evaluation process. The global utility of a strategy is a function of the reports of the individual agents, the agents' credibility when giving such reports according to the other agents, and all agents' opinions on each others reliability. The intuition for the formula expressing the global expected utility is that all assessments concerning the utilities of the strategy $S_i$ are taken into account, each of which is balanced with the different credibility assessments. The credibility assessments are in turn balanced with the reliability assessments. For instance, $u_{ij}c_{ij}$, i.e. the utility of strategy $S_j$ in the opinion of agent $A_i$ weighted by the credibility of agent $A_i$ according to itself, is weighted by the reliability of agent $A_1$ according to all agents in the system. The resulting expression of this is

\[ u_{ij}c_{ij}r_{ij} + u_{ij}c_{ij}r_{i1} + \ldots + u_{ij}c_{ij}r_{ik}. \]

Similar expressions are received for all terms and the resulting expression when all assessments are taken into account is then (except for a normalisation factor)

\[ u_{ij}c_{ij}r_{ij} + u_{ij}c_{ij}r_{i1} + \ldots + u_{ij}c_{ij}r_{i1} + \ldots + u_{ik}c_{ik}r_{ik}. \]

In the remaining parts of this section, the information frame $(S, C, R)$ is supposed to represent a system with $k$ agents and $m$ strategies.

Definition 1

Given an information frame $(S, C, R)$, the global expected utility $G(S_j)$ of a strategy $S_j$ is:

\[ G(S_j) = \frac{1}{k} \sum_{j=1}^{k} \sum_{i=1}^{k} \sum_{m=1}^{m} u_{ij}c_{ij}r_{ij} \]

where $u_{ij}$, $c_{ij}$, and $r_{ij}$ are variables in $S, C$, and $R$ respectively.
Definition 2

A list of numbers \([n_1, \ldots, n_k]\) is a solution vector to a base \(X\) containing variables \(x_i\), where \(i = 1, \ldots, k\), if \(n_i\) can be consistently substituted for \(x_i\) in \(X\). The set of solution vectors to a base constitutes the solution set. This set is a convex polytope, i.e. an intersection of a finite number of closed half spaces.

The solution sets to the bases can be determined by ordinary linear programming (LP) methods. In order to define an evaluation principle, a notation for instantiations of the global expected utility of a strategy is introduced below.

Definition 3

Given an information frame \((\mathcal{S}, \mathcal{K}, \mathcal{R})\). Let \(a, d, e\) be three vectors of real numbers \((a_1, \ldots, a_k), (d_1, \ldots, d_k), (c_1, \ldots, c_k)\).

\[
G(S_i) = \frac{1}{k} \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{m=1}^{k} a_{ij} d_{mj} c_{nm}
\]

where \(a_{ij}, d_{mj}, c_{nm}\) are numbers substituted for \(a_{ij}, d_{mj}, c_{nm}\) in \(G(S_i)\).

The evaluation of an alternative can be based on the concept of admissibility. This intuitively means that a strategy is admissible iff no other strategy is better w.r.t. the information frame under consideration. The \(t\) in the expressions below is a real number in the interval \([0, 1]\) introduced for studying how large the differences between the expected utilities are.

Definition 4

Given an information frame \((\mathcal{S}, \mathcal{K}, \mathcal{R})\) and a real number \(t\) in the interval \([0, 1]\). \(S_i\) is at least as \(t\)-good as \(S_j\) iff \(a d e G(S_i) - a d e G(S_j) > t \geq 0\).

\(S_i\) is \(t\)-better than \(S_j\) iff \(S_i\) is at least as \(t\)-good as \(S_j\) and \(a d e G(S_i) - a d e G(S_j) > t \geq 0\), for some \(a_i, d_i, e_i, a_j, d_j, e_j\), where \(a_i\) and \(a_j\) are solution vectors to \(\mathcal{S}, d_i\) and \(d_j\) are solution vectors to \(\mathcal{K}\), and \(e_i\) and \(e_j\) are solution vectors to \(\mathcal{R}\).

\(S_i\) is \(t\)-admissible iff no other \(S_j\) is \(t\)-better.

These definitions (for \(t = 0\)) conform to statistical decision theory (Lehmann 1959), and can be used below when comparing different alternatives. In the general case, determining admissibility is a fairly demanding task from a computational viewpoint. Consequently, general methods are not suitable for interactive use of the evaluation procedure. Instead, we use procedures for reducing problems of this particular kind (i.e.: when the bases are separated) to linear systems, solvable with LP methods (Danielson & Ekenberg 1996).

Using the above definition, the strategies are evaluated with respect to \((\mathcal{S}, \mathcal{K}, \mathcal{R})\) and one of three situations may occur. For a real number \(t\):

(i) No strategy is \(t\)-admissible
(ii) One strategy is \(t\)-admissible
(iii) More than one strategy is \(t\)-admissible

Case (i) has no solution. For case (ii) the task is done, since the only remaining strategy is superior. Case (iii) remains, which implies a set of seemingly reasonable strategies containing more than one element. When this case occurs, admissibility seems to be too weak to form a decision rule by itself. We have earlier suggested different principles for further discriminating rules for centralised agent systems (Ekenberg & Danielson 1994). (Ekenberg, Danielson, & Boman 1996a) but they can easily be adapted to the extended information frames. A class of techniques is also discussed in (Danielson & Ekenberg 1996). Using such procedures, incoming reports from agents with varying degrees of reliability can be evaluated in order to reach an informed decision based on possibly deviating report contents.

Decision Trees

The model used above requires much responsibility from the agents in the system since all agents are allowed to participate in the decision process. Consequently, for the agents to produce qualified deliberations, they should be able to evaluate the strategies under consideration in a qualified way. In the section above, the problem was modelled without taking into account how decision making agents arrived at their preferences and there were no requirements on the methods they used in this process. By extending the concept of strategy, and using techniques similar to those proposed above, more general decision models can be evaluated. Consider a traditional decision model in Figure 2.

![Figure 2](image)

\[S_1 \quad S_2 \quad \cdots \quad S_n\]

\[S_1 \quad c_{i1} \quad c_{i2} \quad \cdots \quad c_{in}\]

\[S_2 \quad c_{j1} \quad c_{j2} \quad \cdots \quad c_{jn}\]

\[\vdots \quad \vdots \quad \vdots \quad \cdots \quad \vdots\]

\[S_m \quad c_{m1} \quad c_{m2} \quad \cdots \quad c_{mn}\]

Figure 2  A state-consequence matrix

The possible states \((s_1, \ldots, s_n)\) in the model describe a set of mutually exclusive and exhaustive descriptions of the world, not leaving any relevant state out. These states determine the consequences (such as \(c_{ij}\)) of the different strategies alter-
certain alternatives $S_1, ..., S_m$. The true state is the state that does eventually obtain the description of the actual world. Thus, if we adopt the strategy $S_2$ and if $S_3$ becomes the true state, consequence $c_{32}$ will occur. The preferences among the consequences are supposed to be expressed by some kind of value function, for instance a utility function. If such a function exists, the value of consequence $c_{ij}$ can be mapped onto a value $v_{ij}$, and the matrix can be evaluated with respect to different evaluation rules that have been proposed during the years (cf. (Ekenberg, Danielson, & Boman 1996b)). This model is formulated in terms of a single decision node, i.e., when a decision making agent has only one decision to make. This is unnecessarily restrictive, since an agent often confronts a decision situation involving paths with several decision nodes. (cf., e.g., (Raiffa 1968)). Such a situation can be modelled in a decision tree such as in Figure 3. The directed edges labelled by $S$ in the figure denote alternatives, and the $c_i$'s different consequences. The squares are decision nodes, i.e., where a decision has to be made by an agent. The circles denote chance nodes, from which edges lead to leaves or new decision nodes. Finally, the leaves correspond to ultimate consequences. A directed edge labelled by $p$ denotes the probability of the node where the edge terminates. Given that the alternative (leading to the chance node where $p$ begins) occur.

![Figure 3 A Decision Tree](image)

This model could be extended in a way similar to before by allowing for imprecise assessments such as “the probability of consequence $c_{ij}$ obtaining is in the interval [40%, 60%]” or comparative assessments such as “consequence $c_{ij}$ is preferred to consequence $c_{kl}$”. These assessments can be represented in systems of linear inequalities in probability variables $p_{ij}$ and utility variables $u_{ij}$. Similar to before, the first assessment can be represented by the two inequalities $p_{ij} \geq 0.4$ and $p_{ij} \leq 0.6$, and the second one by $u_{ij} \geq u_{kl}$. To simplify the presentation in the sequel, it is assumed that, to each chance node, there is at most one directed edge leading to a decision node. The general case is very similar.

Definition 5

Given a decision tree. A set $[e_{c_1}, ..., e_{c_k}, D_{p_{ij}u_{ij}}]$ is an alternative associated with a chance node $C_j$, if the elements of the set are exhaustive and pairwise disjoint with respect to $C_i$. (This notation will be used even if an alternative does not contain an element $D_{p_{ij}u_{ij}}$).

Informally, this means that exactly one of $e_{c_1}, ..., e_{c_k}, D_{p_{ij}u_{ij}}$ will occur given that the alternative, represented by the directed edge to $C_i$ is chosen.

Definition 6

Given a decision tree, a sequence of edges $[S_1, ..., S_j]$ is a strategy. If for all elements in the set, $S_{i-j}$ is a directed edge from a decision node to a chance node $C_i$, and there is a directed edge from $C_{i-j}$ to a decision node from which $S_{i-1}$ is a directed edge. Explanation...
strategy can be combined with the framework described above and the total decision situation can be evaluated with respect to all credibilities, reliabilities, strategies, probabilities and utilities involved in the decision situation under consideration. Each agent in a multi-agent system may assert probability and utility assessments with respect to the tree. In this sense the probability and utility bases are local to each agent. What remains is to substitute the strategy utilities in section two with the expected utility of a strategy as defined in this section.

**Definition 8**

Given a set of agents \( \{A_1, \ldots, A_k\} \), \( k \) decision trees – each associated with exactly one agent, and a strategy \( \{S_1, \ldots, S_k\} \), the global expected utility of \( \{S_1, \ldots, S_k\} \), \( G(S_1, \ldots, S_k) \), is defined as:

\[
G(S_1, \ldots, S_k) = \frac{1}{k} \sum_{j=1}^{k} \sum_{n=1}^{k} \sum_{n=1}^{k} E(A_j(S_1, \ldots, S_k)) \cdot c_{ij} \cdot r_{nj}
\]

where \( c_{ij} \) and \( r_{nj} \) are as in Definition 1.

Note that the definition does not presume that the decision trees for the different agents are identical. For some domains the tree could be the same for all agents and only the probability and utility assessments may differ. In other domains the agents may have constructed different decision trees involving the strategies under consideration. Similar to the previous section, the strategies are evaluated with respect to the information in the credibility and reliability bases. The difference here is that the strategy base is replaced by a set of probability and utility bases.

Consider the prerequisites in the definition above. Each \( S_i \) in the strategy \( \{S_1, \ldots, S_k\} \) is an alternative on the form \( \{c_i_1, \ldots, c_i_k\} \), for each agent \( A_i \). Each \( S_i \) is associated with a chance node \( C_i \). Assume that the directed edge leading to \( C_i \) emanates from the decision node \( D_i \), which is a node in the strategy \( \{S_1, \ldots, S_k\} \). A local decision frame \( \{P(D_i), C(D_i)\} \) corresponds. Such a frame contains constraints representing the probability and utility assessments of agent \( A_i \). Consequently, \( E(A_j(S_1, \ldots, S_k)) \) is associated with the set \( \{\langle P(D_i), C(D_i)\rangle\} \), \( j = 1, \ldots, r \), in the same way as the strategy variables used in the previous section are associated with the strategy base.

**Definition 9**

Given an agent \( A_i \), a decision tree \( t \), and a strategy \( \{S_1, \ldots, S_k\} \) in \( T \), where each \( S_i \) is an alternative \( \{c_{i_1}, \ldots, c_{i_k}\} \) associated with a chance node \( C_i \). Let \( a_1, \ldots, a_k \) be a vector of real numbers \( \{a_{i_1}, \ldots, a_{i_{k+1}}\} \), \( 1 \leq i \leq r \), and let \( b_1, \ldots, b_k \) be a vector of real numbers \( \{b_{i_1}, \ldots, b_{i_{k+1}}\} \), \( 1 \leq i \leq r \).

Now, \( a^b_E(S_i) \) is defined by the following:

\[
a^b_E(S_i) = \sum_{j=1}^{k} a_{i_j} \cdot b_{i_j}
\]

when \( S_i \) is an alternative \( \{c_{i_1}, \ldots, c_{i_k}\} \), and

\[
a^b_E(S_1, \ldots, S_k) = \left( \sum_{j=1}^{k} a_{i_j} \cdot b_{i_j} \right) \cdot \sum_{j=1}^{k} a_{i_j} \cdot b_{i_j}
\]

when \( S_i \) is an alternative \( \{c_{i_1}, \ldots, c_{i_k}\} \).

This may now be combined with the notation for instantiations of the global expected utility of an strategy in the previous section into the following:

**Definition 10**

Given a set of agents \( \{A_1, \ldots, A_k\} \), \( k \) decision trees – each associated with exactly one agent, and a strategy \( \{S_1, \ldots, S_k\} \).

Let \( a_1, \ldots, a_k, b_1, \ldots, b_k \) be vectors of vectors \( \{a_{i_1}, \ldots, a_{i_{k+1}}\} \). The latter are vectors of real numbers \( \{b_{i_1}, \ldots, b_{i_{k+1}}\} \).

Also let \( d \) be vectors of real numbers \( \{d_{i_1}, d_{i_2}, \ldots, d_{i_k}\} \)

\[
a^b_E(S_1, \ldots, S_k) = \left( \sum_{j=1}^{k} a_{i_j} \cdot b_{i_j} \right) \cdot \sum_{j=1}^{k} a_{i_j} \cdot b_{i_j}
\]

where \( a_{i_j}, b_{i_j}, d_{i_j} \), and \( c_{i_j} \) are numbers substituted for \( d_{i_1}, d_{i_2}, \ldots, d_{i_k} \).

The different strategies can then be evaluated, for instance with respect to admissibility as before.

**Definition 11**

A general decision frame \( t \) is a structure \( \langle T, S, L, K, R \rangle \). \( T \) is a set of \( T \)'s – decision trees associated with the agents \( A_j \), \( j = 1, \ldots, k \). \( S \) is the set of possible strategies modelled in the trees. \( L \) is a set of local decision frames \( \{P(D_i), C(D_i)\} \) corresponding to \( D_i \) and agent \( A_j \) where \( D_i \) is a node in the tree \( T_i \). \( K \) and \( R \) are credibility and reliability bases as in the previous section.

**Definition 12**

Given a general decision frame \( \mathcal{G} \) and a real number \( t \) in the interval \([0, 1]\).

The strategy \( \{S_1, \ldots, S_k\} \) is at least \( t \)-good as the strategy \( \{S_1, \ldots, S_k\} \) iff

\[
a^b_E(S_1, \ldots, S_k) \geq \frac{1}{k} \sum_{j=1}^{k} \sum_{n=1}^{k} \sum_{n=1}^{k} E(A_j(S_1, \ldots, S_k)) \cdot c_{ij} \cdot r_{nj}
\]

Ekenberg 69
for all \( d_i, e_i, d_j, e_j \) where \( d_i \) and \( d_j \) are solution vectors to \( R \) and \( e_i \) and \( e_j \) are solution vectors to \( R \). Furthermore, each \( a_i \) in \( a \) and each \( h_i \) in \( f \) are solution vectors to \( P^A(D) \), and each \( b_i \) in \( b \) and each \( g_i \) in \( g \) are solution vectors to \( V^A(D) \).

The strategy \( \{S_1, ..., S_q\} \) is \( t \)-better than the strategy \( \{S_1', ..., S_q'\} \) if \( S_1 \) is at least as \( t \)-good as \( S_1' \) and

\[
\forall i \in a, g, b, \forall j \in f, e, h, \forall k \in c \frac{G(S_1', ..., S_q') - t \cdot h_i}{G(S_1, ..., S_q) - t > 0}
\]

for some \( d_i, e_i, d_j, e_j \) where \( d_i \) and \( d_j \) are solution vectors to \( R \) and \( e_i \) and \( e_j \) are solution vectors to \( R \), and for every \( a_i \) in \( a \) and \( h_i \) in \( f \) and \( e_i \) in \( e \) and \( g_i \) in \( g \) and \( b_i \) in \( b \) and \( g_i \) in \( g \) are solution vectors to \( V^A(D) \).

The strategy \( \{S_1, ..., S_q\} \) is \( t \)-admissible iff no other strategy \( \{S_1', ..., S_q'\} \) in \( \mathcal{S} \) is \( t \)-better. \( \blacksquare \)

If the set of admissible strategies is too large, contraction methods similar to those suggested in the previous section can be used for investigating the stability of the result.

### Concluding Remarks

We have shown how a set of agents can analyze and evaluate vague and numerically imprecise reports made by the individual agents when determining which strategies are reasonable to choose among. The approach considers a decision problem with respect to the credibilities of the reports as well as the reliability of the agents making the reports. These three aspects are modeled into an information frame consisting of three systems of linear expressions stating inequalities and interval assessments. The strategies are evaluated relative to generalizations of the principle of maximising the expected utility. We also demonstrated how decision trees can be integrated into the framework and suggested an evaluation rule taking into account all strategies, credibilities, reliabilities, probabilities and utilities involved in the framework. The rule is based on the classical concept of admissibility. The need for a central co-ordinating agent is eliminated and the framework is in this respect suitable for use in co-operating autonomous agent architectures. However, the approach has two shortcomings that should be emphasized.

(i) As was mentioned above, the model is restricted in the sense that there are no good justification why the agents should not have opinions on the reliability of the other agents in a more general sense. To model such a perspective would theoretically imply an infinite regress. Consequently, we have to assume that the assessments converge after a reasonable number of levels.

(ii) Another problem is that despite the development of fast algorithms for solving the kinds of problems that are described in this paper (Danielson 1996), solving very large problems may still be quite time-consuming. A natural line of research is to follow aspects discussed in (Good 1952), (Horwitz & Klein 1995), (Horwitz, Cooper & Hecker 1989), where the cost of the deliberation also is taken into account, and to investigate under what restrictions on the different bases the expected utility of a strategy converge. This will be treated in a forthcoming paper.

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### References


