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Minimizing Complexity in Cellular Automata Models of Self-Replication

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Abstract

Understanding self-replication from an information processing perspective is important because, among other things, it can shed light on molecular mechanisms of biological reproduction and on prebiotic chemical evolution. Intuition, biological knowledge, and early computational models of self-replication all suggested that self-replication is an inherently complex process. In this paper we describe recent computational studies that challenge this viewpoint. We summarize our recent work with cellular automata models of simple yet non-trivial self-replicating structures called *unsheathed loops*. For example, one unsheathed loop consists of only six components and requires only 20 rules to specify the local intercomponent interactions needed to bring about replication. The implication of this work is that, when viewed as an emergent property of numerous local, concurrent interactions between components, self-replicating systems can be substantially simpler than is generally recognized.[1]

Developing molecular systems that can self-replicate is currently an area of intense research interest to organic chemists and others. Although it has not proven possible at present to realize any "informational replicating systems" in the laboratory [Orgel, 1992], substantial progress has been made in developing some self-replicating molecules. Much of this work has been motivated by a desire to demonstrate the feasability of self-replicating molecular systems important in the origins of life [Oro et al, 1990; Ponnamperuma et al, 1992].

In conjunction with this experimental work, it seems reasonable to explore the feasibility of developing computational models of self-replicating systems. While a number of approaches have been taken, the most frequently used is that of cellular automata. In this framework a system is created which has a stored instruction sequence that directs the system to replicate itself, raising the question of how complex such instructions must be. In this paper we briefly review the history of cellular automata models of information-carrying, self-replicating systems. We summarize our recent work in developing relatively simple models known as unsheathed loops [Reggia et al, 1993], and demonstrate how these systems work through examples. It is seen that non-trivial self-replicating structures can be substantially simpler than is generally recognized.

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Past Cellular Automata Models of Self-Replication

Past cellular automata models of self-replicating structures can be divided into two groups: a first generation of models requiring universal computability, and a second generation referred to here as sheathed loops. The mathematician John von Neumann first conceived of using cellular automata to study the logical organization of self-replicating structures or "machines" [Burks, 1970; von Neumann, 1966]. In his and subsequent two-dimensional cellular automata models space is divided into cells, each of which can be in one of n possible states. At any moment most cells are in a distinguished "quiescent" or inactive state (designated by a period or blank space in this article) whereas the other cells are said to be in an active state. A self-replicating structure is represented as a configuration of contiguous active cells, each of which represents a component of the machine. At each instance of simulated time, each cell or component uses a set of rules called the transition function to determine its next state as a function of its current state and the state of immediate neighbor cells. Thus, any process of self-replication captured in a model like this must be an emergent behavior arising from the strictly local interactions that occur. Based solely on these concurrent but local interactions an initially-specified self-replicating structure goes through a sequence of steps to construct a duplicate of itself (the replica being displaced and perhaps rotated relative to the original).

Von Neumann's original self-replicating structure was a "universal constructor-computer" embedded in a two-dimensional cellular automata space that consisted of 29-state cells. It was literally a simulated digital computer that used a "construction arm" in a step-by-step fashion to construct a copy of itself from instructions on a "tape". As such, it defined and is representative of the first generation of cellular automata models of self-replication. Von Neumann's work demonstrated that an artificial information-carrying system capable of self-replication was theoretically possible. It established a logical organization that is sufficient for self-replication, but left open the question of the minimal logical organization necessary for such behavior [Burks, 1970]. Subsequent analysis led to several variations (reviewed in [Reggia et al, 1993]) Most influential among this early work has been Codd's demonstration that if the components or cell states meet certain symmetry requirements, then von Neumann's configuration could be done in a simpler fashion using cells having only eight states [Codd, 1968]. While structures in this first generation of cellular automata models do self-replicate, these structures generally consist of tens of thousands of components or active cells, and have thus never actually been simulated computationally because of their tremendous size and complexity.

The complexity of these early cellular automata models seemed consistent with the remarkable complexity of biological self-replicating systems: they appeared to suggest that self-replication is, from an information processing perspective, an inherently complex phenomenon. However, more recently a much simpler second generation of self-replicating structures based on 8-state cells, referred to as *sheathed loops* here, was developed (see Fig. 1b) [Byl, 1989; Langton, 1984]. The term "sheathed" here indicates that these structures are surrounded by a covering or sheath (X's in Fig. 1). For clarity, in Fig. 1 and the remainder of this article self-replicating structures are labelled by their type (SL = sheathed loop, UL = unsheathed loop) followed by the number of components, the rotational symmetry of the individual cell states (S=strong, W=weak; explained below), the number of possible states a cell may be in, and the neighborhood (V=von Neumann). For example, the sheathed loop in Fig. 1b is labelled SL86S8V because it spans 86 active cells, has strongly-symmetric cell states with each cell assuming one of 8 possible states, and its transition function is based on the von Neumann neighborhood [2].

In creating sheathed loops the biologically implausible requirement of universal computability used in earlier models, i.e., requiring the ability to function as a general purpose computer (Universal Turing Machine), was abandoned. To avoid certain trivial cases, sheathed loops are still required to have a readily-identifiable stored "instruction sequence" that is used by the underlying transition function in two ways: as instructions that are interpreted to direct the construction of a replica, and as uninterpreted data that is copied onto the replica [Langton, 1984]. Thus, sheathed loops are truely "information replicating systems" in the sense that this term is used by organic chemists [Orgel, 1992].

The original sheathed loop was a modified version of a *periodic emitter*, a storage element and timing device in Codd's model [Codd, 1968]. Motivated by the way that neurons send signals over myelin-coated axons in the nervous system, Codd had introduced the concept of a sheathed data path as a means for signal propagation. A sheathed data path consists of a series of adjacent cells in state O called the core, covered on both sides by a layer of cells in state X called the sheath (Fig. 2a). Signals used to form an instruction sequence (e.g., the "+" in Fig. 2), represented by cells in other states embedded in the core of a data path, propagate along the path by being passed from one core cell to the next. Codd's periodic emitter was a non-replicating loop similar to that in Fig. 1a that contained a sequence of signals that continuously circulated around the loop. Each time the signal sequence passed the origin of the arm (lower right in Fig. 1a, 1b) a copy of the signal would propagate out the arm and, among other things, could cause the arm to lengthen or turn.

```
a.   XXXXXXXX          b.   XXXXXXXX
    XOOOOOOOOX             XO+ OL OLX
    XOXXXXXXOX             X XXXXXX X
    XOX    XOX             X+X    XOX
    XOX    XOX             XOX    XOX
    XOX    XOX             X X    XOX
    XOX    XOX             X+X    XOX
    XOXXXXXXOXXXXX         XOXXXXXXOXXXXX
    XOOOOOOOOOOOOOX        X +O +O +OOOOOX
     XXXXXXXXXXXXX          XXXXXXXXXXXXX
```

Figure 1. Sheathed loops in two dimensional cellular automata. Cells in the quiescent state are indicated by blank spaces in this and subsequent figures. a). Non-replicating loop plus arm (lower right) consisting of a core of cells in state O and a sheath of cells in state X. b). Initial state of the sheathed loop SL86S8V [Langton, 1984]. The instruction sequence + + + + + + L L is embedded in the core of O's of the non-replicating loop shown in (a) (reading clockwise around the loop starting at the lower right corner).

[2] The von Neumann neighborhood of a cell consists of itself and the four contiguous vertical and horizontal cells (designated NESW).

```
a)                       b)
X X X X X X X X X X      X X X X X X X X X X
0 0 0 0   + 0 0 0 0      0 0 0 0 0   + 0 0 0
X X X X X X X X X X      X X X X X X X X X X
```

Figure 2. Small segment of sheathed data path a) at time t, and b) at time t+1 where the '+' signal has propagated one position to the right.

Langton showed that a loop like Codd's could be made self-replicating by storing in it a set of instructions that direct the replication process [Langton, 1984]. These instructions cause the arm to extend and turn until a second loop is formed, detaches, and also begins to replicate. This replicating sheathed loop consists of 86 active cells as pictured in Fig. 1b, and its transition function has 207 rules. Subsequently, two smaller self-replicating sheathed loops containing as few as 12 active cells in one case have been described [Byl, 1989].

Unsheathed Loops: A Third Generation of Self-Replicating Structures

After studying models of self-replicating structures developed in Codd's eight-state framework [Byl, 1989; Codd, 1968; Langton, 1984], we hypothesized that even simpler and smaller self-replicating structures were possible. For example, we conjectured that removal of the sheath surrounding data paths could produce a third generation of self-replicating structures we call *unsheathed loops*. It was not obvious in advance that complete removal of the sheath would be possible. It has been believed that the sheath is essential for indicating growth direction and for discriminating right from left in a strongly rotation-symmetric space [Codd, 1968; Byl, 1989]. In fact, we have discovered that having a sheath is not essential for these tasks.

```
a.   0+-0L-0L          b.   00<LL000
     -     -                v     0
     +     0                0     0
     0     0                0     0
     -     0                v     0
     +     0                0     0
     0     0                0     0
     -+0-+0-+0000           >00>00>00000
```

Figure 3: Two unsheathed loops: (a) UL32S8V; (b) UL32W8V.

To understand how the sheath (surrounding covering of X's) can be discarded, consider the unsheathed version UL32S8V (shown in Fig. 3a) of the original sheathed loop (Fig. 1b). The cell states and transition rules of this unsheathed loop obey the same symmetry requirements as those of the sheathed loop, and the signal sequence +–+–+–+–+–+–L–L– directing self-replication is similar (read off of the loop clockwise starting at the lower right corner and omitting the "core" cells in state O)[3] . As illustrated in Fig. 4, the instruction sequence circulates counterclockwise around the loop, with a copy passing onto the construction arm. As the elements of the instruction sequence reach the tip of the construction arm, they cause it to extend and turn periodically until a new loop is formed. At t=3 (Fig. 4a) the sequence of instructions has circulated 3 positions counterclockwise around the loop with a copy also entering the construction arm. At t=6 (Fig. 4b) the arrival of the first + state at the end of the construction arm produces a *growth cap* of X's. This growth cap, which is carried forward as the arm subsequently extends to produce the replica, is what makes a sheath unnecessary by enabling directional growth and right-left discrimination. Successive arrival at the growth tip of +'s extends the emerging structure and arrival of L's cause left turns, resulting in eventual formation of a new loop. An intermediate state in this process is shown at t=115 (Fig. 4c). By t=150 (Fig. 4d) a duplicate of the initial loop has been formed and separated (on the right; compare to Fig. 3a); the original loop (on the left, construction arm having moved to the top) is already beginning another cycle of self-directed replication.

The unsheathed loop UL32S8V not only self-replicates but it also exhibits all of the other behaviors of the sheathed loop: it and its descendents continue to replicate, and when they run out of room, they retract their construction arm and erase their coded information. After several generations a single unsheathed loop has formed an expanding "colony" where actively replicating structures are found only around the periphery. Unsheathed loop UL32S8V has the same number of cell states, neighborhood relationship, instruction sequence length, rotational symmetry requirements, etc., as the original sheathed loop and it replicates in the same amount of time. However, it has only 177 rules compared to 207 for the sheathed loop and is less than 40% of the size of the original sheathed loop (32 active cells vs. 86 active cells, respectively). The rules forming the transition function for UL32S8V are given in Appendix A. Each rule is of the form CNESW $\rightarrow C'$ where C is the state of the center cell of the neighborhood, the next four characters are the states of the four noncenter neighbors taken clockwise (north, east, south, west), and C' designates the new state of the center cell.

[3] This and all other self-replicating unsheathed loops described in this article are non-trivial, having a readily identifiable instruction sequence that both directs construction of a replica and is copied onto the replica.

a)
```
 OL-OL-OO
 -     O
 +     O
 O     O
 -     +
 +     -
 O     O
 -+O-+O-+O-+O
```

b)
```
 OL-OOOOO
 -     +
 L     -
 O     O
 -     +
 +     -
 O     O  X
 -+O-+O-+O-+OX
            X
```

```
                              O
                              +
c)                     d)     -
                              O
OL-OL-OO  -O+-O+-O     OOOO+-O+  O+-OL-OL
-     O  +       +     O     -   -      -
+     O XOX      -     -     O   +      O
O     O  X       O     L     +   O      O
-     +          L     O     -   -      O
+     -          -     -     O   +      O
O     O          O     L     +   O      O
-+O-+O-+O-+OOOOO-L     O-+O-+O-  -+O-+O-+OOOO
```

Figure 4. Successive states of unsheathed loop UL32S8V.

```
a)  OO        b)  OO
    L+OO          L>OO
```

Figure 5. Small self-replicating unsheathed loops. a) ULO6S8V, b) ULO6W8V.

Successful removal of the sheath makes it possible to create a whole family of self-replicating unsheathed loops using 8-state cells [Reggia et al, 1993]. Each of these structures is implemented under exactly the same assumptions about number of cell states available (eight), rotational symmetry of cell states, neighborhood, isotropic and homogeneous cellular space, and so forth, as sheathed loops, i.e., within Codd's framework [Codd, 1968]. The smallest unsheathed loop in this specific family using 8-state cells, UL06S8V in Fig. 5a, is listed in line 2 of Table 1; it is more than an order of magnitude smaller than the original sheathed loop SL86S8V. Consisting of only six components and using the instruction sequence +L, it replicates in 14 units of time or iterations (column Replication Time in Table 1). This very small structure uses a total of 174 rules (Total Rules in Table 1) of which only 83 are needed to produce replication (Replication Rules); the remaining rules are used to detect and handle "collisions" between different growing loops in a colony, and to erase the construction arm and instruction sequence on loops during the formation of a colony. If one counts only those rules that cause a change in state of the cell to which they are applied, this structure uses a total of 49 rules to produce replication (State Change Replication Rules). This latter measure is taken here to be the preferred measure of the information processing complexity of a transition function because it includes only rules needed for replication and only rules that cause a state change.

Table 1: Replication Time and Number of Rules*

Label	Repl. Time	Total Rules	Repl. Rules	State Change Repl. Rules	Red. Total Rules	Red. Repl. Rules
UL32S8V	150	177	166	104	74	71
UL06S8V	14	174	83	49	66	32
UL32W8V	151	134	98	52	66	42
UL06W8V	10	101	58	31	33	20

*Repl. = Replication; Red. = Reduced

The smallest previously described sheathed loop that persistently self-replicates [Byl, 1989], designated SL12S6V here, uses 6-state cells, has 12 components, and requires 60 state change replication rules. We have created unsheathed loops, designated UL06S6V and UL05S6V, using 6-state cells with half as many components and requiring only 46 or 35 state change replication rules, respectively. The initial state of UL06S6V is identical to that shown in Fig. 5a; UL05S6V is also identical except it has one less component in its arm; the complete transition functions are given in [Reggia et al, 1992]. To our knowledge, UL05S6V is the smallest and simplest self-replicating structure created under Codd's framework.

Varying Rotational Symmetry

Cellular automata models of self-replicating structures have always assumed that the underlying two dimensional space is homogeneous (every cell is identical except for its state) and isotropic (the four directions NESW are indistinguishable). However, there has been disagreement about the desirable rotational symmetry requirements for individual cell states as represented in the transition function. The earliest cellular automata models had transition functions satisfying *weak rotational symmetry*: some cell states were directionally oriented [Burks, 1970; von Neumann, 1966]. These oriented cell states were such that they permuted among one another consistently under successive 90° rotations of the underlying two-dimensional coordinate system[4]. For example, the cell state designated ↑ in von-Neumann's work is oriented and thus permutes to different cell states →, ↓, and ← under successive 90° rotations; it represents one oriented component that can exist in four different states or orientations. However, Codd's simplified version of von Neumann's self-replicating universal constructor-computer

[4]Weak and strong rotational symmetry are formally defined in [Codd, 1968], pp. 15-16.

[Codd, 1968] and the dramatically simpler sheathed loops [Byl, 1989; Langton, 1984] are all based upon more stringent criteria called ***strong rotational symmetry***. With strong rotational symmetry all cell states are viewed as being unoriented or rotationally symmetric. The transition functions for the unsheathed loops discussed so far also all use this strong rotational symmetry requirement too (indicated by S in their labels). Their eight cell states are labeled $.O\#L - *X+$ where the period designates the quiescent state. All of these states are treated as being unoriented or rotationally symmetric by the transition function[5].

The fact that the simplest self-replicating structures developed so far have all been based on strong rotational symmetry raises the question of whether the use of unoriented cell states intrinsically leads to simpler algorithms for self-replication. Such a result would be surprising as the components of self-replicating molecules generally have distinct orientations. To examine this issue we developed a second family of unsheathed loops, whose initial state and instruction sequence are similar to those already described (e.g., Fig. 3b). However, for these structures weak symmetry is assumed, and the last four of the eight possible cell states $.O\#L\wedge > \vee <$ are treated as oriented according to the permutation $(.)(O)(\#)(L)(\wedge > \vee <)$. In other words, the cell state $\wedge$ is considered to represent a single component that has an orientation and is thus permuted to $>$, $\vee$ and $<$ by successive 90° rotations of the coordinate system, while the remaining four cell states do not change. For example, in Fig. 3b the states $>$, $\vee$, and $<$ appear on the lower, left and upper loop segments, respectively, to represent the instruction sequence $<<<<<<LL$. While cells in such a model have 8 possible states and are thus comparable in this sense with the above work on sheathed and unsheathed loops, they also can be viewed as simpler in that they have only 5 distinct possible components. As can be seen with the examples in Table 1 (lines 3-4), where the presence of oriented cell states or weak symmetry is indicated by W in the structure labels, relaxing the strong rotational symmetry requirement like this consistently led to transition functions requiring fewer rules than the corresponding strong symmetry version; this is true by any of the measures in Table 1, and also holds for unsheathed loops with other sizes [Reggia et al, 1993]. This decrease in complexity occurred in part because the directionality of the oriented cell states intrinsically permits directional growth and right-left discrimination, making even a growth cap unnecessary.

This simplicity and speed of replication made possible by weak rotational symmetry are illustrated in Fig. 6 where the complete first replication cycle of UL06W8V is shown. Only 31 rules are needed to direct replication of this small structure which makes use of only 5 possible components. Colony formation also occurs and continues indefinitely. The complete set of transition function rules needed for one replication of UL06W8V are listed in the top half of Appendix B. Additional observations about unsheathed loops can be found in [Reggia et al, 1993].

```
                                       <
00     0<     vL     L0 0    00^0<   0< vL
L>00   0L>0   00L>   >00L^   L>00L   0Lv00


          0        0         0       0
 #<       0        0         0       ^
vL L0    L0 00    00 0<     0^ vL   vL L0
00 >0    >0 L^    L^ 0L     0L 00   00 v0
  >        >#         0         0       0
                      0         0       0
```

Figure 6. UL06W8V uses only five unique components and its replication can be governed by either of the two relatively small sets of rules in Table 2. Starting at $t = 0$, the initial state (upper left) passes through a sequence of steps until at $t = 10$ an identical but rotated replica has been created (bottom right).

Introducing Placeholder Positions Into Rules

As noted earlier, a complete transition function includes rules that are extraneous to the actual self-replication process (such as instruction sequence erasure) and many rules which simply specify that a cell state should not change. The state change rules alone are completely adequate to encode the replication process. For this reason, we believe that the number of state change rules used for one replication is the most meaningful measure of complexity of transition functions supporting self-replication. As illustrated in Table 1, this measure indicates that, from an information processing perspective, algorithms for self-directed replication can be relatively simple compared to what has been recognized in the past, especially when oriented components are present.

The simplicity of unsheathed loop transition functions when oriented components are used is even more striking if one uses unrestricted placeholder positions in encoding their rules. We implemented a search program [Reggia et al, 1992] that takes as input a set of rules representing a transition function (e.g., top part of Appendix B) and produces as output a smaller set of reduced rules containing "don't care" or "wildcard" positions (bottom part of Appendix B). This program systematically combines the original rules, replacing multiple rules when possible with a single rule containing positions where any cell state is permissible (designated by the underline character _). Introduction

[5] Care should be taken not to confuse the rotational symmetry of a cell state as interpreted by the transition function with the rotational symmetry of the character used to represent that state. Here the character L is not rotationally symmetric, for example, but the cell state it represents is treated as such.

of such wildcard positions is done carefully so that the new reduced rules do not contradict any of the original rules, including those that do not change a cell's state. The size of the reduced rule sets that result from applying this program to the complete original set of rules and to only the replication rules of each of the models described above is shown in the rightmost two columns of Table 1. For example, with UL06W8V the single new rule L___ → O that means "state L always changes to state O" replaces seven original replication rules, while the single rule >__.L → L indicating that L follows > around a loop replaces three original replication rules. With UL06W8V this procedure reduces the complete rule set from 101 to 33 rules, and the set of rules needed for one replication from 58 to 20. Thus, by capturing regularities in rules through wildcard positions, it is possible to encode the replication process for unsheathed loop UL06W8V in only 20 rules (Appendix B, bottom half). Computer simulations verified that these 20 rules can guide the replication of UL06W8V in exactly the same way as do the original rules. Similar reductions occur with other self-replicating structures, and we anticipate that additional modifications to rule format might provide even simpler encodings.

DISCUSSION

Cellular automata models of self-replication have been studied by mathematicians, computer scientists and others for over thirty years. Much of this work has been motivated by the desire to understand the fundamental information processing principles and algorithms involved in self-replication. Understanding these principles is not just an intellectual exercise: it could, for example, shed light on prebiotic chemical evolution and the origins of life, lead to novel manufacturing methods for atomic-scale devices [Drexler, 1989], and provide insight into the mechanisms of molecular replication in biology.

A significant limitation of these computational models of self-replication is that, at present, they are not intended as realistic models of known biochemical processes, and thus they have only a vague correspondence to real molecular structures. The "tape" in early self-replicating models has been related to DNA molecules, for example, and information-carrying loops might be loosely correlated with a circular oligonncleotide where a "protein" (the construction arm) reads the encoded replication algorithm to create the replica. An important direction for future research is to extend cellular automata models to incorporate more chemical/biochemical realism, and some work along these lines is already underway [Chou et al, 1992; Navarro-Gonzalez et al, 1993].

In spite of their limitations, work on cellular automata models of self-replication has produced a number of significant results. First, the earliest models established that artificial self-replicating systems are possible in principle. Although they were never actually implemented, they demonstrated that self-replication could be viewed as an algorithmic process. Second, more recent work, including that described here, has indicated that self-replicating systems can be far simpler than is generally realized. For example, the unsheathed loop UL06W8V consists of only six components and requires only 20 rules to characterize the self-replication process. Finally, we have shown that if one controls for a number of factors such as initial configuration, number of cell states, etc., then the use of orientable components (weak symmetry) in general leads to simpler self-replicating systems than use of unoriented components (strong symmetry).

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(Appendices on following two pages.)

Appendix A: The Transition Function for UL32S8V

```
CNESW -> C'     CNESW -> C'     CNESW -> C'     CNESW -> C'     CNESW -> C'
----------      ----------      ----------      ----------      ----------
..... -> .      ...+. -> .      ...++ -> .      ...+0 -> .      ...-. -> .
...-- -> .      ...-* -> .      ...*. -> X      ...X. -> .      ...XX -> X
...0. -> .      ...0X -> 0      ...00 -> .      ...L. -> .      ...LL -> .
...#0 -> #      ..-.* -> .      ..-.# -> .      ..-0X -> 0      ..*+. -> X
..*0. -> .      ..X.+ -> .      ..X.- -> .      ..X.0 -> .      ..X.L -> .
..X+. -> .      ..X-. -> .      ..X0. -> .      ..X00 -> #      ..XL. -> .
..0.* -> .      ..0.L -> .      ..0.# -> .      ..0X# -> .      ..L.# -> .
..#.. -> .      ..#-. -> .      ..#L. -> .      .+..- -> .      .+.-. -> *
.+X.. -> .      .-X.. -> .      .-0.. -> .      .X..# -> .      .X.0* -> .
.X+0. -> .      .X-+. -> .      .X-L. -> .      .X#.. -> .      .0+.. -> .
.0-.. -> .      .L..- -> .      .L..0 -> .      .LX.. -> .      .L0.. -> .
.#..- -> .      +...- -> *      +.-.0 -> -      +.-0. -> -      +.*.0 -> -
+.#-0 -> *      +-.*. -> -      +-0#0 -> -      +XXX- -> -      +0.L. -> -
+00.- -> -      +0#-. -> -      +#X.- -> .      -...0 -> 0      -..0L -> 0
-.0.+ -> 0      -.0.0 -> 0      -.0.L -> 0      -.#.+ -> 0      -+.-. -> 0
-++.0 -> 0      --.0L -> 0      -X+.0 -> 0      -X*.0 -> X      -XL.0 -> 0
-0+.. -> 0      -0+XX -> 0      -LL.0 -> 0      -#.0L -> 0      *...0 -> 0
*..0+ -> #      *..#. -> 0      *.XL. -> #      *+X*X -> +      **.*X -> 0
**0.- -> -      *X..L -> .      *0.*. -> *      X.... -> .      X...* -> .
X..-. -> X      X..** -> .      X..XX -> .      X..0. -> X      X..L. -> .
X..#. -> *      X.-.+ -> .      X.*X. -> .      X.X.. -> #      X.X+. -> .
X.XL. -> X      X.X#X -> 0      X.0-. -> X      X+X.. -> .      XX..# -> .
XX.X+ -> +      XX.XL -> L      XX+0. -> .      XX*-. -> .      XX#.. -> .
X0..X -> #      XLX.. -> .      0...+ -> +      0...0 -> 0      0...L -> L
0..+0 -> +      0..0- -> 0      0..00 -> 0      0.+-. -> +      0.+0X -> 0
0.+#. -> .      0.-.* -> +      0.-0X -> 0      0.*.+ -> +      0.*.L -> L
0.00X -> 0      0.L.- -> L      0.L.0 -> L      0.L-. -> L      0.L-X -> 0
0.L0X -> .      0+.-. -> +      0+-X. -> +      0-.0. -> 0      0--.+ -> +
0--.0 -> 0      0--.L -> L      0*..+ -> +      0*.0. -> #      0*-.+ -> *
0X-.+ -> +      0X-.L -> L      0XXX+ -> +      0XXXL -> L      0XX#X -> #
00.+. -> +      00.0. -> 0      00.0- -> 0      000.+ -> +      000.0 -> 0
0#.0. -> 0      0#.L- -> L      L...- -> -      L..-0 -> -      L.-.0 -> -
L+.-0 -> -      L**-0 -> -      LXXX- -> -      LX0.- -> -      L00.- -> -
L#.-. -> -      #...+ -> .      #..++ -> .      #.-.. -> X      #+..# -> .
#*.0. -> +      #X.-. -> #      #XXL- -> L      #XX#X -> 0      #0.0. -> #
#0#.. -> .      ##.0. -> 0
--------------------------------------------------------------------------
       Rule neighborhoods are as follows (no rotations listed):
                         N
                       W C E      ->     C'
                         S
```

(Appendix B follows on next page.)

Appendix B: The Transition Function for UL06W8V*

```
CNESW -> C'  CNESW -> C'  CNESW -> C'  CNESW -> C'
-----------  -----------  -----------  -----------
Replication rules:
..... -> .   >.0.L -> L   0..L. -> 0   .0>.. -> .
...0. -> .   0...> -> >   ..000 -> ^   ..>>0 -> .
..L.. -> .   ...v. -> .   0.^>0 -> <   .<L.L -> .
.>... -> .   L..0v -> 0   0^0.> -> v   L#.0v -> 0
...00 -> .   0LL.0 -> 0   00L.0 -> 0   ...L< -> .
0.0L. -> 0   ...LL -> .   0.<0^ -> v   <...# -> .
L0>.. -> 0   ....> -> ^   ^.000 -> .   ..#v. -> .
>00.L -> L   ...>. -> 0   ...v< -> .   #.<L. -> 0
0..>0 -> <   >...L -> L   L<v.0 -> 0   ...#. -> 0
0.0.> -> >   L.>.0 -> 0   v.0.L -> .   ..L>0 -> .
0...0 -> 0   000.> -> >   0v0.v -> >   ..0.0 -> .
0.<0. -> v   0.L.0 -> 0   .<vv< -> .   00.0L -> 0
<..L0 -> L   L0^.0 -> 0   <.... -> <   ..0L. -> .
L<>.0 -> 0   ^...L -> L   ..<<. -> #   00.0. -> 0
...>< -> .   ...^0 -> <

Reduced set of replication rules:
^.0__ -> .   0._.> -> >   0^___ -> v   L____ -> 0
v.0__ -> .   0v0._ -> >   ...^0 -> <   #____ -> 0
<...# -> .   000._ -> >   0._>_ -> <   >__.L -> L
....> -> ^   0.<0. -> v   ...>. -> 0   <..L_ -> L
..000 -> ^   0.<0^ -> v   ...#. -> 0   ..<<_ -> #
-----------------------------------------------------
```

* Same conventions as Appendix A except each rule is interpreted assuming weak rotational symmetry as described in the text.