Independence Diagrams: A Technique for Visual Data Mining

Stefan Berchtold
AT&T Laboratories

H. V. Jagadish
AT&T Laboratories

Kenneth A. Ross
Columbia University

Abstract

An important issue in data mining is the recognition of complex dependencies between attributes. Past techniques for identifying attribute dependence include correlation coefficients, scatterplots, and equi-width histograms. These techniques are sensitive to outliers, and often are not sufficiently informative to identify the kind of attribute dependence present.

We propose a new approach, which we call *independence diagrams.* We divide each attribute into ranges; for each pair of attributes, the combination of these ranges defines a two-dimensional grid. For each cell of this grid, we store the number of data items in it. We display the grid, scaling each attribute axis so that the displayed width of a range is proportional to the total number of data items within that range. The brightness of a cell is proportional to the density of data items in it.

As a result, both attributes are independently normalized by frequency, ensuring insensitivity to outliers and skew, and allowing specific focus on attribute dependencies. Furthermore, independence diagrams provide quantitative measures of the interaction between two attributes, and allow formal reasoning about issues such as statistical significance.

KEYWORDS: Visualization, Two-Dimensional, Histogram, Scatter Plot

Introduction

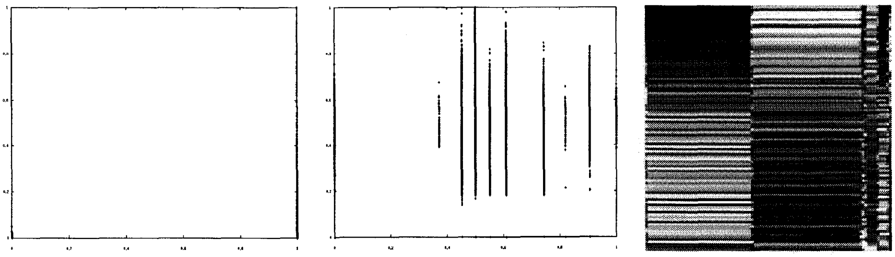
A correlation coefficient can often capture concisely the dependence between two attributes. However, it is possible for two attributes to be highly dependent, and yet have a small correlation coefficient. For example, consider a data set with data points that take values from the set $\{(-1,0),(0,1),(1,0)\}$ with equal probability. The correlation coefficient is zero. Yet, there is a strong dependence: given the value of the X-attribute, the Y-attribute is determined uniquely. There is much information obtainable through visual inspection that cannot be condensed into a single parameter, leading to the popularity of scatterplots among statisticians.

Consider the scatterplot shown in Fig.1(a), showing the relationship between two attributes in a data set from AT&T's business Due to the skew in the distribution of the X-attribute values, most of the data points are piled up along the right edge of the figure, making any patterns very hard to discern. A first attempt at correcting the skew, by taking the logarithm of the X-attribute, results in Fig. 1(b). One can now see that there are only a few distinct values of the X-attribute that occur, but it is hard to determine how often each occurs or what the dependence of the Y-attribute may be.

We propose a new kind of data visualization called an *independence diagram.* Figure 1(c) shows an independence diagram for the same data – as we shall explain later, it is now apparent that two values of the X-attribute (the two broad columns) account for most of the data points, and that the behavior of the Y-attribute is very different for these two values.

Independence diagrams enable the visual analysis of the dependence between any two attributes of a data set. Our technique is not affected by data skew and by outliers. It does not require any transformation of the data (such as the logarithm applied in Fig 1(b)) to be specified by an expert. It provides the ability to focus purely on the dependence between two attributes, stripping away effects due to the respective univariate distributions.

Our basic idea is to divide the attributes independently into slices (*i.e.*, rows or columns) such that each slice contains roughly the same number of data items and additionally split slices having a large extension. This might be seen as a combination of an equi-depth and an equi-width histogram. Each intersection of row and column defines a two-dimensional bucket, with which we store a count. We call this kind of two-dimensional histogram an *equi-slice* histogram. We map the equi-slice histogram to the screen such that the width of a slice on the screen is proportional to the number of data items in the slice. Finally, the brightness of a bucket is proportional to the count of a bucket divided by its area.



(a) A scatter plot (b) Log scale on X (c) Independence Diagram

Figure 1: Finding Dependencies Between Two Attributes in a Real Data Set

This kind of visualization is amenable to interpreting dimension dependence effects, because the one-dimensional distributions have already been "normalized" by depicting the slices in an equi-depth fashion. Equally populated slices occupy equal space in the image, meaning that resolution is spent in proportion to the data population, and that the image is not sensitive to outliers. Returning to Figure 1(c), we see that there are two values for the X-attribute that account for roughly 40% of the data each. The left one of the two has a higher concentration of Y-attribute values in the lower half of all Y-attribute values, while the right one has a higher concentration in the upper third of Y-attribute values.

Our approach is to generate images for *all* combinations of attributes. Given d attributes there are $(d^2 - d)/2$ images. These images could be shown together as thumbnails, lining up the images along the same attributes. This is our preferred mode for initial analysis. Images that show interesting effects can be enlarged and analyzed further. Cross-figure comparisons are possible since each attribute is scaled and bucketized identically in each figure in which it occurs.

Independence Diagrams

There are several steps to generating an independence diagram. First, we determine the boundaries of the slices used in each dimension, and count the number of data items in each grid cell (and slice). Second, we scale each dimension and obtain a mapping from buckets to pixels. Third, we determine the brightness for the pixels in each grid cell. We describe each step in turn below.

Attribute Partitioning. There is considerable literature on choosing a good histogram (Jagadish *et al.* 1998; Ioannidis & Poosala 1995; Poosala *et al.* 1996). However, "good" in this context means that the reconstruction error of the histogram is minimized. Unfortunately, for visualization purposes, this may not be a good optimality criterion.

We propose a modified equi-depth histogram that also handles categorical attributes appropriately. The basic idea is to create an equi-depth partition independently in each dimension, but to be flexible about placement of bucket boundaries to allow for categorical attributes and for large differences. Our two criteria are: (a) The width of any bucket should be smaller than a given constant w_{max}. This is the criterion used to build an equi-width histogram. (b) The depth (count of items) in any bucket should be smaller than a given constant p_{max}. This is the criterion used to build an equi-depth histogram. Exceptions to this rule are allowed when the number of data items with a single value of the attribute exceeds this limit.

The basic idea of the algorithm is first to sort the database according to attribute, j, of interest. (Approximate quantiles could also be used (Alsabti, Ranka, & Singh 1997; Agrawal & Swami 1995).) Then, in one pass, we consider each data item in turn, and place it in the current bucket, if we can do so without violating the criteria above. When either criterion is violated, a new bucket must be started (and marked the current bucket). To handle the situation that a value occurs more frequently than the specified minimum depth, we delay finalizing the bucket boundary until we see a larger value of the attribute. At this point, we can move the bucket boundary to the left or right, if needed, to obtain buckets closest to even without splitting points with identical values.

Scaling. Data distributions are often skewed, and it is not clear that having each axis in the visualization linearly represent the attribute value space is ideal. It is common for a statistician to transform an attribute value by applying some mathematical function, such as taking the logarithm, the square root, or the exponential of a data value, so that attribute values are "properly scaled" to produce the maximum information value to the (visual) analyst. The specific function used is selected subjectively based on the circumstances – there is limited formal procedure.

A better alternative is an *equi-depth* scaling of each axis. The idea is that each unit of length along the axis represents an equal number of data points, rather than an equal range in attribute value. Thus, outliers get squeezed into a small portion of the visual space, and the bulk of the area is devoted to the bulk of the points, even if these are clustered tightly. A desirable consequence is that patterns and correlations within this bulk of points now become easily visible – something that is just not possible in the presence of outliers. Another benefit is that all dimensions can be interpreted in a similar fashion.

Given a bucketization for an attribute, we would like to display the bucket with a width proportional to the number of data points in it. However, we are constrained in electronic displays to widths measurable as integral numbers of pixels. Utmost care is required in this quantization, particularly since one is often working in a regime where the number of buckets along an attribute dimension is comparable to the number of pixels.

Among some alternatives, the following algorithm showed the best results: For each bucket, we compute the exact (fractional) pixel width, and the corresponding density (of data points per pixel). Pixels completely included in the bucket all are assigned the same data point count. Pixels at the boundary get partial contributions from two (or more) buckets. The result is to "soften" the bucket boundaries by one-pixel wide "transitions".

Shading Pixels. We now need to map a count value for a cell into a grey-scale value for the display. Our goals are as follows. (a) There should be a single, simple grey-scale map for the whole picture. (b) This grey-scale map should adapt to the distribution of the relative populations in the buckets. (c) The grey-scale value should be more perceptible if the relative population is higher or lower than the average relative population. (d) Grey-scale values should not be wasted: an all-black image with a single white spot should not occur.

The most straightforward way to scale from counts to pixels is to map the highest count to white, the lowest count to black, and scale all other counts in linear proportion. In practice, this approach leads to mostly-black images because often there are counts that are outliers in the count-space. To overcome this effect, we sort the counts in an image, and use the 5% and 95% quantile count values as the darkest and lightest values respectively. The precise quantiles can be adjusted; in a production-quality interactive tool these values should be adjustable via the user interface. The 5% and 95% quantile values gave good results on most (but not all) of our data sets.

A more sophisticated approach might be to use a grey-scale mapping table as proposed in (Levkowitz 1997). Such tables have been constructed so that visual gradations in brightness correspond subjectively to humans as linear gradations in some scale. We tried such mappings, but found that they did not give sufficiently good grey-scale distinction between buckets. Good grey-scale distinction is more critical in this application than subjective interpretation of grey-scales. Similar issues arise if we attempt to create a color image rather than a grey-scale image.

A legend is generated to explain the numerical value of each grey-scale value. This legend associates a grey-scale value with a ratio of the number of data points in a bucket to the number of points that would be expected in that bucket if the dimensions were independent. Both high ratios (white) and low ratios (black) are potentially interesting. Fig. 2 shows an independence diagram for a synthetic data set, along with its legend. The attributes are independent, except for a range of X-attribute values. The black "finger" in the upper half of the range indicates the very low ratio in that region. The brightening of the lower half shows that there are more points than expected.

Morphing. We could choose to map a given two-dimensional equi-slice histogram in an equi-width manner, or an equi-depth manner. In fact, we could map to an image in which the width for the given bucket is interpolated between two visualization modes. By generating a sequence of images between an equi-width and an equi-depth mapping, we can "morph" one image into another. Morphing the images can aid the interpretation of the data. Details will be given in the full version of this paper.

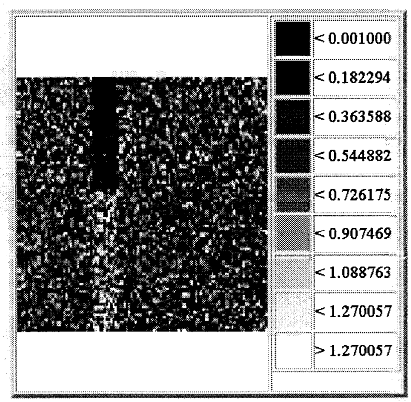


Figure 2: An independence diagram with legend.

Experiments

In this section we present several examples, including an extended example based on a large AT&T data set of more than 800,000 tuples. The system was implemented in C++ on an SGI Unix machine. The time to compute the one-dimensional partitions of the 13-dimensional data set was about 5 minutes (including a sort and one pass through the data set for each dimension, followed by one pass to compute counts for all grid cells in all 78 (=13*12/2) independence diagrams). After this preprocessing step, the time to produce a single independence diagram and write it to disk in PPM format was about 0.23 seconds. Thus, it took about 5.5 minutes to produce *all* 78 independence diagrams. Displaying a single image on screen can be done in less than a second.

To interpret independence diagrams, we must remember that each vertical slice of the image having width w (measured as a fraction of the image width) exactly represents the same number of data items, namely a proportion w of the total data set. A similar property holds for horizontal slices. This makes independence diagrams amenable to a quantitative analysis and can even be used to extract rules of the form:

> "While a fraction x of all points lie in range R_A in attribute A, among those data points lying in range R_B in attribute B, the fraction is y."

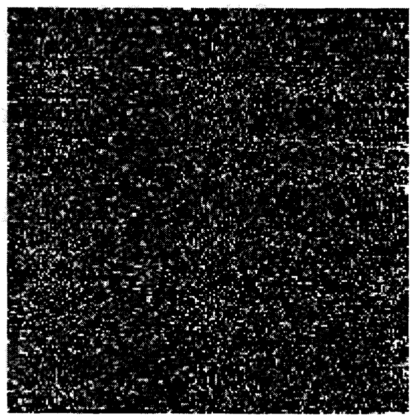


Figure 3: An independence diagram showing real independent data.

In order to get such a rule, a user selects – based on visual characteristics – a rectangular "interesting" region of the image; for example one might choose R_A and R_B to define a region around a bright spot (or a dark spot) in the image. The brightness of this region directly gives us $Pr(R_A \cap R_B)/(Pr(R_A) * Pr(R_B)) = Pr(R_A|R_B)/Pr(R_A) = y/x$. The width of the region gives us $x = Pr(R_A)$. Thus, one can also generate the counts for each range and the intersection region, which allows one to estimate the statistical significance of the difference between x and y. (See (DuMouchel *et al.* 1996) for an example of a technique for evaluating statistical significance in a two-dimensional matrix.)

In a first example, we show what an uninteresting independence diagram looks like. Recall that uninteresting in our sense means "contains no attribute dependencies". Figure 3(b) contains an example of a real data set having two attributes, one of which has some outliers, the other is very skewed. The image, however, does not show any interesting patterns, only random noise, because the attributes are independent.

In contrast to other techniques, even exotic correlations and arbitrarily shaped dependencies can be observed. Figure 4 shows an example: Obviously, the two attributes are strongly correlated and this could be captured even by means of a simple correlation coefficient. However, there is more information contained in the image. As one can see, the correlation is much stronger at the boundary of the data space. Furthermore, there is a second process appearing as a smooth line. These complex dependencies are not likely to be captured by any simple metric.

Even more complex distributions exist. Look at Figure 5(a). From a scatterplot or even an equi-width presentation, one would get the impression that we have a simple Zipfian distribution with some outliers. However, as one can see from the independence diagram in Figure 5(b), the dependency is much more complex. Note that one can see: clusters, empty regions in space, a lower bound for a process as a sharp line (right bottom of the image), *etc.* Little of this information could be conveyed by a scatterplot or equi-width histogram under presence of outliers and skew.

Related Work

A variety of data visualization techniques such as *scatterplots* (Andrews 1972), *hyperslice* (van Wijk & van Liere 1993), *parallel coordinates* (Inselberg 1985) and *circle segments* (Ankerst, Keim, & Kriegel 1996) have been proposed. No previous technique specifically addressed visualizing attribute dependence effects while normalizing out the one-dimensional distributions. See (Keim 1997) for a survey.

There is much literature on finding histogram bucket boundaries (Ioannidis & Poosala 1995; Poosala *et al.* 1996; Jagadish *et al.* 1998). There is even some literature on creating histogram bins in multiple dimensions (Poosala & Ioannidis 1997; Berchtold, Jagadish, & Ross 1998). However, all of this work is focused on finding histograms that minimize the error in the estimate of the attribute value, and our objective is quite different in this paper.

The quantitative rules we can generate resemble association rules (Agrawal, Imielinski, & Swami 1993; Srikant & Agrawal 1996). A fundamental difference between our approach and the association-rule approach is that we aim to communicate regions that are either above *or below* the expected cardinality of a cell assuming independence. Association-rule algorithms try to find regions R_A and R_B such that $Pr(R_A|R_B)$ (the "confidence") is above a threshold, whether or not the confidence measure is close to the expected value assuming independence.

Visual techniques for finding association rules are presented in (Keim, Kriegel, & Seidl 1994; Fukuda *et al.* 1996). These techniques also look for confidence above a certain threshold, and do not normalize the space to focus on attribute dependence effects alone.

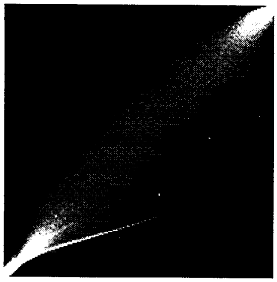
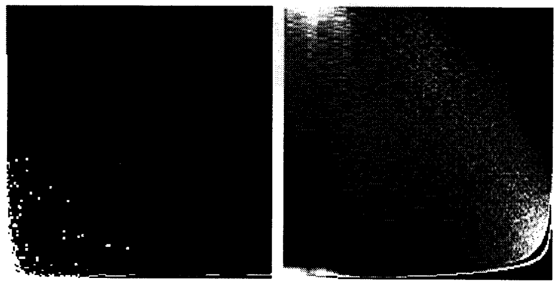


Figure 4: Independence diagram.



(a) Equi-width histogram (b) Independence diagram
Figure 5: A complex distribution.

Conclusions

We introduced independence diagrams as a means to visually determine dependencies between two attributes. Finding such dependencies is at the heart of most data mining.

Our approach is computationally tractable, even when computing images for all combinations of dimensions. We have implemented it and evaluated it using a large AT&T data set, from which the images in this paper are drawn. It is easy to identify when no two-dimensional effects are present. This task may be performed by a nonexpert. Large effects occupy a correspondingly large fraction of an image. With a little training in how to read the pictures, it is possible to make quantitative judgements based on the images without extensive domain knowledge. These quantitative judgements can be supported by measures of statistical significance. Our approach is insensitive to outliers, and is robust under extreme distributions.

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