We want to define a consequence relation |~ between formulas of L, which will tell us which consequences we can "reasonably" draw from each fact, given the background knowledge in Δ. Desirable formal properties for |~ have been largely discussed, for instance by Gabbay (1985), Kraus, Lehmann, and Magidor (1990), Lehmann and Magidor (1992), and Gärdenfors and Makinson (1994). In particular, Kraus, Lehmann, and Magidor (1990) proposed a set of postulates that is commonly regarded as the minimal core of any "reasonable" nonmonotonic system, and defined a nonmonotonic system labelled System **P** based on the following six postulates:

(a) Reflexivity: α |~ α;

(b) Left Logical Equivalence (LLE):

from $\alpha \models \alpha'$, $\alpha' \models \alpha$ and α|~ β deduce α'|~ β;

(c) Right Weakening (RW):

from $\beta \models \beta'$ and α |~ β deduce α |~ β';

(d) OR: from α |~ γ and β |~ γ deduce $\alpha \vee \beta$ |~ γ;

(e) Cautious Monotony (CM):

from α |~ β and α |~ γ deduce $\alpha \wedge \beta$ |~ γ;

(f) Cut: from $\alpha \wedge \beta$ |~ γ and α |~ β deduce α |~ γ.

From these rules, a consequence relation $|\sim_P$ can be defined for any given Δ by: $\phi \mid\sim_P \psi$ iff ϕ |~ ψ can be derived from Δ using the rules of System **P**. We define Preferential closure of Δ denoted by Δ^P, to be the set of conditional assertions obtained from Δ by only applying rules of System **P**.

Another rule which has found wide (yet not unanimous) consensus, while not being a consequence of System **P**, is Rational Monotony (RM): from α |~ δ and not (α |~ $\neg\beta$) deduce $\alpha \wedge \beta$ |~ δ. This rule has been proposed by Lehmann and Magidor (1992) in order to minimise the amount of information lost when we add a new consistent piece of information γ to a pre-existing α.

Moreover, it is commonly agreed on that a nonmonotonic consequence relation should satisfy the five following properties, as summarised in Benferhat, Saffiotti, and Smets (2000):

(a) Specificity: Results obtained from specific classes should override results obtained from generic classes;

(b) Irrelevance: If δ is a plausible consequence of α, and if β is unrelated, "irrelevant" to α or δ, then δ should also be a plausible consequence of $\alpha \wedge \beta$;

(c) Property inheritance: A subclass that is exceptional with respect to some property should still inherit other properties from its super-classes, unless some contradiction obtains;

(d) Ambiguity preservation: In a situation where we have one argument in favour of a proposition and one independent argument in favour of its negation, we should not conclude anything about this proposition; and

(e) Syntax independence: The consequences of a knowledge base should not depend on the syntactical form used to represent the available knowledge; in particular, they should not be sensitive to duplications of rules in the knowledge base. We will refer to the "majority" property, if an inference relation is sensitive to the number of "arguments" that support a conclusion.

Possibility Theory and Default Rules

Possibility theory (see Dubois, Lang, Prade, 1994, for more details) is based on the notion of a possibility distribution π which is a mapping from the set Ω to the interval [0,1]. $\pi(\omega) > 0$ means that ω is only somewhat plausible, while $\pi(\omega) = 0$ means that ω is impossible. π restricts the set of interpretations according to the available knowledge about the normal course of things. $\pi(\omega) > \pi(\omega')$ means that ω is more plausible than ω'. Two set-functions are associated with π:

(a) The possibility degree $\prod(\phi) = \sup\{\pi(\omega) \mid \omega \models \phi\}$ which evaluates to what extent ϕ is consistent with the available knowledge expressed by π.

(b) The necessity (or certainty) degree $N(\phi) = 1 - \prod(\neg\phi)$ which evaluates to what extent ϕ is entailed by the available knowledge.

Comparative possibility distributions

The unit interval [0,1] can be understood as a mere ordinal scale, which means that possibility theory is a qualitative theory of uncertainty. Therefore, to each possibility distribution π, we can associate its comparative counterpart, denoted by $>_\pi$, defined by $\omega >_\pi \omega'$ if and only if $\pi(\omega) > \pi(\omega')$. $>_\pi$ is called a comparative possibility distribution, which can also be viewed as a well-ordered partition (E1,…,En) of the set of classical interpretations Ω. E1 gathers the interpretations which are the most plausible ones and En gathers the interpretations which are the least plausible ones.

We denote by $[\phi]_\pi$ the set of π-preferred models of the formula ϕ where ω is a π-preferred model of ϕ iff:

(i) $\omega \models \phi$, and

(ii) there is no ω' such that $\omega' \models \phi$ and $\omega' >_\pi \omega$.

Given $\geq_\pi$, we define $\phi \geq_\pi \psi$ (resp. $\phi >_\pi \psi$) iff there exists $\omega \in [\phi]_\pi$ such that for each $\omega' \in [\psi]_\pi$, we have $\omega \geq_\pi \omega'$ (resp. $\omega >_\pi \omega'$).

A formula ψ is a possibilistic consequence of a consistent formula ϕ (namely, ϕ is satisfied by at least one interpretation) w.r.t. the comparative possibility distribution $\geq_\pi$, denoted by $\phi \models_\pi \psi$, iff each π-preferred model of ϕ satisfies ψ, i.e., $\phi \models_\pi \psi$ iff $\forall\ \omega \in [\phi]_\pi$, $\omega \models \psi$ iff $\phi\wedge\psi >_\pi \phi\wedge\neg\psi$

System P as a family of comparative possibility distributions

It has been proposed in Benferhat, Dubois, Prade (1997) to model default rules of the form $\alpha\to\beta$ by "$\alpha\wedge\beta$ is more possible than $\alpha\wedge\neg\beta$". This minimal requirement, called the auto-deduction principle, is very natural since it guarantees that each rule $\alpha_i\to\beta_i$ in the default base is preserved, namely if α (and only α) is observed then β should follow.

A set of default rules $\Delta=\{\alpha_i\to\beta_i,\ i=1,n\}$ with consistent α_i's, can thus be viewed as a family of constraints restricting a family $\Pi(\Delta)$ of comparative possibility distributions. Elements of $\Pi(\Delta)$ are said to be compatible with Δ. Namely, $\geq_\pi$ is compatible with Δ iff it satisfies the auto-deduction principle, i.e., for each $\alpha_i\to\beta_i$ of Δ, we have $\alpha_i\wedge\beta_i >_\Pi \alpha_i \wedge\neg\beta_i$.

A conditional assertion $\alpha\to\beta$ is a universal possibilistic consequence of Δ, denoted by $\Delta \models_{\forall\Pi} \alpha\to\beta$, if and only if, β is a possibilistic consequence of α in each $\geq_\pi$ of $\Pi(\Delta)$.

Let $\Delta^\pi = \{\alpha\to\beta\text{: } \alpha\models_\pi\beta\}$ be the set of conditional assertions which are inferred from a comparative possibility distribution $\geq_\pi$. It turns out that the universal possibilistic consequence relation leads to the exact same conclusions given by System **P** (Dubois and Prade, 1995), namely:

$$\Delta^P = \cap_{\geq\pi\, \in\, \Pi(\Delta)}\Delta^\pi$$

MSP closure inference

The possibilistic universal consequence is cautious, since there generally exist several comparative possibility distributions compatible with a given default base. A more adventurous entailment consists in selecting one comparative possibility distribution. Selecting one particular rational extension means to accept "rational monotony" as a natural property for default reasoning. The problem is then to find the "best" possibility distribution compatible with Δ that defines a rational extension of Δ. One possible way is to use the minimum specificity principle (MSP), since it considers each interpretation to be as normal as possible, namely it assigns to each interpretation ω the highest possibility level without violating the constraints. More formally, $>_\pi = \{E_1,\ldots,E_n\}$ is said to be less specific than $>_{\pi'} = \{E'_1,\ldots, E'_m\}$ iff: $\forall\omega$ if $\omega\in E'_i$ then $\omega\in E_j$ with $j\leq i$.

It can be checked (Benferhat et al., 1997) that there is exactly one comparative possibility distribution in $\Pi(\Delta)$ that is the least specific one, denoted by $\geq_{\pi spe}$. Moreover, this way of selecting a single possibility distribution is equivalent to the rational closure of Lehmann and Magidor (1992), and to Pearl's System Z (1990). The inference relation based on $\geq_{\pi spe}$ is called MSP closure Inference.

We denote by Δ^{spe} the rational closure of Δ obtained using the least specific possibility distribution $\geq_{\pi spe}$. A syntactic algorithm that checks if $\phi\to\psi$ belongs to Δ^{spe} has been provided in Benferhat et al. (1997). First, a set of default rules Δ is transformed into a stratified knowledge base $\sum=\Delta 1\cup\ldots\cup\Delta m$ such that $\Delta 1$ contains the more general rules in Δ while Δm contains the most specific ones. Once the stratification is produced, the possibilistic logic machinery is applied. Namely $\phi\to\psi\in\Delta^{spe}$ iff there exists $m\geq i\geq 0$ such that: $\{\phi\}\cup\Delta i\ldots\cup\Delta m$ classically entails ψ and that $\{\phi\}\cup\Delta i\ldots\cup\Delta m$ is consistent.

The inference based on the least specific possibility distribution suffers from the "blocking of property inheritance" problem. It corresponds to the case when a subclass is exceptional with respect to some property of its superclass. The least specific possibility distribution does not then allow to conclude whether this subclass is normal with respect to other properties. Let us consider the following example, where we assume that scientists have discovered some new life forms in the arctic ocean, called **Glacyceas**. Assume that we have the following default base $\{T \to LC, Cr \to T, Cr \to \neg LC, T \to M1\}$ where the rules respectively mean : "Translucent Glacyceas generally live in large colonies", "Crust Glacyceas are generally translucent", "Crust Glacyceas generally live in small colonies", "Translucent Glacyceas are generally shorter than 1 mm". From this example, we cannot infer the expected result "Crust Glacyceas are generally shorter than 1 mm".

Lexicographical closure (LC)

Another way to select one particular rational closure of Δ is inspired from the approaches based on a lexicographic order. The idea is to start from a stratification $\sum=\Delta_1\cup\ldots\cup\Delta_m$ of Δ proposed in Benferhat et al. (1997), and consider each formula in the Δ_i layer as being equally important, and more important than any set of formulas in subsequent layers. An interpretation ω is said to be *lex-prefered* to ω' iff there is $1\leq i\leq m$ such that $m\geq j>i$, $|[\omega]_j|=|[\omega']_j|$, and $|[\omega]_i|>|[\omega']_i|$, where $|[\omega]_i|$ is the number of rules in Δi satisfied by ω.

This approach favors interpretations satisfying a maximal number of rules in Δ. The lexicographical preference induces a total pre-order on interpretations (hence a comparative possibility distribution denoted by $\geq_{lex}$) which refines the one obtained with the minimum specificity principle, which means that $\Delta^{lex} \supseteq \Delta^{spe}$, where Δ^{lex} is obtained using the comparative possibility distribution $\geq_{lex}$. An algorithm to syntactically check whether $\phi\to\psi$ belongs to Δ^{lex} or not is proposed in Benferhat et al. (2001).

The main limitation of the lexicographic inference is that it does not satisfy the requirement of syntax-independence. In particular, it follows the majority property. Namely, the repetition of the same default in Δ, or the presence of different arguments in favor of a conclusion, may change the result. Indeed, consider the following example $\Delta = \{H \rightarrow So, N \rightarrow \neg So, Bu \rightarrow So\}$, where the rules respectively stand for "Hermaphrodite Glacyceas generally live in a solid environment", "Necrophageous Glacyceas do not generally live in a solid environment", and "Bulging Glacyceas generally live in a solid environment". Lexicographical inference allows the conclusion that "Hermaphrodite, Necrophagous and Bulging Glacyceas live in a solid environment", since this conclusion is supported by two arguments.

System LCD within the possibility theory framework

There is another system which allows to go beyond System P, and which deals with the "blocking of property inheritance" problem. It is System LCD, developed in Benferhat et al.(2000) (see also Kern-Isberner, 2001a, 2001b; Lang 1996), which is based on the formalism of belief functions. The idea is to consider "extreme" belief functions that take values that are infinitely close to 0 or to 1, much in the spirit of Adams' (1975) system. A default rule $\alpha i \rightarrow \beta i$ is interpreted as the conditional belief $bel(\beta i|\alpha i)$ being close to 1. System LCD makes use of two mechanisms that are peculiar to the theory of belief functions: the least-commitment principle, as a way to select minimally informative models; and Dempster's rule of combination, as a way to aggregate (default) information.

This section shows that LCD can be rephrased within the formalism of possibility theory.

The first step is to consider each default in Δ as being one item of evidence provided by one of several distinct sources of information, each associated with a weight that indicates its reliability, i.e., the relative "stiffness" of the default rule it provides. More precisely, for each default rule $d = \alpha \rightarrow \beta$ we associate a possibity distribution π_d of the form:

$$\pi_d(\omega) = 1 \text{ if } \omega \models \neg\alpha \vee \beta$$
$$= \varepsilon_d \text{ otherwise}$$

where ε_d is an infinitesimal associated to d, which accounts for the "violability" of rule d.

The second step is to combine these possibility distributions with the product operator to obtain a representation of the aggregate effect of all the defaults in Δ. The product operator can be viewed as the counterpart of Dempster's rule in a possibility theory framework. Given a default base $\Delta = \{d_1,...,d_n\}$ and a set $E = \{\varepsilon_1,..., \varepsilon_n\}$ of infinitesimals ε_i associated with d_i, we build a combined possibility distribution π_* by the product rule as follows:

$$\pi_* = *\{\pi_d \mid d \in \Delta\} \qquad (1)$$

We denote by $\Pi_*(\Delta)$ the family of all possibility distributions that can be built from Δ using the product rule according to (1); the elements of $\Pi_*(\Delta)$ differ in the choice of the set E of infinitesimals associated to the defaults in Δ.
The last step is to restrict $\Pi_*(\Delta)$ to elements satisfying the two following principles:
i) *Auto-deduction principle*, namely $\Pi(\Delta) \supseteq \Pi_*(\Delta)$, and
ii) *Minimum specificity principle*. We want to consider only the least specific possibility distributions in $\Pi_*(\Delta)$ which still satisfy all the defaults in Δ. The subset of possibility distributions in $\Pi_*(\Delta)$ that satisfies these two principles is denoted by $\Pi_{*MSP}(\Delta)$. The inference relation based on all possibility distributions in Π_{*MSP} is equivalent to LCD system. Namely, a conditional assertion $\alpha \rightarrow \beta$ is true in System LCD iff $\forall \pi \in \Pi_{*MSP}(\Delta)$, $\alpha \models_\pi \beta$.

As in MSP-closure and the lexicographical closure, System LCD correctly addresses the problems of irrelevance. Yet, contrary to MSP-closure, System LCD does not suffer from blocking of inheritance. And contrary to lexicographical closure, System LCD is not sensitive to majority and to the number of different rules in a default base that support the same conclusion. Regarding rational properties, System LCD satisfies all the rationality postulates of System P, except rational monotony.

The following section will provide an experimental analysis of these three systems: MSP-closure (MSP), lexicographical closure (LC) and System LCD.

Experimental Studies

Our main objective in this section is to evaluate the psychological plausibility of lexicographic closure (LC), minimum specificity inference (MSP) and epsilon-belief functions (LCD) rephrased in a possibility theory framework. In order to reach this objective, we aim at determining the set of properties satisfied by human inference in default reasoning, among Rationality (System **P** + Rational Monotony), Property Inheritance, Ambiguity Preservation and two forms of Majority property.

Previous experimental studies suggested that, on the whole, human nonmonotonic inference was compatible (although not totally confirmed) with System **P** (Da Silva Neves, Bonnefon, & Raufaste, 2002; Benferhat, Bonnefon, Da Silva Neves, 2004), allowed properties to be inherited (Elio & Pelletier, 1993; Hewson & Vogel, 1994), and was not sensitive to majority (Hewson & Vogel, 1994). However no psychological experiment relevant to Ambiguity Preservation has been found and no conclusive result about rational monotony has been obtained. Moreover, as it is said above the compatibility with System **P** is not totally confirmed. Lastly, no psychological work addressed the plausibility of these properties considered altogether.

Thus, previous results are not conclusive in regard to the psychological plausibility of either LC or MSP inferences, and, more generally, about the psychological plausibility of all considered properties altogether. Moreover, no previous experimental studies have been done regarding LCD system. In the following, two experiments are conducted for a direct comparison of these consequence relations and the psychological testing of these properties. The first one extends experimental results obtained in (Benferhat et al., 2004), by exploring for instance the psychological plausibility of LCD system. The second experiment particularly focuses on, in one hand, monotony, cautious monotony and rational monotony, and, in the other hand, on ambiguity and majority.

Experiment 1

Participants. Fifty-seven first-year psychology students at the University of Toulouse-Le Mirail, all native French speakers, contributed to this study. None of them had previously received any formal logical training.

Material. Twenty concrete but non-familiar default rules and 18 questions were involved in the experimental test of 13 arguments (see Tables 1 and 2), built to study Rationality, Property Inheritance, Ambiguity preservation and Majority. Questions were introduced by the following scenario (appearing on a computer screen).

> Consider the following facts. Scientists have discovered some new life forms in the Arctic Ocean (North Pole), some life forms that can develop in extreme cold. As a generic term, they are called “Glacyceas”. Glacyceas come into two main varieties, “Crusts” and “Worms”. While scientific knowledge about Glacyceas is still scarce and not totally reliable, scientists reckon that the following is true.

Design and procedure. Instructions and questions were displayed on a computer screen. Instructions were broken into several pages (participants were able to go back and forth pages). On the first page, participants were informed about the aim of the experiment and about some characteristics of default rules. Next, a brief scenario (see above) was introduced, followed by a first set of default rules. Questions were introduced one after another, in the same order for all the participants. Questions were framed as follows:

> Suppose you have to examine [e.g. a hermaphrodite Glacycea].
>
> Would you expect that [e.g. this Glacycea]:
>
> □ A. [e.g. lives in a solid environment]?
>
> □ B. [e.g. does not live in a solid environment]?
>
> Or do you think that:
>
> □ C. there is no way to tell?

Table1: Rules and questions for the test of the Rationality properties, plus the monotony property (MN) and RM. For the test of RM, two questions have been made. The first one is about the “$\alpha\wedge\beta$ |~ γ” consequence (labeled RM) and the second one is about the “α |~ $\neg\beta$” premise (labeled RMc).

MN	Worms are generally non-translucent.
	Do you expect this Worm, with more than one year of life expectancy, to be non-translucent?
LLE	All the Crusts and only Crusts move fast Crusts are not hermaphrodite, generally.
	Do you expect this fast moving Glacycea to be a hermaphrodite?
RW	All Worms have a high pressure tolerance. Translucent Glacyceas are generally Worms.
	Do you expect this translucent Glacycea to have a high pressure tolerance?
OR	Hermaphrodite Glacyceas generally live in small colonies. Crusts generally live in small colonies.
	Do you expect this Glacycea (which is either a Crust or a hermaphrodite) to live in a large colony?
CM	Hermaphrodite Glacyceas generally live in a solid environment. Hermaphrodite Glacyceas do not, generally, have mandibles.
	Do you expect this hermaphrodite Glacycea, living in a solid environment, to have mandibles?
CUT	Crusts generally live in small colonies. Varities of Crusts living in small colonies have generally been in existence for some 5 millions years.
	Do you expect this variety of Crust to have been in existence for 5 millions years?
RM	Hermaphrodite Glacyceas are not generally translucent.
	Do you expect this hermaphrodite Glacycea with mandibles to be translucent? (RM) Do you expect this hermaphrodite Glacycea to have mandibles? (RMc)

In addition, participants were asked how sure they were that their answer was true (by choosing a single modality on the following ordinal scale: “Quite sure; Almost sure; Rather sure; A bit sure; Weakly sure; Not sure at all”). Participants could access the Glacyceas/Crust information anytime by clicking on a link – when they did so, the information appeared on the right half of the screen, the problem remaining on the left side. Participants were able to modify their answers to a given question as long as they had not moved to the next question. The second part of the experiment was similar to the first, except that participants had access to some additional information and were asked a different set of questions (in order to test Majority and to reduce the mental load associated to the presentation of the premises altogether).

Rationale. Given the percentages of “Yes”, “No” and “there is no way to tell” answers to each question, we computed a Chi-square coefficient in order to test the differences between these percentages. We concluded that human inference was consistent with some property if the modal response was the predicted one (depending on the considered system) and the null hypothesis H0 (“there is no significant difference between the percentages of responses”) was rejected, i.e. the probability of obtaining a value as large as the observed Chi-square was not greater than .05, as it is usual in experimental psychology.

Results. Eight out of the 57 participants were excluded from the analyses, for they systematically answered “there is no way to tell” when one premise included “generally”.

System **P**: Table 2 shows that 23 participants out of 49 reasoned monotonically about the monotonic argument (MN), and that the remaining participants (26) concluded that “there is no way to tell”. There is no significant difference between these proportions. On the contrary, a significant majority of participants selected the predicted response of LLE, RW, OR and CUT patterns. With CM, the same proportions of participants selected the positive and negative answers, while only a few participants selected the “no way to tell” response.

Table2: Rules and questions for the test of Ambiguity Preservation (AMBd and AMBi), Property Inheritance (INH and INHg) and Majority (MAJ). “dnc” stands for“does not conclude”. “diff” stands for “predicts a difference in Yes responses”.

AMBd	Necrophagous Glacyceas do not, generally, live in a solid environment. Hermaphrodite Glacyceas generally live in a solid environment.	LC	MSP	LCD
	Do you expect this hermaphrodite, necrophagous Glacycea to live in a solid environment?	dnc	dnc	dnc
AMBi	Translucent Glacyceas generally live in large colonies. Hermaphrodite Glacyceas generally live in small colonies. Hermaphrodite Glacyceas generally live in a solid environment. Hermaphrodite Glacyceas do not, generally, have mandibles. Glacyceas living in a solid environment generally have mandibles.			
	Do you expect this hermaphrodite, translucent Glacycea to live in a large colony?	No	No	dnc
INH	Translucent Glacyceas generally live in large colonies. Translucent Glacyceas are generally shorter than 1 mm. Crusts are generally translucent. Crusts generally live in small colonies.			
	Do you expect this Crust to be longer than 1 mm?	No	dnc	No
INHg	Translucent Glacyceas generally live in large colonies. Non-translucent Glacyceas are generally shorter than 1 mm. Crusts are generally translucent. Crusts generally live in small colonies.			
	Do you expect this non-translucent Crust to be shorter than 1mm?	Yes	dnc	No
MAJ	Hermaphrodite Glacyceas generally live in a solid environment. Necrophagous Glacyceas do not generally live in a solid environment. Bulging Glacyceas generally live in a solid environment.			
	Do you expect this necrophagous, hermaphrodite and Bulging Glacycea to live in a solid environment?	diff.	no diff.	no diff

Rational Monotony: In this study, the rule “Hermaphrodite Glacyceas are not generally translucent” was asserted as plausible, and participants had to answer to the following questions: 1. “Do you expect a hermaphrodite Glacycea with mandibles to be non-translucent?” (RM) and 2. “Do you expect a hermaphrodite Glacycea to have mandibles?” (RMc). Question 1 without question 2 would only allow for the test of Monotony. More precisely, the endorsement of the RM pattern presupposes that the agent has not the belief “α |~ $\neg\beta$”. As such, the critical answer to this question, formulated under the form “α |~ β”, is “there is no way to tell”. Given this answer, any subject that accepts the “$\alpha\wedge\beta$ |~ γ” consequence, exhibits an answer consistent with RM.

On the contrary, any subject who rejects this consequence or concludes “there is no way to tell” contributes to the falsification of the RM hypothesis. Table 2 shows that, on the whole, participants reasoned monotonically (18 participants out of 22 choose the A response).

This appears to be independent from the response to question 2 (RMc), given that 9 subjects out of 18 accepted “α |~ $\neg\beta$” (which is consistent with CM), and the other 9 subjects did not accepted “α |~ $\neg\beta$” (while the four remaining subjects answered “there is no way to tell” to the “α |~ $\neg\beta$” and to “$\alpha\wedge\beta$ |~ γ”). Thus, results do not appear to be conclusive about RM.

Inheritance: Regarding the INH pattern, Table2 shows that half participants’ answers were consistent with LC and LCD, that is, the exceptional subclass inherited the property from the super-class despite the contradiction observed with another property (no inheritance blocking). Blocking of inheritance (consistent with MSP) is observed for the remaining participants. With respect to the more general case of inheritance (INHg), Table2 shows a dominance of LC and LCD over MSP.

Ambiguity: Table2 shows that, whether ambiguity was direct (AMBd) or indirect (AMBi), modal response was: “there is no way to tell.” This result is consistent with both LC, MSP and LCD for AMBd, but not for AMBi. Such a result could be due to the neglect of the additional premises implied in the test of AMBi. Indeed, the AMBd pattern is more simple and included in the AMBi one, and subjects could have limited their reasoning to the point corresponding to AMBd.

Majority: According to the hypothesis that participants’ reasoning is affected by majority, we should observe a decrease in the proportion of “there is no way to tell” responses, or, at least, a decrease in the mean of certainty judgements. Results show no significant difference between the proportions of responses to the AMBd question and to the MAJ question. However, a comparison based on the “Student t test” between mean certainty judgements for the two questions shows that when participants answer “there is no way to tell”, they are significantly less confident with MAJ question than with AMBd question ($t(21) = 2$; $p = .05$, i.e. the probability of rejecting by error the null hypothesis that there is no difference between the two means is .05). It denotes an impact of Majority. Thus, our results are consistent with MSP and LCD if we consider the proportion of C responses, but consistent with LC if we consider the mean degree of expressed certainty.

In sum, participants’ inferences appear to be more consistent with LC than with MSP and LCD. However, results do not appear to be absolutely conclusive, in particular because the tests of Rational Monotony and majority were not satisfactory, possibly because of the difficulty of the task

Experiment 2

A second experiment has been run in order (a) to study more deeply RM comparing it to MN and CM, and (b) to study majority, in particular when there is more than one premise.

Table 3: Observed frequencies and Chi^2 values by pattern of inference. The exponent values notify the predicted responses: L for LC; M for MSP, E for LCD, P for System **P**. N = 49, except for MAJ and MRc (N = 22); df = 2 except for MN, RM, AMBd and AMBi (df = 1).

	Responses frequencies			Chi^2	p
	A	B	C		
MN	23	0	26	.18	ns
LLE	4	31^{P}	14	22.8	.000
RW	31^{P}	2	16	25.8	.000
OR	1	39^{P}	9	49.1	.000
CM	21	21^{P}	7	8	.02
Cut	31^{P}	1	17	27.6	.000
RM	18^{LM}	0	4^{E}	27.8	.000
RMc	9	9	4	2.3	ns
AMBd	0	2	20^{LME}	29.4	.000
MAJ	2	2^{L}	18^{ME}	23.3	.000
AMBi	0	4^{LM}	18^{E}	48.3	.000
INH	3	21^{LE}	25^{M}	16.8	.000
INHg	1	31^{LE}	17^{M}	27.6	.000

Participants. Forty first-year psychology students at the University of Toulouse-Le Mirail, all native speakers in French, contributed to this study. None of them had previously received any formal training in logic.

Table4: Rules, questions and expected responses for the test of monotony, cautious monotony, rational monotony, ambiguity, and two forms of majority (MAJ1 and MAJ2). “dnc” means “does not conclude”. For the test of RM, the predicted response to the second question (RM) is “yes” for only participants that respond dnc to the first question (RMc).

MN	Worms are generally translucent.			
	Do you expect this Worm, with more than one year of life expectancy, to be translucent?		yes	
CM	Hermaphrodite Glacyceas generally live in a solid environment. Hermaphrodite Glacyceas do not, generally, have mandibles.			
	Do you expect this hermaphrodite Glacycea, living in a solid environment, to have mandibles?		no	
RM	Worms are generally translucent.	**LC**	**MSP**	**LCD**
	Do you expect this worm is necrophagous ? (RMc)	dnc	dnc	dnc
	Do you expect this necrophagous worm to be translucent? (RM)	yes	yes	dnc
AMBd	Necrophagous Glacyceas generally live in a liquid environment. Hermaphrodite Glacyceas generally live in a solid environment.			
	Do you expect this hermaphrodite, necrophagous Glacycea to live in a solid environment?	dnc	dnc	dnc
MAJ1	Necrophagous Glacyceas generally, live in a liquid environment. Hermaphrodite Glacyceas generally live in a solid environment. Bulging Glacyceas generally live in a solid environment.			
	Do you expect this necrophagous, hermaphrodite, bulging Glacycea to live in a solid environment?	yes	dnc	dnc
MAJ2	Necrophagous Glacyceas generally, live in a liquid environment. Hermaphrodite Glacyceas generally live in a solid environment. Bulging Glacyceas generally live in a solid environment. Speedy Glacyceas generally live in a solid environment.			
	Do you expect this necrophagous, hermaphrodite, bulging, speedy Glacycea to live in a solid environment?	yes	dnc	dnc

Material. A set of 7 concrete but non-familiar default rules and of 7 questions were involved in the empirical test of the 6 arguments (see Table 3) constructed for the study of Monotony, Cautious Monotony, Rational Monotony, Ambiguity, and majority.

Design and procedure. The experiment was presented through two booklets. One featured the instructions: Participants were informed about the aim of the experiment, and about some characteristics of default rules. Next, the following scenario (slightly different from the one used in the first experiment) was introduced:

> Consider the following facts. Scientists have discovered some new life forms in the Arctic Ocean (North Pole), some life forms that can develop in extreme cold. As a generic term, they are called “Glacyceas”. Glacyceas come into two main varieties, “Crusts” and “Worms”. Moreover, Glacyceas live either in a solid or a liquid environment. They are either hermaphrodites (at the same time male and female) or sexually differentiated, are flat or bulging, are necrophagous or not, are opaque or translucent, have mandibles or not, have more than one year of life expectancy or not, and move slowly or fast.

On the third page, participants were presented with a set of 7 default rules. Potential effect of affirmative versus negative framing of premises was controlled for.
A second booklet of seven pages, with one question (see Table 3) per page, was given to participants. Questions were framed as in Experiment 1.

Rationale. The rationale was the same as in Experiment 1.

Results. Table 5 shows that 33 participants out of 40 reasoned monotonically about the monotonic argument (MN). This proportion is even higher with CM : 36 out of 40. A comparison between the average certainty judgements to the MN and CM questions shows that participants were significantly less confident ($t(38) = -3.55$; $p < .001$) with the MN question (mean (m)= 3.23, standard deviation (sd) = 1.11, n = 39) than with the CM question (m = 3.92, sd = 1.03, n = 39). Thus, the additional premise involved in CM enables to conclude monotonically with more confidence.

RM was tested according to the same rationale as in Experiment 1. 28 participants out of 40 provided an answer consistent with the "$\alpha\wedge\beta|\sim\gamma$" consequence, and 26 out of these 28 (and out of 36) concluded "there is no way to tell" to the question about "$\alpha|\sim\neg\beta$". These results thus support the hypothesis that rational monotony has good psychological validity. However, 10 out of the 12 subjects that did not provide an answer consistent with the "$\alpha\wedge\beta|\sim\gamma$" consequence also concluded "there is no way to tell" to the question about "$\alpha|\sim\neg\beta$", which contributes to falsify the hypothesis that RM is consistent with human inference. As such, our results are not decisive about RM.

In addition, a comparison between the average certainty judgements to the MN and RM questions showed no significant difference ($t(38) = -1.1$; non sigificant) between MN (m = 3.20, sd = 1.04, n = 25) and RM (m = 3.44, sd = 1.16, n = 26). Finally, a comparison between the average certainty judgements to the CM and RM questions showed a significant difference ($t(39) = 3.11$; $p = .003$) between CM and RM. CM led to more confident conclusions than RM.

Table 5: Observed frequencies and Chi^2 values by pattern of inference. The exponent values notify the predicted responses: L stands for LC; M for MSP and E for LCD. N = 40; df = 2.

	Responses frequencies A	B	C	Chi^2	p
MN	33	2	5	43.8	< .001
CM	1	36^{ELM}	3	57.9	< .001
RM	28^{LM}	4	8^{E}	21.8	< .001
RMc	*2*	*2*	*36*	*57.8*	*< .001*
AMB	1	6	33^{ELM}	44.4	< .001
MAJ1	25^{L}	1	14^{EM}	21.6	< .001
MAJ2	25^{L}	1	14^{EM}	21.6	< .001

Concerning majority, Table5 shows that the modal response to the ambiguity question was: "There is no way to tell". This result is consistent with LC, MSP and LCD patterns. According to the hypothesis that participants' reasoning is affected by majority, we should observe some decrease in the proportion of "There is no way to tell" responses as the number of premises increases. Such is indeed the case when one additional premise is included (MAJ1), but further additional premises have no effect (MAJ2).

Comparisons between the average certainty judgements show that participants are significantly more confident about the conclusion related to the ambiguity pattern (m = 3.95, sd = 1.2, n = 40) than to the majority ones (m_{MAJ1} = 3.08, sd = 1.21, n = 40); m_{MAJ2} = 3.05, sd = 1.24, n = 40). These results are consistent with LC only. Previous results have been less clear regarding to the effect of majority. The effect of majority on the proportions of C responses can be explained by the fact that this new task is less cognitively demanding because of the lesser number of premises involved in the test.

To conclude, these results show that LC clearly appears to have better psychological plausibility than MSP and LCD.

Conclusion

We began by presenting three different forms of possibilistic nonmonotonic consequence relations: MSP, LC and LCD. We have shown that LCD can be encoded in possibility theory framework. Taking advantage from the fact that the three nonmonotonic consequence relations we considered had different inferential properties, we proceeded to an experimental test of their psychological plausibility: our rationale was that the relation which showed the most overlap (in terms of properties) with actual human nonmonotonic reasoning ought to be the most psychologically plausible.

The results of our experimental investigation provided clear support to LC. In particular, participants clearly manifested sensitivity to majority. Results of this paper can be useful for other nonmonotonic inference relations which are not based on possibility theory. Indeed, it must be stressed that any nonmonotonic consequence relations that satisfy the same properties should be viewed as a psychologically relevant one in the light of our results.

These results demonstrate how the experimental investigation of formal models can benefit to AI (by complementing researchers' intuitions about nonmonotonic reasoning) as well as to psychology (by offering some new insights and perspectives on actual human reasoning).

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References

Adams, E. W. (1975) *The logic of conditionals*. Reidel, Dordrecht.

Benferhat S., Bonnefon J. F., and Da Silva Neves R. (2004). An overview of possibilistic handling of default reasoning: applications and empirical studies. To appear in Synthese. A short version appears in 1st Salzburg workshop on Paradigms of Cognition July 18-20, 2002.

Benferhat S., Cayrol C., Dubois D., Lang J., Prade H. (1993). Inconsistency management and prioritized syntax-based entailment. Proceedings of the 13th International Joint Conference on Artificial Intelligence (IJCAI'93) Chambéry, France, pp, 640 -645.

Benferhat S., Dubois D., Prade H. (1997). Nonmonotonic reasoning, conditional objects and possibility theory. Artificial Intelligence Journal, 92, pp. 259-276, 1997.

Benferhat S., Saffiotti A., Smets P.(2000) Belief functions and default reasoning. Artificial Intelligence Journal, vol. 122/1-2, 2000, pp. 1-69.

Benferhat S., Kaci S., Le Berre D., Williams M. A. (2001b). Weakening conflicting information for iterated revision and knowledge integration. Proc. of the 17th Intern. Joint Conf. on Artificial Intelligence IJCAI01, , p. 109-115, 2001.

Da Silva Neves R.M., Bonnefon J.F. and Raufaste E. (2002). An empirical test of patterns for nonmonotonic inference. Annals of Mathematics and Artificial Intelligence, 34 : 107-130.

Dubois, D., Lang, J. and Prade, H. (1994) Possibilistic logic. In: Handbook of Logic in Artificial Intelligence and Logic Programming, Vol. 3 (D.M. Gabbay, C.J. Hogger, J.A. Robinson, D. Nute, eds.), Oxford University Press, 439-513.

Didier Dubois, Henri Prade. Conditional objects, possibiliy theory and default rules. Dans : Conditionals : From Philosophy to Computer Sciences. Crocco, G. et Fariñas del Cerro, L. et Herzig, A. (Eds.), Oxford University Press, 301--336, 1995.

Elio, R., and Pelletier, F.J. (1993). Human Benchmarks on AI's benchmark problems. In Proc. of the 15th Conference of the Cognitive Science Society, Boulder, CO, pp. 406-411.

Gabbay, D. M. (1985) Theoretical foundations for nonmonotonic reasoning in expert systems. In: K.R. Apt (Ed.) Logics and models of Concurrent Systems. Springer Verlag, Berlin, 439-457.

Gärdenfors, P. and Makinson D. (1994) Nonmonotonic inference based on expectations. Artificial Intelligence, 65, 197-245.

Hewson, C. and Vogel, C. (1994). Psychological evidence for assumptions of path-based inheritance reasoning. In Proc. of the 16th Conference of the Cognitive Science Society, Atlanta, pp. 403-414.

Gabriele Kern-Isberner (1991a) Handling Conditionals Adequately in Uncertain Reasoning. Proceedings of 6th European Conference Symbolic and Quantitative Approaches to Reasoning with Uncertainty, ECSQARU 2001, pp. 604-615, Lecture Notes in Computer Science 2143 Springer.

Gabriele Kern-Isberner (1991b) Representing and Learning Conditional Information in Possibility Theory. Computational Intelligence, Theory and Applications, International Conference, 7th Fuzzy Days, Lecture Notes in Computer Science 2206 Springer, pp. 194-217.

Kraus, S., Lehmann, D. and Magidor, M. (1990) Nonmonotonic reasoning, preferential models and cumulative logics. Artificial Intelligence 44, 167-207.

J. Lang (1996) Conditional desires and utilities -- An alternative logical framework for qualitative decision theory. Proceedings of the 12th European Conference on Artificial Intelligence (ECAI'96), 318-322

D. Lehman (1995) Another perspective on default reasoning, Annals of Mathematics and Artificial Intelligence: 15(1) (1995) pp. 61-82

Lehmann, D. and Magidor, M. (1992) What does a conditional knowledge base entail? Artificial Intelligence, 55, 1-60.

Pearl J. (1990) System Z: A natural ordering of defaults with tractable applications to default reasoning. Procs. of Theoretical Aspects of Reasoning about Knowledge, 121-135.

Pelletier F.J., and Elio R. (1997). What should reasoning be, by default ? Computational Intelligence, Volume 13, Number 2, pp. 165-187.