Logics for Spatial Reasoning

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Over the last decade temporal logics been extensively and successfully used as formal tools both in AI and software engineering but logics describing space have not been developed and exploited to the same degree, although we believe they are at least as important. Reasoning about space requires the explicit representation of topological, metrical and geometrical information. In contrast to our work, most Qualitative Reasoning research about space has concentrated upon geometrical information, e.g. for reasoning about kinematics, with less emphasis on topological and metrical information and has not been grounded in a logic.

Our basic formal theory assumes a primitive dyadic relation: $C(x, y)$ read as 'x connects with y' which is defined on non null regions. C is reflexive and symmetric. In terms of points incident in regions, $C(x, y)$ holds when regions x and y share a common point. Using the relation C, a basic set of dyadic relations are defined. These relations are DC (is disconnected from), P (is a part of), PP (is a proper part of), = (is identical with), O (overlaps), DR (is discrete from), PO (partially overlaps), EC (is externally connected with), TP (is a tangential part of), NTP (is a nontangential part of), TPP (is a tangential proper part of), NTPP (is a nontangential proper part of), TPI (is the identity tangential part of), and NTPI (is the identity nontangential part of). The relations P, PP, TP, NTP, TPP and NTPP support inverses. Of the defined relations, the set DC, EC, PO, TPP, NTPP, TPI, NTPI, and the inverses for TPP and NTPP form a mutually exhaustive and pairwise disjoint set of "base relations". The theory also includes an additional primitive function 'the convex hull of x', which is axiomatised and is used to generate a further set of dyadic relations. These additional relations are used to describe regions that are either inside, partially inside or outside other regions. We have also developed a variety of inference mechanisms for dealing specifically with space and time. Eg transitivity tables have been constructed and put to use in several ways.

We have also specified envisioning axioms which describe direct topological transitions that can be made between pairs of regions. These axioms rule out certain transitions - eg no direct transition between DC and PO is allowed. A simulation program has been built for the logic based on Kuiper's QSIM algorithm, except that our ontology of regions, states based on sets of simultaneously satisfiable atomic formulae, and constraint rules that determine direct topological transitions between pairs of spatial regions, marks a clear difference. Using this specialised theorem prover we have simulated a biological process (that of phagocytosis and exocytosis in unicellular organisms) and a mechanical process (the cycle of operations in a force pump).

The original theory assumes an ontology of regions that are (topologically) either open, semi-open or closed. We found that the advantages offered by this explicit characterisation, merely complicated matters in practice and have recently found a way to eliminate this distinction while keeping the same overall useful expressiveness.

References

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