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Information Loss Versus Information Degradation

Deductively valid transitions are truth preserving in the sense that any such transition from true premises yields a true conclusion. It is this truth preserving quality of deductive validity that makes it a much prized quality, truth being one of our central concerns. However deductively valid transitions are not always or even typically information preserving. For instance the transition from

(A) Ken is in New York and Alan is in Sydney and Elena is in Rome and . . . and John is in Venice.

- imagine the above to be a consistent hundred conjunct conjunction - to the conclusion

(B) Elena is in Rome

though deductively valid is in an obvious sense not an information preserving transition. (B), for instance, does not preserve the information about Alan's whereabouts contained in (A).

Information preservation is not always a desirable quality in inferential transitions. Sometimes our particular interests makes the transition from a large information base to a particular bit of information highly desirable. Given an interest focused solely on the whereabouts of Elena a transition from (A) to (B) is more appropriate then a transition from (A) to a conclusion containing the first 50 or so conjuncts of (A) even though the latter conclusion has more information than (B).

However not all losses of information are on a qualitative par. Compare the transition from our hundred conjunct conjunction (A) to

(C) Elena is in Rome and Alan is in Sydney

and the transition from (A) to

(D) Elena is in Rome or Alan is <u>not</u> in Sydney.

Both transitions are deductively valid, both involve a diminution of information. However there is a degradation of information in the (A) to (D) transition of a particular kind that is not shared by the (A) to (C) transition. How are we to separate the first kind of information loss from the second?

In Gemes [1994] I introduced a notion of content defined in terms of a partition of the standard deductive consequence classes of formulae. Thus we have the informal definition,

(D1) α is a content part of $\beta =_{\text{df.}}$ α is deductive a consequence of β and there is no deductive consequence σ of β such that σ is stronger than α and σ contains only (non-logical) vocabulary that occurs in α.

We say σ is stronger than α if and only if σ deductively entails α but α does not deductively entail β.

This conception of content may be used to characterize the difference between the two different kinds of information loss noted above.

One thing that characterizes the (A) to (C) transition is that while our hundred conjunct conjunction (A) deductively entails the conclusion (C), (C) itself does not entail (A). However this is equally true of the second (A) to (D) transition. Indeed this gives a clear criteria of what kind of transition involves a loss of information. A transition from A to B involves a loss of information if A entails B but B does not entail A. Now note, one thing that separates the two transitions is that the transition from (A) to (C) is a transition to a content part of (A), whereas the transition from (A) to (D) is not a transition to a content part of (A). (D) is not a content part of (A) since, for instance, (C) is a consequence of (A) that is stronger than (contains more information than) (D) and is statable in the vocabulary of (D). In an intuitive sense the inference from (A) to (C) has the maximum bang (information) for the buck (richness of vocabulary). On the other hand the transition from (A) to (D) involves a transition to a consequence that does not have maximum bang for the buck. It is not a transition to a consequence which is the strongest consequence statable in its own vocabulary.

Here then is a way of specifying the difference between the mere information loss and information degradation

(D2) The transition from α to β involves an information loss when α entails β and β does not entail α.

(D3) The transition from α to β involves a degradation of information when α entails β but β is not a content part of β.

The transition from (A) to (C) involves information loss without information degradation. The transition from (A) to (D) involves information loss with information degradation.

More polemically we might say that a deductively valid transition that involves information degredation is a transition to an irrelevant consequence. Thus we have the definition

(D4) α is an irrelevant consequence of $\beta =_{\text{df.}}$ α is a consequence of β and there is some consequence σ of β such that σ is stronger than α and all of σ's (non-logical) vocabulary occurs in α. (i.e. α is consequence but not a content part of β).

Generally transitions that involve information loss are tolerable, and indeed often desirable. For instance, think of the culling of a single desired phone number from a data base containing millions of phone numbers. However transitions involving information degradation, transitions to irrelevant consequences, are generally to be avoided. For instance, imagine an operator who when queried about the phone number of Jones checks his information base realizes the number is 882-3232 and so replies in a truthful but information degrading way "The number is 882-3232 or 256-9794."

The (D1) notion of content can also be utilized for the purposes of trimming information banks of irrelevant information. Suppose a detective is seeking information about the whereabouts of Jones. At first he learns that Jones is in Sydney, Melbourne or Hobart. Later he gets information that pinpoints Jones in Sydney. At this point the claim 'Jones is in Sydney, Melbourne or Hobart' may be expunged from the detective's data bank - his belief set - since it is no longer a content part of that data bank. This suggests that belief sets are optimally represented by sets of content parts rather than other sets of consequences of the relevant agent's beliefs.

For formal propositional languages there are well known machine implementable algorithms for testing if the transition from well formed formula (wff) β to wff α is deductively valid (e.g. the truth table method). So there is a mechanical decision procedure for testing if a given transition involves information loss. Gemes [1994] demonstrates that there is a mechanical decision procedure for testing if propositional wff α is a content part of propositional wff β. So there is a mechanical decision procedure for testing for information degradation.

To see if α is part of the content of β put α in (minimal) Boolean normal form and test if some proper sub-disjunct of that form is entailed by β. If not, and provided α itself is entailed by β, α is a content part of β.

For example the (minimal) Boolean normal form (unique up to ordering of conjuncts and disjuncts) of '(pvq)' is '(p&q)v(p&~q)v(~p&q)'. Now '(p&r)' entails a proper sub-disjunction of '(p&q)v(p&~q)v(~&q)', namely '(p&q)v(p&~q)', so our test delivers the correct result that '(pvq)' is not part of the content of '(p&r)'. On the other hand 'p' is part of the content of '(p&r)' since it is entailed by '(p&r)' and no proper sub-disjunction of its minimal Boolean normal form - being 'p' itself - is entailed by '(p&r)'.

Gemes [1994a] shows how the definitions of content may be extended to more complex quantificational languages.

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References

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