Semantic Vagueness and Context-dependence

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Abstract

A new solution is offered for the ancient 'sorites' paradox of vagueness. The sorites paradox arises if it is assumed that if two objects are indistinguishable, then a vague predicate (e.g., *large*, or *small*) can never distinguish between them. The proposed solution to the paradox depends on a systematic *ambiguity* that is displayed by vague predicates as well as vague relations (e.g., the relation of *indistinguishability*), which arises from the influence of discourse context on their interpretation. This solution is described against the background of recent research on the role of context in natural language interpretation.

Introduction

The sorites (i.e., 'stacked reasoning') paradox of vagueness has puzzled philosophers and logicians for a long time. Since it is well-known that the interpretation of vague predicates (e.g., *large*, or *small*) tends to be context-dependent, and since recent work in the semantics of natural language has revealed much about context-dependent interpretation, it seemed promising to see whether this work sheds new light on the issue of vagueness. Now as it turns out, this is indeed the case. In the present paper, a rather strong claim will be defended: that much of what used to be puzzling about the sorites paradox has now seized to be, and that recent work in semantics, properly viewed, already amounts to a solution of the paradox. For starters, here is a representative version of the paradox.

Suppose you are to judge the height of each of a long series of people, looking at them from a distance that makes a difference in height invisible as long as it amounts to less than, say, 1 cm. Of each of them, you are being asked whether they are short or not. The line-up starts from the shortest, who is very short, and ends with the tallest, who is very tall. The difference between subsequent persons is always less than 1cm, and therefore unnoticeable to you.

Now if you decide that the first person (p_0) is short, you must also judge the next one short, since you can perceive no difference between the two. But then, by the same token, the third person (p_2) must be short as well, and so on indefinitely. In particular, this also makes the last person (p_n) short. But, by assumption, p_n is not short, so a contradiction has been derived:

1. p_0 is short, and p_n **is not short** (Assumptions).
2. If p_0 is short then p_1 is short.
3. Therefore, p_1 is short.
4. If p_1 is short then p_2 is short.
(...)
2n+1. Therefore, p_n **is short**. $\perp$

The reasoning is elementary, and relies only on a long series of applications of Modus Ponens, and on one crucial premiss, which may be called 'inductive' because of its similarity to one of the premisses in a proof by mathematical induction:

Inductive Premiss: For all x and y, if x is short and y is indistinguishable from x, then y is short.

Because of their supposed insensitivity to nonperceptible characteristics of their arguments, words such as *short* are sometimes called Perception Predicates. We will assume that it is the use of a Perception Predicate from which the sorites paradox derives its cogency, since it is these predicates that give rise to the relation of indistinguishability.[1] Many proposals have been made to resolve this paradox, most of them involving radical departures from classical logic. Let me briefly point out what I take to be the requirements for a solution of the paradox. A proper solution should

1. Explain the plausibility of the argument that leads up to the paradox, while showing that the argument is nevertheless invalid.
2. Be consistent with what is known about the meaning of vague predicates. Preferably, the solution would be backed-up by empirical evidence.

[1] Thus, if the sorites argument is transferred to a domain without perceptual limitations and where, consequently, no Perception Predicates exist, then no sorites argument can be formulated that has the same cogency as the one presented in the text.

3. Use as few unmotivated departures from classical logic as possible. A departure can only be motivated by an appeal to (2).

I will argue along the following lines: Empirical research in the last ten years has revealed that *context-dependence* is a pervasive phenomenon in natural language that affects expressions of all kinds, including vague ones. In line with this research, it is hypothesized that vague predication must always involve at least two arguments: an object judged *and* a set of elements with which this object is being compared. Consequently, vague predication is incomplete without a 'comparison set', in comparison with whose elements the predication must be understood. Nothing is small or large *per se*, but only in comparison with other things — As will become clear later, this idea is not restricted to vague *predicates*. In particular, the relation of indistinguishability will also prove to be dependent on a 'comparison set' for its interpretation. Predicates and relations alike take their comparison sets from the linguistic and nonlinguistic context of the utterance.

In order to deal with the sorites paradox, the argument itself is viewed as a piece of discourse during which a comparison set is built up. As a result, when the context-dependence of the vague expressions in the argument is taken into account, it turns out that the Inductive Premiss of the sorites argument becomes *ambiguous*. What makes sorites arguments plausible is the fact that, in some of its interpretations, the premiss supports the argument, **but** then it fails to be true by virtue of the meaning of the vague predicate; in other versions, it is true by virtue of the meaning of the predicate, **but** then it does not support the argument. As we will see, this pattern of explanation holds up even in the case of vague expressions that are not normally viewed as context-dependent.

The current approach to the paradox differs from most other accounts that I know of in that it does not propose a drastic change in the underlying logic. In particular, there is no need to assign unusual interpretations to the conditional, the negation, or the universal quantifier. On the other hand, the interpretation of non-logical constants changes, in that they acquire an additional argument place for a comparison set.

Short and *small* will be our paradigmatic vague predicates, but everything that will be said is meant to be applicable to most vague predicates of the kind that, intuitively speaking, 'measure' something. Such predicates have been studied in standard measurement theory, and their properties are well established.[2] The sentence 'x is small' (or 'x is short') will often be abbreviated by the formula $S(x)$. With the comparison set A as an additional argument, we will write $S(x)^A$ to abbreviate 'x is small with respect to A'. For a more elaborate discussion, the reader is referred to Van Deemter (to appear).

Context-dependence in Natural Language

The notion of context has long been recognized as an important concept in linguistics, especially in formal treatments of the semantics of indexical expressions (Montague 1974, Kaplan 1979). Recent years have seen a rapidly increasing number of applications of the notion of context inside as well as outside linguistics.[3] One prominent example is Situation Semantics, which views meaning as a relation between utterance situations (nonlinguistic context, that is) and described situations. An example that is more directly relevant to our current endeavour is the tide of so-called dynamic theories of anaphora (Kamp 1981, Heim 1982, Barwise 1987), which started to explicate anaphora as a primarily contextual phenomenon. According to these theories, an anaphoric pronoun is only appropriate in a context in which a suitable antecedent for it has been introduced. For instance, if the following pieces of discourse constitute the beginning of a story, then (1) is inappropriate in a sense in which (2) is not.

1.* (a) Yesterday, I saw it. (b) A dog barked.
2. (a) Yesterday, I saw a dog. (b) It barked.

One might say that (2a) adds an individual to the context that is then taken up by *it* in (2b). The occurrence of *it* in (1) is not legitimized by previous context. The main task of an anaphoric theory is then to define under what circumstances a would-be antecedent is *accessible* to a given anaphor. This 'contextual' perspective on anaphora is, by now, widely accepted.

This contextual perspective on things anaphoric has gradually been broadened to include more and more kinds of expressions that depend on context for their interpretation. For example, Barbara Partee and others have analysed the context-dependence of implicit arguments of words such as *local* and *contemporary* along 'anaphoric' lines (Partee 1989, Condoravdi and Gawron, to appear). Perhaps most relevant for present

[2]The vague predicates (e.g., 'small') of standard measurement theory are predicates that correspond with perceptual relations (e.g. 'visibly smaller') that form a *semi-order*. A structure $\langle A, R\rangle$, where R is the perceptual relation (later denoted '$\prec$'), is a semi-order iff (1) $\forall x(\neg xRx)$, (2) $\forall xyzu((xRy \& zRu) \rightarrow (xRu \vee zRy))$, and (3) $\forall xyzu((xRy \& yRz) \rightarrow (xRu \vee uRz))$. Note that a semi-order allows for indistinguishable elements, i.e., elements x, y such that $x \neq y$, $\neg(xRy)$ and $\neg(yRx)$. For an introduction, see Suppes 1963.

[3]A recent attempt at constructing a unifying theory of the notion of context has been made by John McCarthy and others (McCarthy 1987, Buvač and Mason 1993, Buvač et al., to appear). We will not use such a general perspective on context, concentrating on one that is more convenient as long as only vagueness is at stake.

purposes is a research track that was started off in Westerståhl 1985, and where generalized quantifiers can be restricted to contextually available 'context sets'. Subsequent work has extended this approach and shown that noun phrases (NPs) of all sorts, and expressions of other categories as well, can be context-dependent (e.g., Carter 1987, Van Deemter 1992). Moreover, the principles that govern the accessibility of context sets turned out to bear close resemblance to those governing the availability of antecedents of anaphoric pronouns. Thus, the possibility begins to present itself of a unified account of context-dependent phenomena, of which pronominal anaphora is just a special case.

Context-based analyses of NPs containing *vague adjectives* have also been forthcoming. An early account[4] is the one in Kamp 1975, which was worked out in much more detail by Ewan Klein. Consider the sentence 'Some small elephant runs'. According to Kamp and Klein, the set of elephants functions as a *comparison set* that somehow sets the standard for what counts as small (Klein 1980). This explains, for example, why a *small elephant* is not necessarily a *small animal*, since elephants are, on average, larger than the average animal.

However, this raises the following question: if a vague adjective can be dependent on *local* (i.e., NP-internal) context, then cannot it also be dependent on global (i.e., NP-external) context? After all, the situation of the sorites paradox is one in which no local comparison set is available, as in predicative use. Consider a sentence of the form 'x is small'. How is the standard for smallness determined if it is not through local context? The facts suggest that global context comes to the rescue: the standards of smallness are determined by a comparison set that has been built up during a discourse. There are hints of this idea in Kamp 1975, though no formal account.

Global context was taken into account in a series of later proposals for the semantics of vagueness, including Kamp 1981 and Veltman 1987. In this work, context does not play the role of a comparison set, but is only used to enforce certain kinds of coherence conditions. For instance, in Kamp 1981, a universally quantified formula is false if the addition of the formula to the context causes the context to become 'incoherent'. One thing that makes a context incoherent is when it contains elements a and b that are indistinguishable, while there is a vague predicate S that is true of a and false of b. Due to this move, Kamp can let each of a series of implications of the form $S(p_i) \rightarrow S(p_{i+1})$ be true, while the universally quantified formula that has these implications as its instantiations may still be false, because its addition to the context would cause the context to become incoherent. Thus, if a suitable domain is chosen, the inductive premiss of the sorites argument becomes false. Yet, all instantiations of the flawed premiss are true, and this explains why the paradox has intuitive appeal. A similar solution has been proposed by Veltman and Muskens[5] (Veltman 1987).

In the remainder of this paper, a solution to the paradox will be defended that makes use of the notion of global context, *modeled as a comparison set.* The starting point is that it makes little sense to ask, of a given object, whether it is small or large unless a comparison set is specified either implicitly or explicitly. Thus, a vague predicate such as *small* is modeled as a *relation* between an individual and a comparison set. For the sake of argument, it will be assumed that this relation is a precise (i.e., non-vague) one. Of course, this simplifying assumption may be challenged (e.g., Zadeh 1975), but what I intend to show is that the solution to the paradox does not depend on this.

Context, in the present proposal, is built up during discourse, in the spirit of dynamic theories of meaning: a discourse is parsed from left to right, and individuals are added to the context as a by-product. To attain the full theory of the semantics of vagueness, many details would have to be sorted out. But fortunately, the structure of a sorites argument makes it an extremely simple kind of discourse: subsequent elements are judged to be small, and then added to the context.[6]

The proposal of this paper makes use of some of the same building blocks as its predecessors, but amounts to a quite different solution of the paradox. At the root of the difference lies the fact that the current proposal takes the similarities between vagueness and anaphora seriously. As was stressed in van Deemter 1994, this anaphoric perspective predicts that the interpretation of a vague predication suffers from an intrinsic *ambi-*

[4]According to Kamp, the insight that a noun can act as a comparison set for a vague predicate, is 'probably too old to be traced back with precision to its origin' (Kamp 1975), p.127.

[5]Veltman and Muskens' proposal, which makes use of an idea from Goodman and Dummett that will also play an important role in the present proposal, is discussed at some length in van Deemter 1994.

[6]It would be worthwhile to study the context-dependency of vague expressions in greater empirical detail. For example, when a complex NP contains a vague adjective, local and global factors interact. Thus, if 'Some small elephants run' refers to the elephants in a zoo, one possible comparison set is the entire set of Elephants, and another is Elephants $\cap$ (Animals-in-the-Zoo). But Animals-in-the-Zoo seems to be a less natural comparison set. Since our present purpose is to explain the sorites paradox, it suffices to deal with predicative uses of vague adjectives, as in 'x is small', and in such cases global context-dependence is the only thing to worry about.

guity problem, since it is sometimes unclear whether a given expression must be interpreted as anaphoric or not. It will be argued that this ambiguity of vague expressions is exploited in the sorites argument, and the paradox will be analysed as a fallacy that is based on the ambiguity of one of its premisses.

Context-dependent versions of the sorites paradox

In the current section, it will be shown that when comparison sets are taken into account, the inductive premiss of the sorites argument becomes ambiguous, and that there exist highly consequential differences between some of its interpretations.

Given our 'contextual' perspective, what are the possible versions of the crucial premiss? There are three clauses in the premiss to consider: the clause in which x is assumed to be small ($S(x)$), the one in which x and y are compared ($x \sim y$), and the one in which y is concluded to be small ($S(y)$). Let us first look at the clause in which x and y are compared. The relation of being visibly smaller than something will be viewed as primitive, abbreviated as $\prec$ (Inverse: $\succ$).

$x \prec y \Leftrightarrow x$ is visibly smaller than y.
$x \succ y \Leftrightarrow y$ is visibly smaller than x.
$x \sim y \Leftrightarrow \neg x \prec y \;\&\; \neg y \prec x$.

How can context be relevant for the interpretation of (in)distinguishability? One interesting answer has been suggested in Goodman 1966 and examined more closely in Dummett 1975: the elements that a context makes available can be viewed as resources that can help distinguish observed individuals. Thus, one might first define 'x is smaller than y with respect to A' (hence $(x \prec y)^A$) and then 'x is indistinguishable from y with respect to A' (hence $(x \sim y)^A$). We would like to think of $(x \prec y)^A$ as expressing a relation that involves observation assisted by reason. Therefore, it is natural to let $(x \prec y)$ have $(x \prec y)^A$ as a logical implication, and since we do not want to prejudge the question of whether x and y themselves must be elements of A, the clause $x \prec y$ will be included as a separate disjunct:

$$(x \prec y)^A \Leftrightarrow_{Def} (x \prec y \vee \exists h \in A : (h \prec y \;\&\; \neg h \prec x) \vee (x \prec h \;\&\; \neg y \prec h)).$$

For example, in the following situation, h helps to tell x and y apart:

$$\underbrace{\overbrace{h \qquad\qquad x}^{\sim} \qquad\qquad \overbrace{x \qquad\qquad y}^{\sim}}_{\prec}$$

Assuming that $h \epsilon A$, the definition implies that, in this example, $(x \prec y)^A$. One way to motivate this is by arguing that y must be bigger than x, which follows if one assumes that a perceptible difference between h and some other element arises if and only if the difference in size between h and this other element exceeds some constant value. This constant will now be assumed to be the same for all h and called a Just Noticeable Difference (JND) (Suppes and Zinnes 1963).

Relativized *in*distinguishability is defined as follows:

$$(x \sim y)^A \Leftrightarrow_{Def} \neg(x \prec y)^A \;\&\; \neg(y \prec x)^A.$$

In effect, 'indistinguishability with respect to A' implies that A provides no help element to tell x and y apart. It is easy to see that $x \sim y \Leftrightarrow (x \sim y)^{\{x\}} \Leftrightarrow (x \sim y)^{\{y\}} \Leftrightarrow (x \sim y)^{\{x,y\}}$. Relativized indistinguishability has many interesting properties such as, for example, transitivity. Also, the Goodman/Dummett construction is stable, in the sense that iteration does not change it.[7]

At this point we have a plausible candidate for a relativized version of the clause that says 'x is indistinguishable from y'. So how about smallness, how can that notion be relativized to a comparison set? Let us simplify and assume that there is no ambiguity in the notion of 'Smallness with respect to a comparison set A', and that it involves a comparison between two sets: the set of elements in A that are visibly smaller than the element x that is judged (also abbreviated $K[x]^A$), and the set of elements in A that are visibly larger than it (also abbreviated $G[x]^A$). This is expressed in the following meaning postulate, which is illustrated by a picture:

Comparison Postulate: $S(x)^A \Leftrightarrow_{Def}$
$|\{y \in A : y \succ x\}|$ is greater than $|\{y \in A : x \succ y\}|$.

$$\underbrace{\cdots\cdots}_{K[x]^A} \quad \cdots\cdots \quad x \quad \cdots\cdots \quad \underbrace{\cdots\cdots}_{G[x]^A}$$

Whether or not x is small thus only depends on the cardinalities of $K[x]^A$ and $G[x]^A$. The elements in between $K[x]^A$ and x are not taken into account, and neither are those in between x and $G[x]^A$. It is argued in Deemter (to appear) that little hinges on whether a particular usage of *small* satisfies the Comparison Postulate, as long as some much more general meaning postulates are obeyed. But note that even if the Comparison Postulate is taken for granted, there are plenty of ways to render the inductive premiss of the sorites argument. As with anaphora, there is a resolution problem: each of the three clauses can, in principle, be relativized to any context set. The following options seem reasonable: each of the three clauses may be interpreted (a) with no context set at all, (b) with respect to some context set A that is given at the beginning of the argument, or with respect to (c) $A \cup \{x\}$,

[7] These observations were also made by Frank Veltman and Reinhard Muskens who show that the Goodman/Dummett notion of indistinguishability does not lead to the sorites paradox (Veltman 1987).

(d) $A \cup \{y\}$, and (e) $A \cup \{x, y\}$. As a result, there are 125 different versions of the inductive premiss.

Getting to grips with all these versions of the paradox may seem a daunting task, but their number can be reduced. Firstly, let us, in accordance with a dynamic perspective on interpretation, assume that elements in a context set are always only assembled from left to right. Consequently, the only three options for the *first* clause, $S(x)$, are (a), (b), or (c). The element y simply hasn't come up yet.

Secondly, let us assume that the formula $x \sim y$ is symmetric, in that it either introduces both x and y into the comparison set, or none of the two. Once more, only three options are left: options (a), (b), and (e). Unfortunately, these two assumptions do not reduce the options for the third clause, $S(y)$, however, so we are still left with 45 versions.

However, this number can be greatly reduced. $(x \sim y)^A$ and $(x \sim y)^{A \cup \{x,y\}}$ are equivalent, and this reduces the number of options for the indistinguishability clause to two: plain $\sim$ and '$\sim$ with respect to A'. To further reduce the number of options for the other two clauses, discard option (a) for the clauses $S(x)$ and $S(y)$, since S requires that some comparison set is specified. At this point, we are left with the following options, in which x, y and A are universally quantified: (Indicated are the sets to which the respective clauses may be relativized. A dash indicates the option of using no context set at all. A, x, and y are universally quantified.)

(S(x)	&	(x ~ y))	--->	S(y)
A	|	-	|	A
A U {x}	|	A	|	A U {x}
	|		|	A U {y}
	|		|	A U {x,y}

But most of these options are logically equivalent if 'smallness with respect to a comparison set' is interpreted as in the Comparison Postulate. It will never make any difference for the truth of the S-clauses whether the comparison set for a certain clause equals A, $A \cup \{x\}$, $A \cup \{y\}$, or $A \cup \{x, y\}$, given that x and y are perceptually indistinguishable. In other words: (b), (c), (d) and (e) are logically identical in the case of the two S-clauses. Consequently, only one version of the two S-clauses needs to be distinguished. For reasons of general plausibility in a dynamic setting, we select a version in which $S(x)$ is evaluated with respect to A, while $S(y)$ is evaluated with respect to $A \cup \{x\}$:

(S(x)	&	(x ~ y))	---->	S(y)
A	|	-	|	A U {x}
A	|	A	|	A U {x}

Neither of these two versions leads to a sorites paradox. The first one is easily falsified. Let A contain z as its only element, and assume x can just barely be discerned to be smaller than z. In other words, the difference in size between them equals a JND. Consequently, if $x \sim y$, while y is larger than x, then y is *not* distinguishably (i.e., discernably) smaller than z. Under these circumstances, $G[x]^A = \{z\}$ and $K[x]^A = \emptyset$, while $G[y]^A = K[y]^A = \emptyset$, and consequently $S(x)^A$, while not $S(y)^A$.

The second version of the premiss cannot be falsified. Such a version of the premiss, which must be true due to the meaning of the terms in the premiss, will be called *valid*. On the other hand, this version does not support the sorites argument. To see that it is valid, assume $S(x)^A$ while not $S(y)^{A \cup \{x\}}$. Thus, we have $S(x)^A$ and not $S(y)^A$. Now given the Comparison Postulate, this can only be true if some element h of A behaves differently towards x and y, in other words, if $h \prec y$ while not $h \prec x$, or if $x \prec h$ while not $y \prec h$. But under such circumstances, h fulfils the function of a help element in the sense of Goodman/Dummett, contradicting $(x \sim y)^A$. So, this version of the premiss is valid. To see that it does not support the paradox, let Size be a function that maps individuals to their (real-valued) size in the relevant dimension. Let x_0 be an element that is judged to be small with respect to the comparison set A_0. Then Size(x_0)+ JND functions as an upper limit on the sizes of elements that can be inferred, by the inductive premiss, to be small. The argument goes

$S(x_0)^{A_0}$ and $(x_0 \sim x_1)^{A_0}$, therefore
$S(x_1)^{A_0 \cup \{x_0\}}$; also, $(x_1 \sim x_2)^{A_0 \cup \{x_0\}}$, therefore
$S(x_2)^{A_0 \cup \{x_0, x_1\}}$; *etcetera*.

Note that x_0 is an element of all comparison sets after the initial one. As soon as one arrives at x_i such that $(x_0 \sim x_{i-1})$ and $(x_0 \prec x_i)$, then $(x_{i-1} \prec x_i)^{A \cup \{x_0, x_1, x_2, \ldots x_{i-2}\}}$. At that point, x_i is not indistinguishable from its predecessor (with respect to the relevant comparison set) and the argument comes to a halt. — A more detailed illustration of how this can happen is offered in the next section.

To sum up, when comparison sets are brought to bear on vague predicates, the inductive premiss of the sorites argument displays an 'anaphoric' ambiguity, that hinges on the choice of comparison sets for the respective clauses of the premiss. The premiss becomes ambiguous between two sorts of interpretations. One is strong enough to support the sorites argument but is invalid; the other is too weak to support the

paradox, but happens to be valid. I believe that this ambiguity is a plausible cause of confusion on the part of language users.

At this point, the *invalid* version of the inductive premiss need not occupy us much. The *valid* version, however, which employs Goodman/Dummett-style indistinguishability calls for some illustration.

Illustration: How the paradox breaks down.

To illustrate how comparison sets cause the version of the sorites argument that makes use of the Comparison Postulate for smallness and the Goodman/Dummett notion of indistinguishability to break down, let us go through an actual sorites argument. We are dealing with the following version of the inductive premiss:

Premiss **P**:
$\forall y \forall y \forall A\ ((S(x)^A\ \&\ (x \sim y)^A)\ \rightarrow S(y)^{A \cup \{x\}})$.

Imagine a line-up of individuals (Martians, say) that are just 1 mm apart, starting at 0.60 m and ending at 1 m. Let us number them from x_0 upwards: x_0 has a length of 0.60, and x_i has length 0.60 + i mm. Let a JND equal 10 mm. We will assume that the initial standard of comparison is something like one's own height. In my case this is sufficiently taller than 1 m, so the first Martian will be called small. Now the following is predicted, where the letter I denotes the speaker:

0. $S(x_0)^A$, since $A = \{I\}$, while $G[x_0]^A = \{I\}$ and $K[x_0]^A = \emptyset$

1. $S(x_1)^A$, since $A = \{I, x_0\}$, while $G[x_1]^A = \{I\}$ and $K[x_1]^A = \emptyset$

.....

9. $S(x_9)^A$, since $A = \{I, x_0, .., x_8\}$, and $G[x_9]^A = \{I\}$ and $K[x_9]^A = \emptyset$

10. $\neg S(x_{10})^A$, since $A = \{I, x_0, .., x_9\}$, and $G[x_{10}]^A = \{I\}$ and $K[x_{10}]^A = \{x_0\}$.

$$\underbrace{\overbrace{x_0 \quad x_9}^{\sim}\ x_{10}}_{\prec}\ \overbrace{\qquad\qquad}^{\prec}\ I$$

Somehow, the sorites argument must break down at, or before, x_{10}. There are prospective applications of **P** from (0) to (1), from (1) to (2),..., and from (9) to (10), with different instantiations of x, y, A. For example, one gets from (0) to (1) by chosing $x := x_0$, $y := x_1$, and $A := \{I\}$, observing that $(x_0 \sim x_1)^{\{I\}}$, since both are distinguishably smaller than the speaker. This works until one arrives at step (10). By assumption, x_{10} is 1 mm taller than x_9, making it, for the first time, distinguishably taller than the initial element x_0. As a result, $\neg((x_9 \sim x_{10})^{\{I, x_0, \ldots, x_8\}})$, since $(x_0 \sim x_9)$, but $(x_0 \prec x_{10})$. In other words: after having inferred that x_9 must be small, relative to the context at hand, there happens to be no suitable object around to instantiate the antecedent of **P**. Of course such an object might have been around, for example, an element $x_{10'}$ that is just 0.5 mm taller than x_9, and then another after $x_{10'}$, and so on. However, in that way one will always only infer the smallness of elements that are smaller than x_{10}, and no paradoxical conclusion is inferred. The value Size(x_0) + JND functions as a limit: only x that are smaller than this value (i.e., smaller than x_{10}) are Goodman/Dummett-style indistinguishable from x_9.

Observe that, in this example, elements stop being called small at precisely the same time when they stop being Goodman/Dummett-style indistinguishable from x_9. Should our initial comparion set have consisted of two elements, I and an element I' that is just 1 mm taller than I, then x_{10} would also have been Goodman/Dummett distinguishable from x_9, but it would still have been small, since $G[x_{10}]^A = \{I, I'\}$ and $K[x_{10}]^A = \{x_0\}$.

Starting from this example, it is easily seen that the paradox can break down in one of the following three ways: **(i)** the case illustrated above, in which the argument breaks down through an element x_i that has the same $G[x_i]^A$ as its predecessor, but an enlarged $K[x_i]^A$; **(ii)** the same $K[x_i]^A$, but a diminished $G[x_i]^A$; or **(iii)** through an element that is located in such a way that $G[x_i]^A$ and $K[x_i]^A$ change at the same time.

So, why is the paradox flawed? Well, firstly, it was assumed that a vague notion such as *small* requires a comparison set. Now *whatever* finite set one takes as the initial comparison set A_0, the result is some finite 'norm', implying some upper limit on what is small with respect to that norm. Of course, the norm will change when newly-judged individuals enter the comparison set. But, as long as the elements that are judged are *small* with respect to the current comparison set, they will only cause the norm to go *down*:

If $S(y)^A$, then $\|S^{A \cup \{y\}}\| \subseteq \|S^A\|$.[8]

As a result, all the individuals $(x_0, x_1, ...)$ that are added in the course of a sorites argument first leave the norm of the initial comparison set intact, and in the somewhat longer run, they cause the norm to become even stricter, causing an even earlier break-down of the argument.

Let me point out that many interesting variations on the theme of this construction can be conjured up, including, for example, notions of indistinguishability that contain a stochastic aspect, to account for the fact that observers tend to be less than completely consistent in their judgments. These and other variations are discussed at length in Van Deemter (to appear).

[8]The extension of *small* with respect to comparison set A is denoted $\|S^A\|$.

Conclusion

In the above, it was shown how the assumption that vague predications involve an explicit or implicit comparison set bears on the sorites paradox, through ambiguities in what constitutes the comparison set for the different clauses of the inductive premiss of the sorites argument. In the introduction of this paper, logical conservativity was defended to be an important *desideratum* for a solution to the sorites paradox. So, in how far can the current solution be regarded as logically conservative?

It has been noted that the only non-classical element in the proposal consists of relativizations of atomic clauses to comparison sets. Thus, a mechanism that is often only implicit in natural languages was made explicit in the logical object language. An alternative approach is also possible, which leaves comparison sets implicit. Mimicking the work of Groenendijk and Stokhof on Dynamic Predicate Logic (Groenendijk and Stokhof 1991), one might use an object-language that is syntactically equal to a variant of predicate logic, while its semantics ensures that comparison sets are provided. One might think of interpretive rules such as the following (where M is a model with universe E and interpretation function I, while $A \subseteq E$ is a comparison set):

$$M, A \models S(a) \& S(b) \Leftrightarrow$$
$$M, A \models S(a) \text{ and } M, A \cup \{I(a)\} \models S(b),$$

where the individual denoted by a enters the comparison set once a has been interpreted. Thus, interpretation would become truly dynamic. Our main reason not to 'go implicit' is that it would have forced us to rule out any ambiguities in the assignment of comparison sets, and ambiguity is a key element in the explanation of the paradox. Less crucially, it would also have tended to over-simplify the way in which comparison sets grow. For example, elements are not only added to a comparison set, but also substracted, since memories of the elements of comparison sets fade away and become irrelevant.

Our choice to keep references to context explicit has as a consequence that the logic is monotonic in the following sense: If premisses are added to the premisses of a valid argument, then a valid argument must again result. If $\Gamma \models S(x)^A$, then also $\Gamma \cup \{\varphi\} \models S(x)^A$, no matter what φ is. The language of relativizations can be embedded in classical logic by writing $S(x)^A$ as a two-place relation: $S(x, A)$. Intuitively, this language is used to model a non-monotonic situation. After all, a natural language sentence of the form 'x is small' can be true at a given point in a discourse (e.g., as a conclusion from premisses Γ), but false at some later point (e.g., as a conclusion from premisses $\Gamma \cup \{\varphi\}$). But non-monotonicity is delegated to the translation from natural language to logic: If 'x is small' is uttered in a context in which A is the comparison set for smallness, it is represented by means of a formula in which A appears; in a context in which a superset A' of A is the comparison set, A is replaced by A'.

This paper has advocated an approach to the sorites paradox of vagueness in which the notion of context-dependence plays a central role. This raises the question of whether vagueness must always involve context-dependence and how possible counterexamples affect the current proposal.

It is at least conceivable that there exist vague predicates that are not context-dependent. For example, their vagueness may result from perceptual or judgmental limitations, while the standards for comparison are invariable. A tentative example would be the vague predicate *healthy*. Arguably, whether a person is healthy is not dependent on the context in which the person's health is considered, but only on the requirements of survival and procreation. Let *smoll* be similar to the adjective *small*, except that it is 'absolute', like *healthy*:

$$Smoll(x) \Leftrightarrow_{Def} x \prec I.$$

Something is *smoll* if and only if it is distinguishably smaller than the speaker. Using the definiens of *smoll*, an informal version of the inductive sorites premiss says

Inductive Premiss: For all x and y, if $x \prec I$ and y is indistinguishable from x, then $y \prec I$.

Let us define, as before, the contextualized version $(x \sim y)^A$ in Goodman/Dummett-style, while noncontextualized $x \sim y$ is equivalent to something of the form

$$|\text{Size}(y) - \text{Size}(x)| < n.$$

Then what we see is the usual pattern:

1. $Smoll(x) \;\&\; x \sim y \;\rightarrow\; Smoll(y)$.
2. $Smoll(x) \;\&\; (x \sim y)^A \rightarrow Smoll\ (y)$.

Version **1** is clearly invalid and supportive. To determine the status of version **2**, one has to assume that $I \in A$, which seems natural enough. Consequently, **2** becomes valid for perfectly familiar reasons: suppose $Smoll(x)$ and $\neg Smoll(y)$, then $x \prec I$, but not $y \prec I$, which implies $\neg(x \sim y)^A$. This shows that a vague predicate need not be dependent on *discourse* context to give rise to the kinds of ambiguities that help us to understand the sorites paradox.

In how far does all this amount to a solution of the sorites paradox? I believe that it fulfils the *first* requirement that was formulated in the Introduction, namely of explaining the invalidity of the paradox, while at the same time accounting for its plausibility. Also, the approach is very much in the spirit of the *third* requirement, since its departures from classical logic are modest, as we have just seen. The only qualification that I can see has to do with the *second* requirement, of empirical adequacy. It might be

argued that the current proposal fails to reflect an intuition that has sometimes been put forward: that an object a that is smaller than a certain small object b is small *to a higher degree* than b. Doing full justice to this purported intuition of graduality, however, tends to lead to a highly nonclassical logic such as Zadeh's *fuzzy logic* (Zadeh 1975). There is no technical obstacle to superimposing graduality – in the form of multiple truth values – on a proposal of the kind that was advocated in this paper. But what I have tried to show is that such a drastic move is not required for the solution of the sorites paradox.

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