Boundary Region Relations

Emilio Remolina and Benjamin Kuipers *
Computer Science Department
University of Texas at Austin
Austin, Texas 78712 USA
e-mail: {eremolin,kuipers}@cs.utexas.edu

Abstract

We are interested in the problem of how an agent organizes its sensorimotor experiences in order to create a spatial representation. Our approach to solve this problem is the Spatial Semantic Hierarchy (SSH), where multiple levels of spatial representation coexists. At the SSH topological level, space is represented by *places* and connectivity relation among them. Places are arranged into *streets* so that the topological representation looks like the street network of a city.

Grouping places into *regions* allows an agent to reason efficiently about its spatial knowledge. Different types of regions can be defined as the agent travels in the environment. Using the language of *Causal Theories*, we give a formal account of how an agent establishes *boundary region relations* while navigating its environment.[1]

Introduction

The basic problem we are interested in solving is how an agent creates its spatial representation from its sensorimotor experiences. Our approach to solve this problem is the **Spatial Semantic Hierarchy (SSH)** (Kuipers & Byun 1988; Kuipers *et al.* 1993; Kuipers 1996; Kuipers & Byun 1991; Kuipers 1978; Kuipers & Levitt 1988). The SSH is an ontological hierarchy, where each level of the hierarchy has its own *ontology* abstracting the ontology of the levels below it. In this paper we are primarily concerned with the SSH topological level. At this level, space is represented by *places* and connectivity relations among them. *Places* are arranged into *streets* so that the topological map[2] looks like the street network of a city.

When people solve route finding problems using a map, they group places into *regions*. *Regions* are then used to guide the search for a route between two specific places. For example, in order to find a route from Austin to Boston, we might first find a route from Texas to Massachusetts, and then use this route to find the actual route from Austin to Boston. In order for an autonomous agent to use this hierarchical planning strategy, it has to create the appropriated space representation from its sensorimotor experiences. In this paper we describe how an agent establishes *boundary region* relations while navigating its environment (see footnote (1)). Once a sufficient number of boundary relations have been accumulated, they provide a useful topological route-finding heuristic. For example, to find a route from A to B, if there exists a street s such that A belongs to the right of s and B belongs to the left of s, look for routes from A to s and from s to B.

The idea of using boundary relations in the context of the SSH was informally proposed in (Kuipers 1978; Kuipers & Levitt 1988). In this paper we give a formal ground to those ideas. Using the formalism of *causal theories* (McCain & Turner 1997) we describe how an agent deduces different boundary relations while navigating its environment. As it will be computationally expensive and cognitively ungrounded to assume that the agents knows the relation between every place and every boundary, we are interested in defining the different states of partial knowledge associated with boundary relations. Moreover, as we do not rely on metrical information, our formalization captures the following default: in order for an agent to go from one side to the other of a boundary, the agent has to cross that boundary. We analyze how the boundary relations are affected when this default is not true, that is, when the agent misses the boundary.

This work has taken place in the Qualitative Reasoning Group at the Artificial Intelligence Laboratory, The University of Texas at Austin. Research of the Qualitative Reasoning Group is supported in part by NSF grants IRI-9504138 and CDA 9617327, by NASA grant NAG 9-898, and by the Texas Advanced Research Program under grants no. 003658-242 and 003658-347.

[1] A *boundary* is a sequence of one or more directed streets. A *boundary region* is the set of places defined to be on one side of a boundary. A *boundary relation* establishes for a given place whether it belongs to the boundary, or to one of the regions associated with the boundary.

[2] We use the term *topological map* to refer to the SSH topological level.

The paper is organized as follows: we first review the ideas behind the Spatial Semantic Hierarchy (SSH) as well as we present the language of Causal theories. In particular, we define the language in which the topological map is described. Then we present our theory describing how the agent assimilates boundary relations. Finally, we define the boundary relations entailed by the environment given the set of actions executed by the agent.

Background

In this section we describe the main ideas behind the Spatial Semantics Hierarchy (SSH) as well as the language of Causal theories (McCain & Turner 1997). We describe in detail the SSH topological level as we are interested in defining how boundary regions are associated with it. Causal theories will be used then to formally specify how boundary relation are established.

The Spatial Semantic Hierarchy

The Spatial Semantic Hierarchy (SSH) (Kuipers & Byun 1988; Kuipers *et al.* 1993; Kuipers 1996; Kuipers & Byun 1991) is an *ontological hierarchy* of representations for knowledge of large-scale space [3]. Each level of the hierarchy has its own *ontology* (the set of objects and relations it uses for describing the world) and its own set of inference and problem-solving methods. The objects, relations, and assumptions required by each level are provided by those below it. Next we describe the different SSH levels.

- The *sensorimotor level* of the agent provides continuous sensors and effectors, but not direct access to the global structure of the environment, or the robot's position or orientation within it.
- At the *control level* of the hierarchy, the ontology is an egocentric sensorimotor one, without knowledge of fixed objects or places in an external environment. A *distinctive state* is defined as the local maximum found by a hill-climbing control strategy, climbing the gradient of a selected feature, or *distinctiveness measure.* Trajectory-following control laws take the robot from one distinctive state to the neighborhood of the next, where hill-climbing can find a local maximum, reducing position error and preventing its accumulation.
- The ontology at the SSH *causal level* consists of views, distinctive states, actions and schemas. A *view* is a description of the sensory input obtained at a locally distinctive state. An *action* denotes a sequence of one or more control laws which can be initiated at a locally distinctive state, and terminates after a hill climbing control law with the robot at another distinctive state. A *schema* is a tuple $((V, dp), A, (V', dq))$ representing the (temporally extended) event in which the robot takes a particular action A, starting with view V at the distinctive state dp, and terminating with view V' at distinctive state dq. In addition, we require that $dp \neq dq$[4].
- At the *topological level* of the hierarchy, the ontology consists of *places, streets and regions*, with connectivity and containment relations among them. Relations among the distinctive states and trajectories defined by the control level, and among their summaries as schemas at the causal level, are effectively described by the topological network. Using the network representation, navigation among distinctive states is not dependent on the accuracy, or even the existence, of metrical knowledge of the environment.
- At the *metrical level* of the hierarchy, the ontology for places, paths, and sensory features is extended to include metrical properties such as distance, direction, shape, etc. Geometrical features are extracted from sensory input, and represented as annotations on the places and paths of the topological network.

The SSH Topological Level. As mentioned above, the ontology at the SSH topological level consists of *places, streets and regions*, with connectivity and containment relations among them. A *street* is an ordered sequence of places. Associated with each street there are two directions, *pos* and *neg*, that discriminate between the two directions one might be facing along a street. For example, in figure (1a), street $s2$ consists of three places A,B and C. When facing the positive direction of $s2$, the places are ordered as A, B, C. When facing in the opposite direction, the places are ordered as C, B, A. We use the following schemas to indicate the relations above:[5]

1. **inStreet(p,s)** : place p is in street s.
2. **nextS(p,q,s,dir)** : place q is the next place when traveling from p on the direction *dir* of street s. In addition we require that $nextS(p, q, s, pos) \equiv nextS(q, p, s, neg)$.[6]

At each place, an order is defined among the different streets containing the place. This order specifies the next street the agent will face if it rotates to the right or

[3] In large-scale space the structure of the environment is revealed by integrating local observations over time, rather than being perceived from a single vantage point.

[4] For example, we do not allow turns of 360 degrees.

[5] The causal theory we are to define uses an underlying propositional language. However, we use schemas to present our theory. Schemas allow us to see the theory as a many-sorted first-order causal theory in which the domain closure and unique names assumptions are made. By assuming a finite set of constants for the different sorts in the theory, it is possible to ground the theory to produce an equivalent propositional theory. We have sorts for *places, streets, regions, directions, truth values, fluents, actions*, and *time*.

[6] We also require that the next place is unique, that is, $nextS(p, q, s, d) \wedge nextS(p, r, s, d) \supset q = r$.

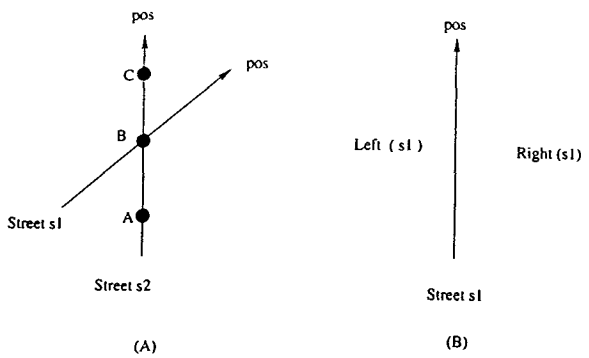

Figure 1: (a) A street has associated two directions: positive (pos) and negative (neg). Places in a street are ordered. Streets at a place are ordered (see text). (b) The right and left regions associated with a street are defined with respect to the positive direction of the street. The positive direction of a street is indicated by an arrow in the figure.

left of its current location.[7] For example, in figure (1a), if the agent is at place B facing the positive direction of street $s1$, then, by rotating to the right it will be facing the *negative* direction of street $s2$. We use the following schemas to indicate the order among streets at a given place:

1. **nextR(p,s,d,s',d')** : if the agent is at place p facing direction d of street s, after rotating to the right, the agent will be facing direction d' of street s'.[8]
2. **nextL(p,s,d,s',d')** similar to the above. In addition we require that $nextR(p,s,d,s',d') \equiv nextL(p,s',d',s,d)$.[9]

A *region* is a set of places. A *boundary* is a sequence of one or more directed streets.[10] A *boundary region* is the set of places defined to be on one side of a boundary. A street s has associated two boundary regions: **right(s)** and **left(s)**. These regions are defined with respect to the *positive* orientation of a street, as suggested in figure (1b).

Causal theories

Causal theories introduced by McCain and Turner (McCain & Turner 1997) is a mathematically simple language for expressing commonsense reasoning tasks associated with action domains. Next we summarize the main aspects of causal theories. The presentation is necessarily incomplete and we refer the reader to the original source (McCain & Turner 1997).

A *causal law* is an expression of the form $\phi \Rightarrow \psi$, where ϕ and ψ are formulas in an underlying propositional language. ϕ and ψ are called the *antecedent* and the *consequent* of the causal law, respectively. Note that $\Rightarrow$ is <u>not</u> the material implication $\supset$. A *causal theory* is a set of causal laws.

The semantics of a causal theory is defined as follows. An interpretation I for a propositional language is identified with the set of literals L such that $I \models L$. For every causal theory D and interpretation I, let

$$D^I = \{\psi : for\ some\ \phi,\ \phi \Rightarrow \psi \in D,\ I \models \phi\}$$

That is, D^I is the set of consequents of all causal laws in D whose antecedents are true in I. I is **causally explained** according to D if I is the unique model of D^I. A formula ϕ is a *consequece* of D, if ϕ is true in every causally explained interpretation according to D. We write this as $\vdash_D \phi$.

The consequence relation for causal theories is *non-monotonic*. For example, p is a consequence of the causal theory $\{p \Rightarrow p\}$ but is not a consequence of the causal theory $\{p \Rightarrow p,\ \neg p \Rightarrow \neg p\}$. In order to represent action domains the following methodology is given in (McCain & Turner 1997):

1. Specify the set of causal laws describing the direct effect of actions.
2. The specification of the action theory is augmented by the following schemas, where a is a metavariable for action names,
$$a_t \Rightarrow a_t \quad (1)$$
$$\neg a_t \Rightarrow \neg a_t \quad (2)$$
These schemas represent the fact that action occurrences may be *exogenous* to the causal theory.
3. The initial values of fluents may be exogenous to the causal theory. This is expressed by the following schemas, where f is a meta-variable for fluent names,
$$f_0 \Rightarrow f_0 \quad (3)$$
$$\neg f_0 \Rightarrow \neg f_0 \quad (4)$$
4. Frame axioms for fluents are added by the following schemas,
$$f_t \wedge f_{t+1} \Rightarrow f_{t+1} \quad (5)$$
$$\neg f_t \wedge \neg f_{t+1} \Rightarrow \neg f_{t+1} \quad (6)$$
5. Ramification and Qualification constraints are formalized, respectively, by schemas of the form,
$$True \Rightarrow \phi_t \quad (7)$$
$$\neg \phi_t \Rightarrow False \quad (8)$$

Schemas (1)-(6) are called **standard schemas** since they are included in many formalizations of action domains.

[7] The location of the agent is specified by indicating the place it is at, the street it is at, and the street's direction it is facing.

[8] The relation $nextR(p,s,d,s',d')$ is false when place p does not belong to either street s or street s'.

[9] As for nextS, we require the next street to the right to be unique.

[10] That is, we have to specify the street and the street's direction we consider when traveling the boundary.

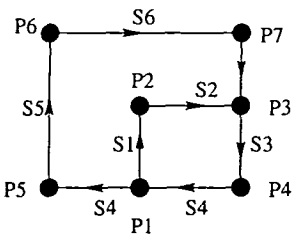

Figure 2: Starting at place $P1$ facing the positive direction of street $s1$, the agent visits places $P2, P3, P4, P1$ by interleaving the execution of *travel* and *turnRight* actions. Street's positive direction is indicated by arrows in the figure.

Boundary Regions

We are to formalize how an agent accumulates boundary region information while navigating its environment. The intuitive ideas are as follows. At any time, the agent keeps track of all the regions it is in. Action sequences of the form *turn*; *travel* are used to define when the agent enters a region. In order for the agent to leave a region, the agent has to cross the boundary of such region. The next example illustrates these ideas.

Example 1 *Consider the environment in figure 2. The agent starts traveling at place $P1$ facing the positive direction of street $S1$. The agent visits places P2,P3,P4,P1 by interleaving the execution of travel and turnRight actions.*

After turning right on $P2$ and traveling to $P3$, the agent concludes that it is in region $right(s1)$. By the time the agent reaches $P4$, it concludes that it is in regions $right(s1)$ and $right(s2)$. That the agent is in $right(s2)$ is deduced from turning right at $P3$ and traveling to $P4$. That the agent is in $right(s1)$ is deduced by default, since the agent did not cross $s1$ and it was before in region $right(s1)$.

Finally, by the time the agent reaches $P1$ again, it concludes that it is in regions $right(s2)$ and $right(s3)$. Moreover, the agent concludes that it is not in region $right(s1)$ since place $P1$ belongs to street $s1$. Notice that many states of incomplete knowledge are possible in this description. For example, the agent does not conclude that $P2$ is in region $right(s3)$ even though that is the case.[11]

Suppose the agent continues its traveling by visiting places $P5, P6$, and $P7$. It will conclude that $P7$ belongs to $right(s2)$ even though that is not the case. We will show how our formalization handles this kind of situation in the presence of more information.[12]

We present our theory in two parts: navigation theory and boundary relations theory. The navigation theory describes how the location of the agent changes as actions are executed. The region theory describes how the agent updates its record of what regions it is in.

Navigation theory. The location of the agent at time t is represented by the following fluents:

1. $atPlace(p)_t$: the agent is at place p at time t.
2. $atStreet(s,d)_t$: the agent is at street s facing direction d at time t.
3. $atRegion(r)_t$: the agent is in region r at time t.

We guarantee that the agent is at at most one location while navigating its environment,

$$atPlace(p)_t \wedge p \neq q \Rightarrow \neg atPlace(q)_t \tag{9}$$

$$atStreet(s,d)_t \wedge \neg(s = s' \wedge d = d') \Rightarrow \neg atStreet(s',d') \tag{10}$$

We consider three kind of actions, *travel*, *turnRight* and *turnLeft*, whose qualifications and effects are described by the following causal laws:[13]

$$\{travel_t \wedge atPlace(p)_t \wedge atStreet(s,d)_t \wedge \neg\exists q\, nextS(p,q,s,d)\} \Rightarrow false \tag{11}$$

$$\{travel_t \wedge atPlace(p)_t \wedge atStreet(s,d)_t \wedge nextS(p,q,s,d)\} \Rightarrow atPlace(q)_{t+1} \tag{12}$$

$$\{turnRight_t \wedge atPlace(p)_t \wedge atStreet(s,d)_t \wedge \neg\exists s',d'\, nextR(p,s,d,s',d')\} \Rightarrow false \tag{13}$$

$$\{turnRight_t \wedge atPlace(p)_t \wedge atStreet(s,d)_t \wedge nextR(p,s,d,s',d')\} \Rightarrow atStreet(s',d')_{t+1} \tag{14}$$

Causal laws (11) and (13) describe the conditions under which *travel* and *turnRight* can be executed. For example, we require that in order to travel, there must exists a next place to which to go to. Causal laws (12) and (14) describe how the location of an agent changes as the result of executing actions in accordance to the map of the environment.[14] In addition, we do not allow concurrent actions to take place,

$$a_t \wedge b_t \wedge \neg(a_t = b_t) \Rightarrow false \tag{15}$$

[11] We could handle this case by writing causal laws describing the effect of action sequences of the form *travel*; *turnRight*; *travel*.

[12] For example, if from P7 the agent continues the traveling by visiting places $P3 and P4$, it will cross street $s2$ from "right to left" and conclude that $p4$ is in $left(s2)$. Since by that time the agent "knows" that $p4$ is in $right(s2)$, this information will lead the agent to conclude that the boundary relation of $p7$ and street $s2$ is unknown. See example (2).

[13] A formula of the form $\exists x\, \phi(x,y)$ is equivalent to $\bigvee_{a \in sort(x)} \phi(a,y)$ (we require $sort(x)$ to be finite).

[14] Causal laws similar to (13) and (14) are included for the action *turnLeft*.

where a and b are meta-variables for actions.

Action sequences of the form *turn*; *travel* are used to determine when the agent enters a region,

$$atStreet(s, pos)_t \wedge turnRight_t \wedge travel_{t+1} \Rightarrow atRegion(right(s))_{t+2} \quad (16)$$

Similar axioms are included depending on whether the agent is facing the negative direction of a street and whether it turns left instead of turning right.[15]

We also consider action sequences of the form *travel*; *travel* in order to detect when the agent crosses a boundary from one side to the other,

$$\{atRegion(right(s))_t \wedge travel_t \wedge atPlace(p)_{t+1} \wedge inStreet(p, s) \wedge travel_{t+1}\} \Rightarrow atRegion(left(s))_{t+2} \quad (17)$$

A similar causal law is included when crossing a boundary from left to right.

As it is possible that the agent misses a boundary while executing a *travel* action, we assert that it is possible to leave a region after a travel action occurs,[16]

$$travel_t \wedge atRegion(r)_t \wedge \neg atRegion(r)_{t+1} \Rightarrow \neg atRegion(r)_{t+1} \quad (18)$$

Boundary Relations theory. Boundary relations are captured by the fluent **inRegion(p,r,tv)**$_t$, where *tv* is a truth value. Informally, if $inRegion(p, r, true)_t$ is true, then at time t the agent "knows" that place p is in region r. Similarly, if $inRegion(p, r, false)_t$ is true, then at time t the agent 'knows" that place p is <u>not</u> in region r. Known boundary relations have to be consistent, that is, we have the constraint,

$$true \Rightarrow \{inRegion(p, r, true)_t \supset \neg inRegion(p, r, false)_t\} \quad (19)$$

Whenever both, $inRegion(p, r, true)_t$ and $inRegion(p, r, false)_t$, are false, then the boundary relation between p and r is unknown at time t.

Whether or not a place belongs to a region is independent of time. However, the agent does not necessarily know all boundary relations. The agent can go from not knowing a boundary relation to knowing it. Once the agent knows a boundary relation, this knowledge does not change afterwards.

A street s splits the environment into three separated regions: the street itself, $right(s)$ and $left(s)$. Accordingly, the following axioms constraint the set of possible boundary relations.

$$inRegion(p, left(s), true)_t \Rightarrow inRegion(p, right(s), false)_t \quad (20)$$

$$inRegion(p, right(s), true)_t \Rightarrow inRegion(p, left(s), false)_t \quad (21)$$

$$inStreet(p, s) \Rightarrow \{inRegion(p, left(s), false)_t \wedge inRegion(p, right(s), false)_t\} \quad (22)$$

Boundary region relations are established by declaring a place to belong to the regions the agent is in at any time of the navigation. In the same vein, known boundary relations are used to establish at which regions the agent is in at a given time,

$$atRegion(r)_t \wedge atPlace(p)_t \Rightarrow inRegion(p, r, true)_t \quad (23)$$

$$inRegion(p, r, false)_t \wedge atPlace(p)_t \Rightarrow \neg atRegion(r)_t \quad (24)$$

Finally, known region relations are kept through out the navigation,

$$inRegion(p, r, true)_t \Rightarrow inRegion(p, r, true)_{t+1} \quad (25)$$

$$inRegion(p, r, false)_t \Rightarrow inRegion(p, r, false)_{t+1} \quad (26)$$

Discussion

Let **BRT**[17] denote the causal theory consisting of the causal laws (9)-(26) in addition to the standard schemas. *BRT* describes the general knowledge the agent has about how to assimilate boundary relations.

The theory *RBT* allows us to draw some useful conclusions about the state of the navigation. For example,

$$\exists p, s, d\ \{atPlace(p)_0 \wedge atStreet(s, d)_0\} \supset \exists p, s, d\ \{atPlace(p)_t \wedge atStreet(s, d)_t\} \quad (27)$$

$$atRegion(left(s))_t \supset \neg atRegion(right(s))_t \quad (28)$$

$$inStreet(p, s) \wedge atPlace(p)_t \supset \{\neg atRegion(left(s))_t \wedge \neg atRegion(right(s))_t\} \quad (29)$$

[15] For example,

$$atStreet(s, neg)_t \wedge turnRight_t \wedge travel_{t+1} \Rightarrow atRegion(left(s))_{t+2}$$

[16] This causal law allows the agent to make useful conclusions even in the presence of conflicting information. See example (2).

[17] *BRT* stands for Boundary Region Theory.

Formula (27) states that, given that the agent is at a location at time 0, it will always be at a location afterwards. Formulas (28) and (29) state that the agent will never be at two known disjoint regions simultaneously.

In order to assimilate boundary relations we have to specify the following facts:[18]

1. The map of the environment, that is, we have to specify the truth value for the predicates $nextS$, $nextR$, and $nextL$.
2. The initial location of the agent.
3. Any known boundary relations.
4. The sequences of actions executed by the agent: $a_0, \ldots, a_T$.

Next we define a default criteria for an agent to deduce boundary relations.

Definition 1 (Boundary relations) *Let D denote the conjunction of facts (1)-(4) above. Assuming that $D \wedge RBT$ is consistent, we say that* ***place p belongs to a region r****, according to D, if the following two conditions are true:*[19]

$$\nvdash_{D \wedge RBT} \neg inRegion(p, r, true)_T \qquad (30)$$
$$\vdash_{D \wedge RBT} \neg inRegion(p, r, false)_T$$

Similarly, p **does not belong to** r if the following two conditions are true:

$$\nvdash_{D \wedge RBT} \neg inRegion(p, r, false)_T \qquad (31)$$
$$\vdash_{D \wedge RBT} \neg inRegion(p, r, true)_T$$

If neither condition (30) nor condition (31) are satisfied, then we say that the boundary relation between p and r is **unknown**.

Definition (30) establishes a default conclusion for p to belong to r. In order to conclude that p belongs to r there must exists at least one causally explained model of $D \wedge RBT$ in which $inRegion(p, r, true)_T$ is true. Intuitively this means that there is an environment described by the topological map in which p is in r. We also require that in all causally explained models of $D \wedge RBT$, $\neg inRegion(p, r, true)_T$ is true. Intuitively this means that in all posible environments described by the topological map it is the case that p does not belongs to r.

We could have been more strict in definition (30) and require that $\vdash_{D \wedge RBT} inRegion(p, r, true)_T$. However this requeriment will preclude the agent from establishing some useful boundary relations. Being a default conclusion, definition (30) implies also that the agent might assimilate wrong boundary relations. Similar remark applies to condition (31). Next we illustrate the boundary relations predicted by the theory in the environment of example (1).

[18] See example (2).

[19] The time T refers to the time on which the last action was executed. See (4) above.

Example 2 *Let's consider the same situation as in example (1). We specify the initial conditions (except the map) as follows:*

1. *Initial location,*
   $atPlace(p1)_0 \wedge atStreet(s1, pos)_0$
2. *We assume not kown boundary relations except the ones implied by causal law (22)*
   - $\neg atRegion(r)_0$
   - $\neg inRegion(p, r, true)_0$
   - $\neg inStreet(p, s) \supset$
     $\{\neg inRegion(p, right(s), false)_0 \wedge$
     $\neg inRegion(p, left(s), false)_0\}$
3. The actions executed by the agent are such that it visits the places $P1, P2, P3, P4, P1, P5, P6, P7, P3, P4$.

In addition to the boundary relations predicted in example (1), the agent concludes the following boundary relations: $p5$ and $p6$ belong to $left(s1)$ and $right(s2)$, $p7$ belongs to $left(s1)$, the boundary relation of $p7$ and $s2$ is unknown.[20]

By the time the agent travels from $p7$ to $p3$ it "believes" that it is at $right(s2)$ and $left(s1)$. Given that it is known that $p3$ is in $right(s1)$, the agent concludes, by default, that $P7$ is in $left(s1)$. This is the case since the agent has changed from one side to the other of $s1$ without crossing $s1$.

Once the agent travels from $p3$ to $p4$ a contradictory situation is found. Since $p7$ is "believed" to be in $right(s2)$ and $p3$ belongs to the street $s2$, $p4$ should then belong to $left(s2)$ (see (17)). However is it known that $p4$ belongs to $right(s2)$. The agent solves this inconsistency by declaring unknown the boundary relation between $p7$ and $s2$.

Notice that the agent concludes that $p6$ is in $right(s2)$ though that it is not the case (at least in figure (1)). Once the relation of $p7$ and $s2$ is declared unknown, the agent has not evidence that $p6$ is not in $right(s2)$. Since it has evidence of $p6$ being in $right(s2)$, the agent deduces, by default, that $P6$ is in $right(s2)$.

Since we only use topological constraints in our theory, we can find environments "topologically equivalents" to the one in figure (1) in which $P6$ is indeed to the right of $s2$ (see figure (3)).

Related work

The idea of using boundary regions in the context of the SSH was proposed in (Kuipers 1978; Kuipers & Levitt 1988). Axiom (16) formally states the description given in these references. Evidence of how people use boundary relations to solve route finding problems is given in (Lynch 1960; Chase 1982;

[20] We have verified these conclusions by using a causal theories model checker.

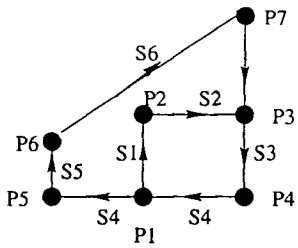

Figure 3: Environment topologically equivalent to the one in figure (1) in which place $P6$ in on the right of street $s2$.

Elliot & Lesk 1982). People's cognitive maps are organized around a "skeleton map" of major streets, within which most problem-solving takes place, with final links from the original start and goal points to the nearest point on the skeleton (Kuipers & Levitt 1988).

By organizing places into regions, hierarchical planning techniques can be applied to solve route finding problems. Regions lend themselves to create a hierarchy of space representations. Reasoning methods and useful properties of this hierarchy can be borrowed from work on "abstraction" theories (Giunchiglia, Villafiorita, & Walsh 1997; Knoblock 1989; 1991). Most work on hierarchical route finding assumes a given hierarchy of space representations. The automatical creation of these hierarchy has been tackled in very few works (Knoblock 1994; Maio & Rizzi 1993).

Causal theories were introduced in (McCain & Turner 1997) for the propositional case, and then extended to the case with variables in (Lifschitz 1997). Whenever a causal theory is *definite*[21] an equivalent classical propositional theory can be associated with the theory (McCain & Turner 1997).[22] Satisfiability checking can be then applied to find the models of a definite causal theory. In turn, model-finding can be viewed as planning (Kautz & Selman 1992; 1996; McCain & Turner 1998).

Future Work and conclusions

Grouping places into regions allows an agent to reason efficiently about its spatial knowledge. For example, hierarchical planning techniques can be used to solve route finding problems. In order to use these techniques, the automatic assimilation of region relations is a desired characteristic for autonomous agents that ground their spatial representation on their sensorimotor experiences. In this paper we have presented an action theory, RBT, that allows an agent to do so.

[21]A causal theory is **definite** if the consequent of each causal law in the theory is either a literal or false.

[22]This is done by an elaboration of the Clark completion method (Clark 1978).

The theory RBT is by no means a complete specification of how boundary region relation are to be established. It states commonsense rules by which an agent enters (causal laws (16) and (17)) or leaves a region (causal laws (18), (22) and (24)) . The theory captures *states of partial knowledge* where boundary relations are unknown. In addition, the theory allows the agent to draw useful boundary relations whenever it misses a boundary.

The use of the language of causal theories allow us to formally specify how region relations are established. The availability of model checkers for the language of causal theories, makes easier the task of formally verify the consequences of the theory. We believe that the use of action languages as a specification language are useful for clarifying many of the ideas in robotics. We are working towards extending our current implementation of the SSH to handle boundary relations as described by the theory presented in this paper.

References

Chase, W. 1982. Spatial representations of taxi drivers. In Rogers, D., and Sloboda, J., eds., *Acquisition of Symbolic Skills.* New York: Plenum.

Clark, K. 1978. Negation as failure. In Gallaire H., M. J., ed., *Logic and Data Bases.* New York: Plenum Press. 293–322.

Elliot, R., and Lesk, M. E. 1982. Route finding in street maps by computers and people. In *AAAI-82*, 258–261.

Giunchiglia, F.; Villafiorita, A.; and Walsh, T. 1997. Theories of abstraction. Technical Report MRG/DIST 97-0051, University of Genova.

Kautz, H., and Selman, B. 1992. Planning as satisfiability. In *ECAI-92*, 359–379.

Kautz, H., and Selman, B. 1996. Pushing the envelope: planning, propositional logic, and stochastic search. In *AAAI-96*, volume 2, 1194–1201.

Knoblock, C. 1989. A theory of abstraction for hierarchical planning. In P., B., ed., *Workshop on Change of representation and Inductive Bias.* Kluwer, Boston, MA.

Knoblock, C. 1991. Search reduction in hierarchical problem solving. In *AAAI-91.* AAAI Press.

Knoblock, C. 1994. Automatically generating abstractions for planning. *Artificial Intelligence* 68:243–302.

Kuipers, B., and Byun, Y. T. 1988. A robust qualitative method for spatial learning in unknown environments. In Kaufmann, M., ed., *AAAI-88.*

Kuipers, B., and Byun, Y. T. 1991. A robot exploration and mapping strategy based on semantic hierarchy of spatial representations. *Journal of Robotics and Autonomous Systems* 8:47–63.

Kuipers, B., and Levitt, T. 1988. Navigation and

mapping in large-scale space. *AI Magazine* 9(2):25–43.

Kuipers, B.; Froom, R.; Lee, W.; and Pierce, D. 1993. The semantic hierarchy in robot learning. In Connell, J., and Mahadevan, S., eds., *Robot Learning*. Kulwer Academic Publishers. 141–170.

Kuipers, B. 1978. Modeling spatial knowledge. *Cognitive Science* 2:129–153.

Kuipers, B. 1996. A hierarchy of qualitative representations for space. In *Working papers of the Tenth International Workshop on Qualitative Reasoning about Physical Systems (QR-96)*. AAAI Press.

Lifschitz, V. 1997. On the logic of causal explanation. *Artificial Intelligence* 96:451–465.

Lynch, K. 1960. *The Image of the City*. Cambridge, Mass:MIT press.

Maio, D., and Rizzi, S. 1993. Map learning and clustering in autonomous systems. *IEEE transactions on Pattern Analysis and Machine Learning* 15(12).

McCain, N., and Turner, H. 1997. Causal theories of action and change. In *AAAI-97*, 460–465.

McCain, N., and Turner, H. 1998. Satisfiability planning with causal theories. In *To appear in KR-98*.