Generating User Interfaces for Pen-based Computers

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Abstract
We provide an overview of the Penguins project. The aim of this project is to develop tools that facilitate the development of software for pen-based graphics editors. The project is based on the intelligent pen and paper metaphor for human-computer interaction. In this metaphor, the user communicates with the computer using an application specific visual language composed of handwritten text and diagrams. As the diagram is drawn with a pen, the underlying graphic editor parses the diagram, performing error correction and collecting geometric constraints which capture the relationships between diagram components. During manipulation these constraints are maintained by the editor in order to preserve the semantics of the diagram. The Penguins system contains tools which, given a grammatical specification of a visual language, automatically construct a core user interface for that visual language which embodies the intelligent pen and paper metaphor. This core user interface consists of a tokenizer, constraint solver, layout controller and constraint-based graphics editor together with an incremental parser for the specified visual language. The specification language is based on constraint multiset grammars.

Introduction
Interactive graphic tablets and pen-based notepad computers are now on the market place. This new technology, however, while offering great potential has not yet been very successful, mainly because software for pen-based computers is still immature. The aim of the Penguins project is to develop tools that facilitate the development of software for pen-based computers which takes full advantage of the pen’s new capabilities.

The project is based on the intelligent pen and paper metaphor for human-computer interaction with pen-based computers. In this interaction metaphor, the user communicates with the computer using an application specific visual language composed of handwritten text and diagrams. Input can be given in free form, modelessly in any order and place, and modified using natural gesture commands. Users are thus able to express themselves directly in the visual lingua franca of their application domain, such as statecharts, structural-chemical formulae, or mathematical equations. As the user draws the diagram, the underlying software recognizes the structure of the diagram, performing error correction and adding constraints between the diagram components. These constraints preserve the diagram semantics during direct manipulation.

The Penguins system contains tools which, given a grammatical specification of a visual language, automatically construct a core user interface for that visual language which embodies the intelligent pen and paper metaphor. The core user interface supports the creation, manipulation and interpretation of diagrams in the specified visual language. The specification language is based on constraint multiset grammars. The core user interface provides a constraint-based graphics editor that supports graphics primitives, such as lines, circles, texts and arrows. In order to provide support for free-hand gestures from a pen, a tokenizer is also provided. This maps input gestures to the appropriate graphics primitives. A constraint solver is incorporated into the core user interface to provide the constraint solving mechanisms necessary for geometric error correction and diagram manipulation. Certain modules in the generated program can be extended to cater for application specific computation. The core user interface, together with the application specific routines is compiled to give the final pen-based application. In the future a layout controller will also be generated from the grammar. Figure 1 shows the main components of the Penguins system.

There are three underlying ideas on which the Penguins system is based - constraint multiset grammars, the metric space model and declarative graph or diagram layout. There is a synergy between these ideas with each model formalizing a different part of the interface – recognition, manipulation and layout. However, as each model uses geometric constraints to represent the semantics of the diagram, they fit naturally together.

The Penguins project is still work in progress. Currently, we have implemented the constraint solver, tokenizer, graphics editor and the parser generator. We have demonstrated their use in the development of several visual user-interfaces, for instance, mathematical equations and state transition diagrams. These exam-
Related Work

The most closely related work are our earlier papers on diagram parsing and the Penguins System. In (Helm, Marriott, & Odersky 1991; Marriott 1994) we introduced constraint multiset grammars. In (Chok & Marriott 1995) we described a prototype of the incremental parser used in the Penguins system, while (Chok & Marriott 1998) provides an overview of the system together with a detailed empirical evaluation.

Several other research groups have investigated the intelligent pen and paper metaphor. In particular, (Gross 1994a; 1994b; Gross & Do 1996) has developed a pen-based system which supports the incremental recognition of components of informal sketches made during the design process. Weitzman and Wittenburg also consider design support, incrementally recognizing document or diagram structure and enforcing the induced semantic constraints during manipulation (Weitzman & Wittenburg 1993; 1994; 1996), although here the input device need not be a pen. The Penguins system extends these by allowing more expressive constraints, such as inequalities, and a more powerful underlying grammatical formalism providing existential and negative constraints. For example, we believe it is unlikely that these other systems could be used to recognize $N$-ary trees or message sequence charts. Nor do these systems have a formal method for handling error correction.

Several authors have looked at automatic construction of syntax-directed diagram editors from a visual language specification (Uskudarli 1994; Minas & Viehstaedt 1995). In this diagramming model the visual language grammar guides the top-down construction of the diagram. One drawback of this approach is that it is impossible for the user to construct incorrect syntactic structures for intermediate diagrams as the editing process is strictly guided by a sequence of structural templates.

There has also been considerable research into automatic generation of static parsers from a visual language specification. The most general approaches are either based on graph grammars (for instance (Ferrucci et al. 1991; Tucci, Vitiello, & Costagliola 1994; Rekers & Schürr 1995)) or attributed multiset grammars (for instance (Golin 1991; Golin & Magliery 1993; Wittenburg 1992)). The constraint multiset grammar formalism is an example of an attributed multiset approach. However, during error correction and semantics preserving diagram manipulation the behavior is more graph grammar-like since the actual attributes are ignored but the relationships between the diagram components are preserved. For a more complete survey, see (Marriott, Meyer, & Wittenburg 1998).

Constraint Multiset Grammars

The Penguins system uses constraint multiset grammars (Helm, Marriott, & Odersky 1991; Marriott 1994) for the specification of the visual language. These are an attributed multiset based grammatical formalism, which provides a high-level and declarative framework for the definition of visual languages. Constraint multiset grammars arose out of early work in logic based formalisms for diagram specification (Helm & Marriott 1986; 1990). An important reason for using a logic or constraint based formalism is that the same grammar can be used to both generate and to recognize diagrams in the language as the constraints are bi-directional. As we shall see constraints also allow for geometric error correction in parsing and manipulation in the graphics editor.
Productions in a constraint multiset grammar have the form

\[ P ::= P_1, \ldots, P_n \text{ where } C \]

indicating that the non-terminal symbol \( P \) can be rewritten to the multiset of symbols \( P_1, \ldots, P_n \) whenever the attributes of all symbols satisfy the constraints \( C \). The constraints enable information about spatial layout and relationships to be naturally encoded in the grammar. We will demonstrate this by mean of the grammar for state transition diagrams.

Productions in the grammar specify how states and transitions are composed from the terminal symbols which are circles, texts, lines and arrows. Every symbol in a constraint multiset grammar has a **symbol type** with associated attributes. Symbol types are either terminal or non-terminal. There is one distinguished non-terminal type called the start type. The declarations for each symbol type must be given in the grammar. Those for our example are as follows:

```plaintext
declare symboltype arc(start:point, end:point, label:string):nonterminal;
define symboltype startArc()
    start:point, end:point):nonterminal;
define symboltype state()
    mid:point, radius:integer, label:string, kind:string):nonterminal;
define symboltype transition()
    start:string, tran:string, end:point):nonterminal;
define symboltype transitions(set:multiset<transition>):nonterminal;
define symboltype states(set:multiset<state>):nonterminal;
define symboltype std(ss:states, ts:transitions):starttype;
```

For instance, the first declaration specifies that the symbol type arc is a non-terminal symbol and that it has four attributes: its start-point, its mid-point, its end-point and its label.

The simplest production in the grammar is that defining the arc. It specifies that an arc in a state transition diagram is composed of a text label \( T \) and an arrow \( A \), and that the mid-points of \( T \) and \( A \) must be the same:

\[ R:\text{arc}() ::= A:\text{arrow}, T:\text{text} \text{ where } (A . \text{mid} = T . \text{mid}) \]

In many visual languages, there are language elements which are superficially similar but with quite different meanings. In the case of state transition diagrams, a start arc is very similar to an arc. The only difference between these two structures lies in the additional text label forming part of an arc structure. This poses potential ambiguity in parsing. For example, if an arrow is drawn, should it be recognized as a start arc or an arc? In static parsing such ambiguity is usually resolved by considering all possible maximal interpretations and then choosing the interpretation which explains all elements in the diagram. In our context such ambiguity is bad for at least two reasons. First, it makes parsing considerably more expensive since multiple parse trees must be constructed. Second, and even more importantly, it makes it extremely difficult to provide immediate feedback about what has been recognized so far during diagram construction since the ambiguity may only be resolvable once the complete diagram has been drawn. This problem can be overcome by the use of a negative constraint which specifies that a start arc is recognized if there is no text label \( R \) with the same mid-point as the arrow \( A \). This essentially makes the grammar deterministic. The following is the defining production for startArc.

```plaintext
S:startArc() ::= A:arrow where 
    not exist R:text where (R . mid = A . mid) 
} 
S.start = A.start; 
S.end = A.end; 
S.mid = A.mid; 
```

A final state is made up of two circles \( C_1 \) and \( C_2 \) and text \( T \) satisfying three underlying geometric relationships. The first specifies that the mid-points of the circle \( C_1 \) and the circle \( C_2 \) are the same. Similarly, the second specifics equality of the mid-points of the circle \( C_1 \) and the text \( T \). The third ensures that \( C_2 \) is the outermost circle. The following is the defining production for a final state.

```plaintext
S:state() ::= C1:circle, C2:circle, T:text 
    where (C1 . mid = C2 . mid && 
          C1 . mid = T . mid && 
          C1 . radius <= C2 . radius) 
} 
S.mid = C1 . mid; 
S.radius = C2 . radius; 
S.label = T . label; 
S.kind = "final"; 
```

A start state is composed of a labelled circle, \( C \) and \( T \), with a start arc \( A \) pointing to it. To avoid ambiguity in reductions involving the production for a start state and the production for a final state, a negative constraint is required. This negative constraint specifies that there must not be another circle \( M \) with the same mid-point, since if such a circle exists, then a final state should be recognized instead. The following shows the defining production for a start state.
A normal state is similar to a start state, except that it does not have a start arc pointing to it. Again, there is a potential ambiguity in reductions. The problem is that a labelled circle that is rightfully part of a start state, may be recognized as a normal state instead. To remove such ambiguity, an explicit negative constraint is put in place, specifying that there must not be a start arc pointing to the labelled circle. Another possible ambiguity is between a normal state and a final state, similar to that between a start state and a final state. This ambiguity can be removed with an explicit negative constraint, specifying that there must not be another circle with the same mid-point.

Apart from negative constraints, constraint multiset grammars also allow existential quantification. Expanding the constraint multiset grammar formalism in this way essentially makes it context-sensitive. It has been proven in Marriott et al (Marriott & Meyer 1997) that some sort of context-sensitivity is required for the specification of graph-based visual languages. The idea of existential quantification of variables is to allow the quantified variables to refer to symbols which have already been reduced as well as to symbols in the current sentence. Consider the following production which specifies a transition in a state transition diagram. The variables $S_1$ and $S_2$ are existentially quantified. This means that they can be assigned symbols from either the current sentence or from the previously reduced symbols.

In many graph-like visual languages it is useful to be able to collect symbols of the same type into a multi-set or set. For instance, in the case of state transition diagrams, we wish to collect all of the states and all of the transitions. Collection productions support this activity. Thus the following productions collect all of the states and transitions in the diagram respectively.
The \texttt{Intersect} constraint specifies that the lines \texttt{LA} and \texttt{LB} intersect. The computation function \texttt{Intersection} determines the exact intersection point of the two lines, used to compute the end-points of the four new lines.

\section*{Constraint Solver}

The constraint solver is a revised version of the toolkit, \textsc{QOCA} (Borning \textit{et al.} 1997; Helm \textit{et al.} 1995; Marriott, Chok, \& Finlay 1998). This is a C++ constraint solving toolkit which supports the \textit{metric space model} over various classes of arithmetic constraints.

In the metric space model, at any time there is a set of constraints over some variables, a metric which gives the distance between two assignments to the variables, and a current solution which is an assignment to the variable that satisfies the constraints. The variables correspond to graphical attributes of the graphics objects in the diagram and the diagram on the graphics display is, therefore, just a visual representation of the current solution.

Interaction in the metric space model can occur in three ways. First, a constraint may be added to the current set of constraints in which case the new solution is the assignment which is closest to the old assignment and which satisfies the new constraint set. Second, a constraint may be deleted from the current set of constraints, in which case the current solution remains unchanged. Finally, the current solution may be manipulated by “suggesting” a value for several of the variables. The new solution is the assignment which is closest to the old assignment and to the requested variable values and which satisfies the constraint set. The constraint set remains unchanged.

The metric space model provides a good formal basis for incremental manipulation of diagrams in a visual language specified by a constraint multiset grammar. The current constraints are generated in the parsing process. They capture the semantics or meaning of the diagram, for example in an equation they capture that in a division formula the numerator is above the denominator. During manipulation of the diagram the constraints ensure that only valid (sub-)diagrams in the language are constructed.

The \textsc{QOCA} toolkit provides three main classes. An instance of the \texttt{CFloat} class, behaves like a float except that if its value is set, this is treated as advice to the constraint solver, not as a true assignment to the variable. Only the constraint solver can set the value of a \texttt{CFloat}. \texttt{LinConstraints} represent the equality or inequality constraints in the problem. They are expressions built on top of \texttt{CFloats}. Actually, neither \texttt{CFloats} nor \texttt{LinConstraints} are directly manipulated by the programmer. Instead, for efficiency and safeness, they are manipulated by means of the reference-counted “handles,” \texttt{CFloatHandle} and \texttt{ConstraintHandle} respectively which can be assigned and constructed in the obvious ways.

To the application programmer the \texttt{CFloat} appears to have a single floating point value, which they set using the method \texttt{SuggestValue} and then read after calling the constraint solver using \texttt{GetValue}. Internally, however, the desired and actual values are kept separately. It also has a \textit{stay weight} and an \textit{edit weight} which, respectively, indicate the importance of leaving the variable unchanged if it is not being edited and the importance of changing it to the desired value when the variable is being edited. Both weights are non-negative floats and are set when the \texttt{CFloat} is created and cannot be subsequently changed.

\textsc{QOCA} currently provides three different solvers. Here we use the constraint solver \texttt{CassSolver}. This uses an incremental version of the Simplex algorithm for constraint solving and for finding the closest solution to the suggested solution which satisfies the equations. Constraints can be added to the solver one at a time using \texttt{AddConstraint}. With each addition the solver checks that the new constraint is compatible with the current constraints. This method is used to add constraints which enforce the geometric relationships found in parsing. \texttt{RemoveConstraint} allows the removal of a constraint which is currently in the solver. This is used when an object is deleted by the user or when invalidation of a negative constraint causes parsing to be undone.

The solver provides four methods for direct manipulation. First the application programmer tells the solver which variables are to be edited using multiple calls to \texttt{SetEditVar}. Next \texttt{BeginEdit} is called. This initializes internal data structures for fast “resolving” of the constraints. Now during manipulation the application programmer repeatedly sets the desired values of the edit variables and then calls the solver function \texttt{Resolve} which efficiently computes the new solution to the constraints which is as close as possible to the old solution and to the new desired values of the edit variables. Finally the application programmer calls \texttt{EndEdit} to signal the end of direct manipulation.

The following (slightly modified) example taken from (Marriott, Chok, \& Finlay 1998) gives some feel for \textsc{QOCA}'s interface. Consider a diagram consisting of a point \((x_m, y_m)\) and a line from \((x_l, y_l)\) to \((x_u, y_u)\) in which the point is constrained to lie at the midpoint of the line. The following program fragment creates the variables and constraints, adds them to the solver and calls \texttt{Solve} to compute the initial position. The constructor for \texttt{CFloatHandles} takes three arguments: the stay and edit weights together with an initial desired value. Note that both \(x_m\) and \(y_m\) have a stay weight of zero indicating that they are “dependent variables,” although they of course have a non-zero edit weight (otherwise, editing would never change their value). Next the program chooses \(x_m\) and \(y_m\) to be the edit variables, and then repeatedly samples the mouse to find the desired values and calls \texttt{Resolve} to compute the new value until the user releases the mouse button, which finishes the edit cycle.
CFloatHandle
   xl(1,1000,45), xm(a,1000,0), xu(1,1000,60),
   yl(1,1000,45), ym(a,1000,a), yu(1,1000,60);
ConstraintHandle
   xeon = (1*xl + 1*xu - 2*xm == 0),
   yeon = (1*yl + 1*yu - 2*ym == 0);
CassSolver solv;
   solv.AddConstraint(xeon);
   solv.AddConstraint(yeon);
   solv.Solve();
   DrawLine(xl.GetValue(),yl.GetValue(),
            xu.GetValue(),yu.GetValue());
   solv.SetEditVar(xm);
   solv.SetEditVar(ym);
   solv.BeginEdit();
   while (mouse.button.down) {
      xm.SuggestValue(mouse.x);
      ym.SuggestValue(mouse.y);
      solv.Resolve();
      DrawLine(xl.GetValue(),yl.GetValue(),
               xu.GetValue(),yu.GetValue());
   }
   solv.EndEdit();

Parser Generator and Parser

From the application programmer's point of view, the parser generator is a black box which reads in a deterministic constraint multiset grammar and generates a parser for the visual language of the grammar in the form of C++ code.

Before the generation of the parser, the parser generator checks that the input grammar is stratified, that is, there is no recursion through negative constraints. The problem is that a production may directly or indirectly create the symbol it depends on negatively, that is, there may be a cycle in the dependency graph which passes through a negative constraint. For example, the following production has such a cycle.

\[ W:\text{negDep}() ::= T:\text{text} \text{ where (not exist } N:\text{negDep where (N.mid == T.mid))} \]

If there is a cycle through a negative constraint in the dependency graph, subsequent reductions will produce new symbols which may invalidate an earlier reduction involving the negative constraint. This may lead to an infinite loop in which a symbol is repeatedly created then deleted. Instead attention is restricted to grammars that are stratified in the sense that a production should only be allowed to depend negatively on those symbol types which are strictly lower in the dependency graph.

Conceptually, in order to determine if a grammar is stratified, one constructs the grammar's dependency graph. In the dependency graph, each node is a symbol type. If the symbol type \( A \) depends on the symbol type \( B \), either through existential quantification or through reduction, then there is a positive dependency between the node for \( A \) and the node for \( B \) represented by the arc \( A \leftarrow B \). Similarly, a "negative" dependency between two symbol types can also be represented by a connection in the dependency graph, except that the symbol \( \leftarrow \) is used instead of the symbol \( \rightarrow \). Figure 3 shows the dependency graph for the state transition diagram grammar presented in the previous section. Note that for collection production, there is an implicit negative constraint associated with the type it collects, since a collection is invalidated once a new symbol of the type it contains is created. It is obvious from the example dependency graph that the grammar is stratified.

It also is important to ensure that a reduction sequence is finite in length, so as to ensure that parsing terminates. To ensure this, the grammar must be cycle-free, in the sense that there are no infinite reductions in which a single symbol is repeatedly reduced. The parser generator always ensures that the restricted portion of the grammar is cycle-free before code generation. Though the condition of a cycle-free can be extended to unrestricted grammars, currently has not been implemented. Therefore it is the programmer's responsibility to ensure this. Examining the dependency graph for the state transition diagram grammar, indicates that there is no cycle that can cause non-termination.

The generator performs two major optimizations to ensure efficiency of the parser. Firstly, it determines an optimal sequence of production evaluation in the constraint multiset grammar during parsing. This is done by examining the dependency graph. The result is a sequence of evaluations which ensures that clusters of strongly dependent productions are evaluated independently and in the right sequence. Since the dependency graph only shows symbol types, all the defining productions for each symbol type are evaluated as a group. As an example, the following shows an optimal sequence evaluation based on the dependency graph given in Figure 3:
are, startArc, [state(normal), state(final), state(start)], transition, states, transitions, std.

Secondly, it minimizes the number of constraints created during parsing. This is done by ensuring that only constraints over attributes of terminal symbols are considered by the embedded constraint solver. Attributes of non-terminal symbols are only a reference to attributes of terminal symbols. Further optimization of this nature can be done by ensuring that non-constraint datatypes do not add constraints to the solver.

In an interactive diagramming environment, incrementality in parsing is crucial since efficiency is a prime factor in such an environment. The generated parser is incremental. Whenever a graphics object is input, the parser updates the parse tree with minimum computation. This is somewhat difficult because negative constraints mean that parsing is non-monotonic – adding a new object may make an earlier reduction invalid. To demonstrate this, consider the simple state transition diagram shown in Figure 4(a). This diagram is initially recognized as a start state, but an additional text label "a" nullifies this initial result. Instead, an arc and a normal state should be identified, as shown in Figure 4(b).

The incremental parser performs parsing in a bottom-up fashion, by taking a multiset of terminal symbols and repeatedly using satisfiable production rules in the grammar to combine symbols into larger and larger parse trees.

During diagram creation whenever a new graphics object is drawn, two events occur. Firstly, the parser checks whether the newly added object invalidates any previously satisfiable negative constraints. If invalidation of negative constraints occurs, then relevant sections of the parse forest will be destroyed and in the process, makes available some symbols for use for the current incremental parse. Secondly, a parse is performed to update the parse forest based on this newly added object, and if any, the made available symbols.

For the purpose of efficient parsing, constraint checking performed in the evaluation of each production, is divided into two stages. The main idea is to minimize the number of constraint additions. This is especially true for the case of a production with multiple constraints, where one constraint is satisfiable does not always imply that the remaining constraints are also satisfiable. The two stages involved are the constraint checking stage and the constraint adding stage.

During the constraint checking stage, constraints in a production are considered satisfiable if the associated errors are within the accepted tolerance. For example, points A and B are considered equal if the distance between them is within the acceptable tolerance. If the constraint checking stage is successful, then the constraint adding stage is performed. In this stage, all the constraints representing the geometric relationships between the attributes of symbols are added into the constraint set.

Diagrams constructed with a stylus or with a traditional graphics editor almost always contain geometric errors such as lines not quite touching the objects they should touch. Therefore, some form of automatic geometric error correction mechanism is crucial, providing immediate feedback to the user about what has been recognized and removing any geometric error.

The error correction mechanism utilizes geometric constraints collected during the constraint adding stage. These geometric constraints represent the geometric relationships between symbols in a diagram. The constraint solver is then activated to perform constraint solving over the new constraint set. The purpose of the constraint solving step is to give new values to the attributes of symbols that satisfy all the constraints in the new constraint set. By doing this, the geometric errors involved in a diagram are removed. After the relevant constraints have been solved, the diagram is simply re-displayed to reflect the new solution of all the attributes of symbols. Figure 5 gives an overview of the steps involved to deliver geometric error correction based in the production for a normal state.

It is often necessary for a grammar to utilize constraint functions that are not already defined. Addi-
tional constraint functions are written as C++ procedures which extend the Constraint class. Each constraint function must implement the constraint checking and the constraint adding stages.

Graphics Editor

The graphics editor combines with the generated incremental parser and the constraint solver to give a user interface based on the specified visual language. The editor allows the user to add or delete symbols and to manipulate components of the diagram.

In general, diagram interactions can be classified into two categories. The first category consists of interactions that modify the semantics of a diagram. These types of interactions include add, delete and separation of sub-diagrams. For example, adding another circle to a normal state in a state transition diagram changes the initial interpretation to a final state. The second category consists of interactions that preserve the semantics of the original diagram. These types of interactions, which are also called diagram manipulation, include moving and resizing of sub-diagrams.

The addition of graphics object has been discussed in the preceding section. Obviously, deletion of symbols from a diagram also changes the semantics of the diagram since previously identified diagram structures that depend on the deleted symbols become invalid. Thus the parse forest must be updated. A naive solution to the problem is to perform a complete re-parse of the entire diagram to obtain the corresponding parse forest. This solution is simple, but very inefficient.

A better solution is to remove only the relevant parts of the parse forest, and then perform an incremental parse. The end result of the incremental parse is a parse forest that reflects the changes made to the diagram. This is related to how negative constraints are handled.

It is useful to be able to separate a section of the diagram that was previously identified as a valid diagram structure. For example, in a state transition diagram, the user may have drawn a circle that belongs to a normal state in a state transition diagram and later decided to change it to a normal state by moving away one of the circles. The algorithm for such a separation operation is similar to that for deletion. The only different is that the selected object is not removed from the parse forest.

During diagram manipulation, the semantics of the diagram should be preserved. This is made possible by the embedded constraint solving mechanism. As previously mentioned, the constraints representing geometric relationships between symbols in a diagram are added into the constraint set during incremental parsing. When a symbol is manipulated, its geometric attributes are changed to reflect its new geometric location. For instance, imagine moving a circle that belongs to a normal state in a state transition diagram. When the circle is moved, the text is also moved to preserve the original semantics of the diagram. This is illustrated in Figure 6.

Figure 6: Example diagram manipulation

Tokenizer

The role of the tokenizer is to accept a sequence of geometric points from a pen or mouse, and to determine the graphic object which best fits the points. Currently, the tokenizer supports recognition of lines, circles, ellipses, rectangles and splines, but also provides a basic framework for supporting additional symbol types.

Tokenization consists of three stages. The first stage is the filtering process designed to narrow down the number of possible symbol types, thus reducing the amount of computation involved in the later stages. This is achieved by examining features of the input points. For example, if the first and last points from the sequence of input points are close to each other, then the possibility of a line will be dismissed since the input is almost certainly best fitted by a closed curve. In the second stage, the best fitting graphic object for each of the candidate graphic object types is computed. This stage also computes the associated geometric error. Based on this geometric error, the final stage selects the graphic object with the best fit. For each type of graphic object, the tokenizer uses a least squares curve fitting algorithm tailored to that type of graphic. As an example, we consider how circles are fitted.

The parametric equations for a circle are

\[ x = r \cos \theta + x_c, \]
\[ y = r \sin \theta + y_c. \]

Before finding a circle that minimizes the associated geometric error, the mid-point of the circle must first be estimated by using a bounding box. The estimated mid-point is required for computing the angle \( \theta_i \) for each input point \( (x_i, y_i) \). The angle \( \theta_i \) is in turn used to calculate the point \( (x_i^*, y_i^*) \) on the ideal circle. The geometric error involved between the input point \( (x_i, y_i) \) and the point \( (x_i^*, y_i^*) \) is separated into the \( e_x \) and \( e_y \) error components representing the geometric error in the \( x \) and \( y \) axis respectively. Figure 7 illustrates circle fitting. The geometric error \( E \) is computed using the following equations:
\[ E = \sum_{i=0}^{n-1} e_{x_i}^2 + \sum_{i=0}^{n-1} e_{y_i}^2 \]

where \( r \) and \((x_c, y_c)\) are the unknown radius and mid-point of the best fitting circle respectively. The angle \( \theta_i \) is simply \( \tan^{-1}\left(\frac{y_i - y_c}{x_i - x_c}\right) \), where \((x_c, y_c)\) is the estimated mid-point of the circle, and \((x_i, y_i)\) is the \(i\)th input point. The radius \( r \), and the mid-point \((x_c, y_c)\) which minimize \( E \) can therefore be found by solving the following partial differential equations:

\[
\frac{\partial E}{\partial x_c} = -2 \sum_{i=0}^{n-1} (x_i - r \cos \theta_i - x_c) = 0
\]

\[
\frac{\partial E}{\partial y_c} = -2 \sum_{i=0}^{n-1} (y_i - r \sin \theta_i - y_c) = 0
\]

\[
\frac{\partial E}{\partial r} = -2 \sum_{i=0}^{n-1} (x_i - r \cos \theta_i - x_c) \cos \theta_i
\]

\[
-2 \sum_{i=0}^{n-1} (y_i - r \sin \theta_i - y_c) \sin \theta_i = 0
\]

In the actual circle fitting algorithms, these differential equations are rewritten in the form of matrices and are used for quickly finding \((x_c, y_c)\) and \( r \) of the best fitting circle.

**Empirical Evaluation**

We have developed six applications, each employing a different visual language. The applications are:

- **State transition diagrams:** This takes a state transition diagram and determines if a string can be recognized by the corresponding finite state automata.

- **Mathematical expressions:** This recognizes a mathematical expression constructed from the standard trigonometric functions and arithmetic operators, and generates the corresponding \LaTeX{} expression.

- **Structured flow charts:** This takes a structured flow chart consisting of if and loop constructs, and generates the corresponding \LaTeX{} program pseudo code.

- **Message sequence charts:** This recognizes a message sequence charts which is commonly used to describe interaction between entities, how messages are interchanged between process instances, and what internal actions they performed.

- **Binary tree:** This recognizes a binary tree diagram representing a mathematical expression constructed from binary operators.

- **N-ary tree:** This recognizes trees with an arbitrary number of children. Although this visual language appears similar to that for binary trees, it is actually quite different, and relies on recursive use of negative constraints.

Diagrams for each of these applications are shown in Figure 8 (taken from (Chok & Marriott 1998)).

Static parsing of quite complex diagrams - actually those shown in Figure 8 - takes less than 30 milliseconds while incremental parsing of a new token takes a few milliseconds. As for parsing with error correction, there is a significant overhead due to the need for constraint solving. In all examples except the mathematical expressions, the performance is still good, taking less than one tenth of a second. The mathematical expression application is slow because of the large number of constraints involved. For a detailed empirical evaluation of the Penguins system, see to (Chok & Marriott 1998).

**Future Work**

The Penguins project is still work in progress and some aspects of the system are yet to be implemented. These include extending the constraint multi-set grammar formalism include specification of layout requirements and layout control generator. From experience, it is often not desirable to specify layout requirements as required constraints in parsing since doing this imposes drastic restrictions a user can draw the diagram. Therefore layout requirements need to be specified separately from those for recognition. They can be viewed as additional geometric constraints which are considered by the constraint solver but not the incremental parser. As an example, consider a simple layout rule which specifies that a labelled circle must be ten pixel units in length.

```plaintext
L:labelledCircle() ::= C:circle,T:text where ( 
  T.mid == C.mid ) ( 
  L.mid = C.mid; 
  L.radius = C.radius; 
  L.label = T.text; 
) layout { 
  L-radius == 10; 
}
```
Diagram layout can be couched as an optimization problem, in which constraints arise from the topology of the graph and there is an optimization function which must be minimized. The optimization function captures aesthetic criteria in layout. In our context, the layout controller will be used for beautification or to redisplay the diagram after the application modifies the parse tree.

The next stage of the Penguins project is to incorporate transformation rules into constraint multiset grammars. Transformation rules allow high-level specification of diagram execution and animation. Unlike production rules, transformation rules are evaluated at user's command. This is to avoid unwanted modification to the diagram. As an example, consider a transformation rule `tCircleToSquare` that transforms a labelled circle to a labelled square.

\[
x = (a \cdot b \cdot y) + b \cdot z \cdot \sin(a - b + 1) / 2
\]
\[
y = \frac{z \cdot \sin(a - b + 1)}{\sin(a - b + 1)} - b \cdot 2
\]
\[
z = \frac{(a \cdot b \cdot y) + b \cdot z \cdot \sin(a - b + 1)}{2}
\]

Another area for future research is the extension of the underlying constraint solver to provide non-linear as well as linear arithmetic constraints. Currently, inherently non-linear constraints such as `OnCircle` must be approximated by the application programmer using linear constraints.

One final area of future work is to perform usability studies on the Penguins system. One such study involves the development of some practical pen-based applications based on different visual languages and to compare the relative ease of diagram creation in these applications with traditional graphics editors.
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References


