Two Algorithms for Learning the Parameters of Stochastic Context-Free Grammars

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Abstract

Stochastic context-free grammars (SCFGs) are often used to represent the syntax of natural languages. Most algorithms for learning them require storage and repeated processing of a sentence corpus. The memory and computational demands of such algorithms are ill-suited for embedded agents such as a mobile robot. Two algorithms are presented that incrementally learn the parameters of stochastic context-free grammars as sentences are observed. Both algorithms require a fixed amount of space regardless of the number of sentence observations. Despite using less information than the inside-outside algorithm, the algorithms perform almost as well.

Introduction

Although natural languages are not entirely context free, stochastic context-free grammars (SCFGs) are an effective representation for capturing much of their structure. However, for embedded agents, most algorithms for learning SCFGs from data have two shortcomings. First, they need access to a corpus of complete sentences, requiring the agent to retain every sentence it hears. Second, they are batch algorithms that make repeated passes over the data, often requiring significant computation in each pass. These shortcomings are addressed through two online algorithms called SPAN and PRESPAN that learn the parameters of SCFGs using only summary statistics in combination with repeated sampling techniques.

SCFGs contain both structure (i.e. rules) and parameters (i.e. rule probabilities). One approach to learning SCFGs from data is to start with a grammar containing all possible rules that can be created from some alphabet of terminals and non-terminals. Typically the size of the right-hand-side of each rule is bound by a small constant (e.g. 2). Then an algorithm for learning parameters is applied and allowed to "prune" rules by setting their expansion probabilities to zero (Lari & Young, 1990). PRESPAN and SPAN operate in this paradigm by assuming a fixed structure and modifying the parameters.

Given a SCFG to be learned, both algorithms have access to the structure of the grammar and a set of sentences generated by the grammar. The correct parameters are unknown. PRESPAN and SPAN begin by parsing the sentence corpus using a chart parser. Note that the parse of an individual sentence does not depend on the parameters; it only depends on the structure. However, the distribution of sentences parsed does depend on the parameters of the grammar used to generate them. Both algorithms associate with each rule a histogram that records the number of times the rule is used in parses of the individual sentences.

PRESPAN and SPAN make an initial guess at the values of the parameters by setting them randomly. They then generate a corpus of sentences with these parameters and parse them, resulting in a second set of histograms. The degree to which the two sets of histograms differ is a measure of the difference between the current parameter estimates and the target parameters. PRESPAN modifies its parameter estimates so the sum total difference between the histograms is minimized. In contrast, SPAN modifies its estimates so the difference between individual histograms is minimized. Empirical results show that this procedure yields parameters that are close to those found by the inside-outside algorithm.

Stochastic Context-Free Grammars

Stochastic context-free grammars are the natural extension of Context-Free Grammars to the probabilistic domain (Sipser, 1997; Charniak, 1993). Said differently, they are context-free grammars with probabilities associated with each rule. Formally, a SCFG is a four-tuple $M = (V, \Sigma, R, S)$ where

1. $V$ is a finite set of non-terminals
2. $\Sigma$ is a finite set, disjoint from $V$, of terminals
3. $R$ is a finite set of rules of the form $A \rightarrow w$ where $A$ belongs to $V$, and $w$ is a finite string composed of elements from $V$ and $\Sigma$. We refer to $A$ as the left-hand side (LHS) of the rule and $w$ as the right-hand side (RHS), or expansion, of the rule. Additionally, each rule $r$ has an

paradigm by assuming a fixed structure and modifying the parameters.
associated probability \( p(r) \) such that the probabilities of rules with the same left-hand side sum to 1.

4. \( S \) is the start symbol.

Grammars can either be ambiguous or unambiguous. Ambiguous grammars can generate the same string in multiple ways. Unambiguous grammars cannot.

Learning Stochastic-Context Free Grammars

Learning context-free grammars is the problem of inducing a context-free structure (or model) from a corpus of sentences (i.e., data). When the grammars are stochastic, one faces the additional problem of learning rule probabilities (parameters) from the corpus. Given a set of sentence observations \( O = \{o_0 \ldots o_{n-1}\} \), the goal is to discover the grammar that generated \( O \). Typically, this problem is framed in terms of a search in grammar space where the objective function is the likelihood of the data given the grammar.

While the problem of incrementally learning the structure of SCFGs is interesting in its own right, the main focus here is on learning parameters. For a thorough overview of existing techniques for learning structure, see (Stolcke, 1994; Chen, 1996; Nevill-Manning & Witten, 1997).

Learning Parameters

The inside-outside algorithm (Lari & Young, 1990; Lari & Young, 1991) is the standard method for estimating parameters in SCFGs. The algorithm uses a general-purpose expectation-maximization (EM) procedure. Almost all parameter learning is done batch style using some version of the inside-outside algorithm. For example, in learning parameters, (Chen, 1995) initially estimates the rule probabilities using the most probable parse of the sentences given the grammar. While the problem of incrementally learning the structure of SCFGs is interesting in its own right, the main focus here is on learning parameters. For a thorough overview of existing techniques for learning structure, see (Stolcke, 1994; Chen, 1996; Nevill-Manning & Witten, 1997).

Table 1: A grammar that generates palindromes

<p>| | | | |</p>
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<thead>
<tr>
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<tbody>
<tr>
<td>( S \rightarrow AA )</td>
<td>( S \rightarrow AC )</td>
<td>( S \rightarrow BB )</td>
<td>( S \rightarrow BC )</td>
</tr>
<tr>
<td>( A )</td>
<td>( A )</td>
<td>( B )</td>
<td>( C )</td>
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<tbody>
<tr>
<td>( B \rightarrow Z )</td>
<td>( B \rightarrow Y )</td>
<td>( C \rightarrow Z )</td>
<td>( D \rightarrow Y )</td>
</tr>
</tbody>
</table>

**Figure 1:** The palindrome grammar rule histograms after only one parse of the sentence \( y y y y \). Because only one sentence has been parse, the mass of the distribution is concentrated in a single bin.

The Algorithms

PRESPAN and SPAN address some of the concerns given in the previous section. Both are unsupervised, incremental algorithms for finding parameter estimates in stochastic context-free grammars.

PRESPAN and SPAN are incremental in two ways. First, in the classical sense, at every iteration they use the previously learned parameter estimation as a stepping stone to the new parameters. (Chen, 1995) initially estimates the rule probabilities using the most probable parse of the sentences given the grammar. While the problem of incrementally learning the structure of SCFGs is interesting in its own right, the main focus here is on learning parameters. For a thorough overview of existing techniques for learning structure, see (Stolcke, 1994; Chen, 1996; Nevill-Manning & Witten, 1997).
would have decreasing impact on the histograms' distributions. This is because the bin counts typically increase linearly while the sample size remains constant. Future work will examine the role of the normalization factor, however, for this work it is kept fixed throughout the duration of the algorithm.

For each rule \( r \) PRESPAN now has two distributions: \( H^L_r \), based on the corpus generated from \( T \), and \( H^C_r \) based on the corpus generated from \( M \). Comparing \( H^L_r \) to \( H^C_r \) seems a natural predictor of the likelihood of the observation corpus given PRESPAN's learning grammar. Relative entropy (also known as the Kullback-Leibler distance) is commonly used to compare two distributions \( p \) and \( q \) (Cover & Thomas, 1991). It is defined as:

\[
D(p||q) = \sum_x p(x) \log \frac{p(x)}{q(x)}
\]

Because two distributions are associated with each rule \( r \), the relative entropies are summed over the rules.

\[
\tau = \sum_r \sum_x p_r(x) \log \frac{p_r(x)}{q_r(x)}
\]

If \( \tau \) decreases between iterations, then the likelihood of \( M \) is increasing so PRESPAN increases the probabilities of the rules used in generating the sample corpus. When \( s \) is large, the algorithm only increases a small subset of the rules used to generate the sample. Likewise, if \( \tau \) increases between iterations, PRESPAN decreases the rule probabilities.

PRESPAN uses a multiplicative update function. Suppose rule \( r \) was selected for an update at time \( t \). If \( p_t(r) \) is the probability of \( r \) at time \( t \) and \( \tau \) decreased between iterations, then \( p_{t+1}(r) = 1.01 \times p_t(r) \). Once the probability updates are performed PRESPAN starts another iteration beginning with the generation of a small sentence corpus from the learning grammar. The algorithm stops iterating when the relative entropy falls below a threshold, or some pre-specified number of iterations has completed.

**A Description of SPAN**

SPAN differs from PRESPAN in the selection of rules to update, the criteria for updates, and the update rule itself. Recall that PRESPAN uses \( \tau \) (Equation 1), the sum of relative entropy calculations for each rule, as a measure of progress or deterioration of grammar updates. Since \( \tau \) is an aggregate value, an unsuccessful change in probability for one rule could overshadow a successful change of another rule. Furthermore, the update rule does not differentiate between small and large successes and failures.

SPAN addresses these concerns by examining local changes in relative entropy and using those values to make rule specific changes. SPAN calculates the relative entropy for rule \( r \) at time \( t \) and compares it with the relative entropy at time \( t - 1 \). If the relative entropy decreases it means SPAN updated the rule probability favorably, if it increases, the distributions have become more dissimilar so the probability

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\*Using only the rules fired during the generation of the last sentence seems to work well.
should move in the opposite direction. This is best explained by examining \textsc{Span}’s update rule:

\[
P_{t+1}(r) = P_t(r) + \alpha \cdot \text{sgn}(P_t(r) - P_{t-1}(r))
+ \beta \cdot f(\Delta RE_t)
+ \beta \cdot (P_t(r) - P_{t-1}(r))
\]

(2)

The update rule is based on the steepest descent method (Bertsekas & Tsitsiklis, 1996). Here, \( \text{sgn} \) is the “sign” function that returns -1 if its argument is negative, 0 if its argument is zero, and +1 if its argument is positive. The first sign function determines the direction of the previous update. That is, it determines whether, in the last time step, \textsc{Span} increased or decreased the probability. The second sign function determines if the relative entropy has increased or decreased. If it has decreased, then the difference is positive, if it has increased, the difference is negative. Together these sign functions determine the direction of the step. The function \( f(\Delta RE_t) \) returns the magnitude of the step. This, intuitively, is an estimate of the gradient since the magnitude of the change in relative entropy is reflective of the slope. The \( \alpha \) parameter is a step-size. Finally the \( \beta \cdot (P_t(r) - P_{t-1}(r)) \) expression is a momentum term.

Once the probability updates are performed for each rule, another iteration starts beginning with the generation of a small sentence corpus from the learning grammar. Like \textsc{PreSpan}, the algorithm stops iterating when the relative entropy falls below a threshold, or some prespecified number of iterations has completed. \textsc{Span} is a more focused learning algorithm than \textsc{PreSpan}. This is because all rules are individually updated based on local changes instead of stochastically selected and updated based on global changes. The algorithmic changes speed up learning by, at times, two orders of magnitude. While these benefits drastically increase learning time, they do not necessarily result in more accurate grammars. Evidence and explanation of this is given in the Experiments section.

**Algorithm Analysis**

In this section both the time and space requirements of the algorithms are analyzed. Comparing the results with the time and space requirements of the inside-outside algorithm shows that \textsc{Span} and \textsc{PreSpan} are asymptotically equivalent in time but nearly constant (as opposed to linear with inside-outside) in space.

The inside-outside algorithm runs in \( O(L^3|V|^3) \) time where \( L \) is the length of sentence corpus and \( |V| \) is the number of non-terminal symbols (Stolcke, 1994). The complexity arises directly from the chart parsing routines used to estimate probabilities. Note that the number of iterations used by the inside-outside algorithm is dominated by the computational complexity of chart parsing.

Both \textsc{Span} and \textsc{PreSpan} chart parse the observation corpus once but repeatedly chart parse the fixed size samples they generate during the learning process. Taken as a whole, this iterative process typically dominates the single parse of the observations sentences, so the computational complexity is \( O(J^3|V|^3) \) where \( J \) is the length of of the maximum sample of any iteration.

Every iteration of the inside-outside algorithm requires the complete sentence corpus. Using the algorithm in the context of embedded agents, where the sentence corpus increases continuously with time, means a corresponding continuous increase in memory. With \textsc{Span} and \textsc{PreSpan}, the memory requirements remain effectively constant.

While the algorithms continually update their learning histograms through the learning process, the number of bins increases only when a sentence parse contains an occurrence count larger than any encountered previously. The sample is representative of the grammar parameters and structure, so typically after a few iterations, the number of bins becomes stable. This means that when new sentences are encountered there is typically no increase in the amount of space required.

**Experiments**

The previous section described two online algorithms for learning the parameters of SCFGs given summary statistics computed from a corpus of sentences. The remaining question is whether the quality of the learned grammar is sacrificed because a statistical summary of the information is used rather than the complete sentence corpus. This section presents the results of experiments that compare the grammars learned with \textsc{PreSpan} and \textsc{Span} with those learned by the inside-outside algorithm.

The following sections provide experimental results for both the \textsc{PreSpan} and \textsc{Span} algorithms.

**Experiments with \textsc{PreSpan}**

Let \( M_T \) be the target grammar whose parameters are to be learned. Let \( M_L \) be a grammar that has the same structure

<table>
<thead>
<tr>
<th>Table 2: A grammar generating simple English phrases</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S \rightarrow NP )</td>
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<tr>
<td>( NP \rightarrow Det )</td>
</tr>
<tr>
<td>( VP \rightarrow Vt )</td>
</tr>
<tr>
<td>( PP \rightarrow Vt )</td>
</tr>
<tr>
<td>( Det \rightarrow THE )</td>
</tr>
<tr>
<td>( Vt \rightarrow Vt )</td>
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<tr>
<td>( Vc \rightarrow IS )</td>
</tr>
<tr>
<td>( Vt \rightarrow ROLLS )</td>
</tr>
<tr>
<td>( N \rightarrow CIRCLE )</td>
</tr>
<tr>
<td>( N \rightarrow TRIANGLE )</td>
</tr>
<tr>
<td>( P \rightarrow ABOVE )</td>
</tr>
<tr>
<td>( A \rightarrow a )</td>
</tr>
<tr>
<td>( THE \rightarrow the )</td>
</tr>
<tr>
<td>( IS \rightarrow is )</td>
</tr>
<tr>
<td>( ROLLS \rightarrow rolls )</td>
</tr>
<tr>
<td>( BOUNCES \rightarrow bounces )</td>
</tr>
<tr>
<td>( CIRCLE \rightarrow circle )</td>
</tr>
<tr>
<td>( SQUARE \rightarrow square )</td>
</tr>
<tr>
<td>( TRIANGLE \rightarrow triangle )</td>
</tr>
<tr>
<td>( ABOVE \rightarrow above )</td>
</tr>
<tr>
<td>( BELOW \rightarrow below )</td>
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</tbody>
</table>
The power of the t-test is 0.95. That is, with probability 0.95 the significance level of 0.05, and an actual delta of 174.79, the sample size of 50, a standard deviation of 240.05, a significance of the test, and the actual difference between the means. Given the fact that the means of the two distributions are minute, a more powerful test is needed.

Since both PRESPAN and the inside-outside algorithm were run on the same problems, a paired sample t-test can be applied. This test is more powerful than the standard t-test. Suppose again the the means of the two distributions are equal. Using this as the null hypothesis and performing the paired sample t-test yields \( p < 0.01 \). That is, the probability of making an error in rejecting the null hypothesis is less than 0.01. Closer inspection of the data reveals why this is the case. Inside-outside performed better than the online algorithm on each of the 50 grammars. However, as is evident from the means and standard deviation, the absolute difference in each case was quite small.

The same experiments were conducted with the ambiguous grammar shown in Table 3. The grammar is ambiguous because, for example, \( z z \) can be generated by \( S \to A \to CC \to zz \) or \( S \to BA \) with \( B \to S \to C \to z \) and \( A \to C \to z \). The mean and standard deviation of the log likelihoods for PRESPAN were \( \mu = -1983.15 \) and \( \sigma = 250.96 \). These values for the inside-outside algorithm were \( \mu = -1979.37 \) and \( \sigma = 250.57 \). The standard t-test returned a \( p \) value of 0.94 and the paired sample t-test was significant at the 0.01 level. Again, inside-outside performed better on every one of the 50 grammars, but the differences were very small.

**Experiments with SPAN**

The same experiments were performed using SPAN. That is, the grammar in Table 2 was used with 50 different target parameter settings and 500 sentences in \( \Omega^T \) for each setting. The mean and standard deviation of the log likelihoods for the SPAN with \( h = s = 100 \) (histogram size and learning corpus size respectively) were \( \mu = -4266.44 \) and \( \sigma = 650.57 \). These values for the inside-outside algorithm were \( \mu = -3987.58 \) and \( \sigma = 608.59 \). Recall that equivalent performance would be a significant accomplishment because the online algorithm has access to much less information about the data. Assuming both the means of the distributions are equal and using this as the null hypothesis of a two-tailed t-test results in \( p = 0.03 \).

The same experiment was conducted with the ambiguous grammar shown in Table 3. The grammar is ambiguous, for example, because \( z z \) could be generated by \( S \to A \to CC \to zz \) or \( S \to BA \) with \( B \to S \to C \to z \) and \( A \to C \to z \). The mean and standard deviation of the log likelihoods for the online algorithm were \( \mu = -2025.93 \) and \( \sigma = 589.78 \). These values for the inside-outside algorithm were \( \mu = -1838.41 \) and \( \sigma = 523.46 \). The t-test returned a \( p \) value of 0.03.

Unfortunately, the above result does not sanction the conclusion that the two distributions are the same. One can, however, look at the power of the test in this case. If the test's power is high then it is likely that a true difference in the means would be detected. If the power is low then it is unlikely that the test would detect a real difference. The power of a test depends on a number of factors, including the sample size, the standard deviation, the significance level of the test, and the actual difference between the means. Given a sample size of 50, a standard deviation of 240.05, a significance level of 0.05, and an actual delta of 174.79, the power of the t-test is 0.95. That is, with probability 0.95 the t-test will detect a difference in means of at least 174.79 at the given significance level. Because the mean of the two distributions is minute, a more powerful test is needed.

<table>
<thead>
<tr>
<th>Table 3: An ambiguous grammar</th>
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<tbody>
<tr>
<td>( S \to A )</td>
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<tr>
<td>( S \to B \ A )</td>
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<tr>
<td>( A \to C )</td>
</tr>
<tr>
<td>( A \to C \ C )</td>
</tr>
<tr>
<td>( B \to S )</td>
</tr>
<tr>
<td>( B \to D )</td>
</tr>
<tr>
<td>( B \to E )</td>
</tr>
<tr>
<td>( B \to F )</td>
</tr>
<tr>
<td>( C \to z )</td>
</tr>
<tr>
<td>( D \to y )</td>
</tr>
<tr>
<td>( E \to x )</td>
</tr>
<tr>
<td>( F \to w )</td>
</tr>
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</table>
the inside-outside algorithm is always higher than that for SPAN. Comparison of the parameters learned shows that SPAN is moving in the direction of the correct parameters but that it never actually converges on them.

**Discussion**

It was noted earlier that SPAN learns more quickly than PRESPAN but the Experiments section shows this improvement may come at a cost. One reason for this may lie in the sentence samples produced from the learning grammar during each iteration. Recall that SPAN learns by generating a sentence sample using its current parameter estimates. Then this sample is parsed and the distribution is compared to the distribution of the sentences generated from the target grammar. Each sentence sample reflects the current parameter estimates, but also has some amount of error. This error may be more pronounced in SPAN because at each iteration, every rule is updated. This update is a direct function of statistics computed from the sample, so the sample error may overshadow actual improvement or deterioration in parameter updates from the last iteration.

Overcoming the sample error problem in SPAN might be accomplished by incorporating global views of progress, not unlike those used in PRESPAN. In fact, a synergy of the two algorithms may be an appropriate next step in this research.

Another interesting prospect for future parameter-learning research is based on rule orderings. Remember that parameter changes in rules closer to the start-symbol of a grammar have more effect on the overall distribution of sentences than changes to parameters farther away. One idea is to take the grammar, transform it into a graph so that each unique left-hand side symbol is a vertex and each individual right-hand side symbol is a weighted arc. Using the start-symbol vertex as the root node and assuming each arc has weight 1.0, one can assign a rank to each vertex by finding the weight of the shortest path from the root to all the other vertices. This ordering may provide a convenient way to iteratively learn the rule probabilities. One can imagine concentrating only on learning the parameters of the rules with rank 1, then fixing those parameters and working on rules with rank 2, and so forth. When the final rank is reached, the process would start again from the beginning. Clearly self-referential rules may pose some difficulty, but the ideas have yet to be fully examined.

**Conclusion**

Most parameter learning algorithms for stochastic context-free grammars retain the entire sentence corpus throughout the learning process. Incorporating a complete memory of sentence corpora seems ill-suited for learning in embedded agents. PRESPAN and SPAN are two incremental algorithms for learning parameters in stochastic context-free grammars using only summary statistics of the observed data. Both algorithms require a fixed amount of space regardless of the number of sentences they processes. Despite using much less information than the inside-outside algorithm, PRESPAN and SPAN perform almost as well.

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