Multiagent Games, Competitive Models, and Game Tree Planning

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Abstract

The paper is based on agent plan computing where the interaction amongst heterogeneous computing resources is via objects, multiagent AI and agent intelligent languages. Modeling, objectives, and planning issues are examined at an agent planning. A basis to model discovery and prediction planning is stated. The new agent computing theories the author defined since 1994 can be applied to present precise decision strategies on multiplayer games with only perfect information between agent pairs. The game trees are applied to train models. The computing model is based on a novel competitive learning with agent multiplayer game tree planning. Specific agents are assigned to transform the models to reach goal plans where goals are satisfied based on competitive game tree learning. The planning applications include OR- Operations Research as goal satisfiability and micro-managing decision support with means-end analysis.

1. Introduction

Modeling, objectives, and planning issues are examined with agent planning and competitive models. Model discovery and prediction is applied to compare models and get specific confidence intervals to supply to goal formulas. Competitive model learning is presented based on the new agent computing theories the author defined since 1994. The foundations are applied to present precise decision strategies on multiplayer games with only perfect information between agent pairs. The game tree model is applied to "train" models. The computing model is based on a novel competitive learning with agent multiplayer game tree planning. Specific agents are assigned to transform the models to reach goal plans where goals are satisfied based on competitive game tree learning. Intelligent and/or trees and means-end analysis is applied with agents as the hidden–step computations. A novel multiplayer game model is presented where “intelligent” agent enriched languages can be applied to address game questions on models in the mathematical logic sense.

The project’s accomplishments have specific application to emerging areas on intent inference. Modeling intent-aware systems with intelligent software agents, systems with multiple autonomous operators, team and adversarial intent inference, benefit from multiplayer games and competitive models presented here. Game tree planning might applied as a basis to modelling intent inference systems with adversarial goals. Applications range from designing complex systems for command and control centers, to business modelling, economic games, and enterprise resource planning (Nourani 2002). For adversarial intent inference, decision support for teams facing intelligent opponents are limited in their utility without planning systems which can encompass adversary goals and actions. Thus, modelling asymmetric multiplayer competitive games is a key capability to intent inference. The models presented are robust and encompass complex multiple operators. Collaborative agents which have opened new avenues in modeling and implementing teams of cooperative agents are ingredients to specific application modeling, for example ERP presented in the author’s projects. The development of intent-aware decision support for multi-operator complex systems is facilitated with the competitive model decision tree planning techniques in the author’s projects since 1994.

2. KR, KB, and PLAN Discovery

Modeling with agent planning is applied where uncertainty, including effector and sensor uncertainty, is relegated to agents, where competitive learning on game trees determines a confidence interval. The incomplete knowledge modeling is treated with KR on predictive model diagrams. Model discovery at KB’s are with specific techniques defined for trees. Model diagrams allow us to model-theoretically characterize incomplete KR. To key into the incomplete knowledge base we apply generalized predictive diagrams whereby specified diagram functions a search engine can select onto localized data fields. The predictive model diagrams (Nourani 1995) could be minimally represented by the set of functions \{f1,…,fn\} that inductively define the model. Data discovery from KR on diagrams might be viewed as satisfying a goal by getting at relevant data which instantiates a goal. The goal formula states what relevant data is sought. We propose methods that can be applied to
planning (Nourani 1991) with diagrams to implement discovery planning. In planning with G-diagrams that part of the plan that involves free Skolemized trees is carried along with the proof tree for a plan goal. Computing with diagram functions allows us to key to active visual databases with agents.

Diagrams are well-known concepts in mathematical logic and model theory. The diagram of a structure is the set of atomic and negated atomic sentences that are true in that structure. Models uphold to a deductive closure of the axioms modeled and some rules of inference. The generalized diagram (G-diagram) (Nourani 1991,94a) is a diagram in which the elements of the structure are all represented by a specified minimal set of function symbols and constants, such that it is sufficient to define the truth of formulas only for the terms generated by the minimal set of functions and constant symbols. Such assignment implicitly defines the diagram. It allows us to define a canonical model of a theory in terms of a minimal family of function symbols. The minimal set of functions that define a G-diagram are those with which a standard model could be defined. Formal definition of diagrams are stated here, generalized to G-diagrams, and applied in the sections to follow.

2.1 Prediction and Discovery

Minimal prediction is an artificial intelligence technique defined since the author's model-theoretic planning project. It is a cumulative nonmonotonic approximation (Nourani 1999c) attained with completing model diagrams on what might be true in a model or knowledge base. A predictive diagram for a theory T is a diagram D (M), where M is a model for T, and for any formula q in M, either the function $f: q \rightarrow \{0,1\}$ is defined, or there exists a formula $p$ in D (M), such that $T \cup \{p\}$ proves $q$; or that $T$ proves $q$ by minimal prediction. A generalized predictive diagram, is a predictive diagram with D (M) defined from minimal specific functions. The predictive diagram could be minimally represented by a set of functions $\{f_1,\ldots,f_n\}$ that inductively define the model. The free trees we had defined by the notion of provability implied by the definition, could consist of some extra Skolem functions $\{g_1,\ldots,g_l\}$ that appear at free trees. The f terms and g terms, tree congruences, and predictive diagrams then characterize partial deduction with free trees. The predictive diagrams are applied to discover models to the intelligent game trees. Prediction is applied to plan goal satisfiability and can be combined with plausibility (Nourani 1991) probabilities, and fuzzy logic to obtain, for example, confidence intervals.

2.2 KR with Keyed Functions

Practical AI systems are designed by modeling AI with facts, rules, goals, strategies, knowledge bases. Patterns, schemas, AI frames (Fikes and Kheler 1985) and viewpoints are the micro to aggregate glimpses onto the database and knowledge bases were masses of data and their relationships-represenations, respectively, are stored. Schemas and frames are what might be defined with objects, the object classes, the object class inheritances, user-defined inheritance relations, and specific restrictions on the object, class, or frame slot types and behaviors. A scheme might be

Intelligent Forecasting

IS-A Stock Forecasting Technique
Portfolios Stock, bonds, corporate assets
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Schemas allow brief descriptions on object surface properties with which high level inference and reasoning with incomplete knowledge can be carried out applying facts and the defined relationships amongst objects. Relationships: Visual Objects A and B have mutual agent visual message correspondence. Looking for patterns is a way some practical AI is a carried on with to recognize important features, situations, and applicable rules. From the proofs standpoint patterns are analogies to features as being leaves on computing trees. Forward chaining is a goal satisfaction technique, where inference rules are activated by data patterns, to sequentially get to a goal by apply the inference rules. The current pertinent rules are available at an agenda store. The carried out rules modify the database. Backward chaining is an alternative based on opportunistic response to changing information. It starts with the goal and looks for available premises that might be satisfied to have gotten there.

Goals are objects for which there is automatic goal generation of missing data at the goal by recursion backward chaining on the missing objects as sub-goals. Data unavailability implies search for new goal discovery. Goal Directed Planning is carried out while planning with diagrams. That part of the plan that involving free Skolemized trees is carried along with the proof tree for a plan goal. If the free proof tree is constructed then the plan has an initial model in which the goals are satisfied.

Let us see what predictive diagrams do for knowledge discovery knowledge management. Diagrams allow us to model-theoretically characterize incomplete KR. To key into the incomplete knowledge base. The following figure depicts selector functions Fi from an abstract view grid
interfaced via an inference engine to a knowledge base and in turn onto a database.

![Diagram of Keyed KR, Inference, and Model Discovery](image)

**Figure 1.** Keyed KR, Inference, and Model Discovery

Generalized predictive diagrams where applying specified diagram functions a search engine can select onto localized data fields. A *Generalized Predictive Diagram*, is a predictive diagram with $D$ ($M$) defined from a minimal set of functions. The predictive diagram could be minimally represented by a set of functions $\{f_1,...,f_n\}$ that inductively define the model. The functions are keyed onto the inference and knowledge base to select via the areas keyed to, designated as $S_i$'s in figure 1. Visual object views to active databases might be designed with the above. The trees defined by the notion of provability implied by the definition might consist of some extra Skolem functions $\{g_1,...,g_n\}$, that appear at free trees. The $f$ terms and $g$ terms, tree congruences, and predictive diagrams then characterize deduction with virtual trees as intelligent predictive interfaces. Data discovery from KR on diagrams might be viewed as satisfying a goal by getting at relevant data which instantiates a goal. The goal formula states what relevant data is sought. We have presented planning techniques, which can be applied to implement discovery planning. In planning with G-diagrams that part of the plan that involves free Skolemized trees is carried along with the proof tree for a plan goal. The idea is that if the free proof tree is constructed then the plan has a model in which the goals are satisfied. The model is the initial model of the AI world for which the free Skolemized trees were constructed. Partial deductions in this approach correspond to proof trees that have free Skolemized trees in their representation. While doing proofs with free Skolemized trees we are facing proofs of the form $p(g(\ldots))$ proves $p(f(g(\ldots))$ and generalizations to $p(f(x))$ proves for all $x$, $p(f(x))$. Thus the free proofs are in some sense an abstract counterpart of the SLD.

### 3. Multiplayer Decision Trees

We have defined specific application areas for multiagent computing to multinational corporations and their strategic management of multinational transactional business models appears in brief at (Nourani 1998a). The areas applied to are global planning, external enterprise assessment, and goal-setting applications for operations research and market forecasting (Nourani 1998b). A specific models starting with a transactional business model is in (Nourani 1999a). The organizational knowledge (van Heijst et.al. 1994) is one of the main bases to competitive advantage. Enterprise modeling includes stock management, payroll, and advanced administrative tasks applying decision support. The following figure is a glimpse onto applying means-end analysis decision support where the hidden steps are designed and computed with parameter agents. The obvious planning goal satisfaction applications are where agents apply backward chaining from objectives.

**Figure 2.** Means-End Decision Trees

### 4. Competitive Models and Games

Planning is based on goal satisfaction at models. Multiagent planning, for example as (Muller and Pischel 1994, Bazier et.al. 1997), in the paper is modeled as a competitive learning problem where the agents compete on game trees as candidates to satisfy goals hence realizing specific models where the plan goals are satisfied. When a specific agent group “wins” to satisfy a goal the group has presented a model to the specific goal, presumably consistent with an intended world model. For example, if there is a goal to put a spacecraft at a specific planet’s orbit, there might be competing agents with alternate micro-plans to accomplish the goal. While the galaxy model is the same, the specific virtual worlds where a plan is carried out to accomplish a real goal at the galaxy via agents are not. Therefore, Plan goal...
selections and objectives are facilitated with competitive agent learning. The intelligent languages (Nourani 1996, 1998) are ways to encode plans with agents and compare models on goal satisfaction to examine and predict via model diagrams why one plan is better than another, or how it could fail. Virtual model planning is treated in the author’s publications where plan comparison can be carried out at VR planning (Nourani 1999d). Games play an important role as a basis to economic theories. Here the import is brought forth onto decision tree planning with agents. Intelligent tree computing theories the author defined since 1993 can be applied to present precise strategies and prove theorems on multiplayer games. Game tree degree with respect to models is defined and applied to prove soundness and completeness. The game is viewed as a multiplayer game with only perfect information between agent pairs. Upper bounds on determined games are presented. The author had presented a chess-playing basis in 1997 to a computing conference. For each chess piece a designating agent is defined. The player P makes its moves based on the board B it views. <P,B> might view chess as if the pieces on the board had come alive and were autonomous agents carrying out two-person games as in Alice in Wonderland. Game moves are individual tree operations.

4.1 Intelligent AND/OR Trees and Search

AND/OR trees Nilsson(1969) are game trees defined to solve a game from a player's standpoint.

```
         n
        / | \           an OR node.
       /   |   \          m
      |     |     \      an AND node
     |     |     |     |
    a1 a2 a3
```

Formally a node problem is said to be solved if one of the following conditions hold.
1. The node is the set of terminal nodes (primitive problem- the node has no successor).
2. The node has AND nodes as successors and the successors are solved.
3. The node has OR nodes as successors and any one of the successors is solved.

A solution to the original problem is given by the subgraph of AND/OR graph sufficient to show that the node is solved. A program which can play a theoretically perfect game would have task like searching and AND/OR tree for a solution to a one-person problem to a two-person game. An intelligent AND/OR tree is and AND/OR tree where the tree branches are intelligent trees. The branches compute a Boolean function via agents. The Boolean function is what might satisfy a goal formula on the tree. An intelligent AND/OR tree is solved iff the corresponding Boolean functions solve the AND/OR trees named by intelligent functions on the trees. Thus node m might be f(a1,a2,a3) & g(b1,b2), where f and g are Boolean functions of three and two variables, respectively, and ai's and bi's are Boolean valued agents satisfying goal formulas for f and g.

```
g is on OR agent
/|
|]
|b1|b2
 f f is an AND agent
/|
| |
a1 a2 a3
```

The chess game trees can be defined by agent augmenting AND/OR trees (Nilsson 69). For the intelligent game trees and the problem solving techniques defined, the same model can be applied to the game trees in the sense of two person games and to the state space from the single agent view. The two person game tree is obtained from the intelligent tree model, as is the state space tree for agents. To obtain the two-person game tree the cross-board-coboard agent computation is depicted on a tree. Whereas the state-space trees for each agent is determined by the computation sequence on its side of the board-coboard. Thus a tree node m might be f(a1,a2,a3) & g(b1,b2), where f and g are Boolean functions of three and two variables, respectively, and ai's and bi's are Boolean valued agents satisfying goal formulas for f and g.

A tree game degree is the game state a tree is at with respect to a model truth assignment, e.g. to the parameters to the Boolean functions above. Let generic diagram or G-diagrams be diagrams definable by specific functions. Intelligent signatures (Nourani 1996) are signatures with designated multiplayer game tree function symbols. A soundness and completeness theorem is proved on the intelligent signature language (Nourani 1996). The techniques allowed us to present a novel model-theoretic basis to game trees, and generally to the new intelligent game trees. The following specifics are
from (Nourani 1999b). Let \( N \) be the set of all functions from \( \omega \) to \( \omega \). Let \( A \) be a subset of \( N \). (Gale and Stewart, 1953) associated with \( A \) a 2-person game of perfect information \( G<A> \). Player I begins by choosing \( n_0 \) in \( \omega \); player two chooses \( n_1 \) in \( \omega \); then I chooses \( n_2 \) in \( \omega \); so on. Let \( a(i) = n_i \). I wins \( G<A> \) if and only if \( a \) in \( A \). We say that \( G<A> \) is determined if one of the players has a winning strategy.

**Proposition** If \( G<A> \) is determined, the complexity upper bound on the number of moves to win is \( A \)'s cardinality.

**Theorem** 1 For every pair \( p \) of opposing agents there is a set \( A<p> \) (N. The worse case bound for the number of moves for a determined game based on the intelligent game tree model is the sum \( \sum_{|A<p>|} \) p agent pairs. \( \sum \) over the proposition (Nourani 1999b).

At the intelligent game trees the winning agents determine the specific model where the plan goals are satisfied.

### 4.2 Two-Person Games

From the game tree viewpoint for what Shannon had estimated a complete tree carried to depth 6-three moves for each player- would already have one billion tip nodes. Yet from an abstract mathematical viewpoint only, the game is a two-person game with perfect information. However, the two-person game view is not a mathematical model for any chess playing algorithm or machine. The real chess game, from the abstract viewpoint, might well be modeled as a multiagent game, being only a two-agent game with perfect information between mutually informable agents. A multiagent chess design or chess computing by any technique does not in reality provide perfect information in a way, which can be applied. The perfect information overall is a massive amount of data to be examined. There are thousands of move trees computed for searches coming close to being exhaustive. The multiagent chess paradigm (Nourani 1997) is not based on an exhaustive two-person game with perfect information model. There is only minimal information for the multiagent plans across the board. The multiagent multiboard model is a realization where the game is partitioned and correlated amongst agents and boards, with a cognitive anthropomorphism to human player's mind. There is an abstract two-person game model, but it does not apply to define a chess-playing machine. It is there to make precise mathematical statements.

### 5. Agent Logic

**Definition 5.1** Let \( M \) be a structure for a language \( L \), call a subset \( X \) of \( M \) a generating set for \( M \) if no proper substructure of \( M \) contains \( X \), i.e., if \( M \) is the closure of \( X \cup \{c[M] : c \text{ is a constant symbol of } L\} \). An assignment of constants to \( M \) is a pair \( <A,G> \), where \( A \) is an infinite set of constant symbols and \( G : A \to M \), such that \( \{G[a] : a \in A\} \) is a set of generators for \( M \). For a fixed assignment \( <A,G> \) of constants to \( M \), the diagram of \( M \), \( D<A,G>[M] \) is the set of basic [atomic and negated atomic]sentences of \( L[A] \) true in \( M \).

Generic diagrams, denoted by \( G \)-diagrams, were what we defined since 1980's to be diagrams for models defined by a specific function set, for example \( \Sigma_1 \) Skolem functions.

**Definition 5.2** A \( G \)-diagram for a structure \( M \) is a diagram \( D<A,G> \), such that the \( G \) in definition 5.1 has a proper definition by specific function symbols.

**Definition 5.3** Let \( (M,a) \in C \) be defined such that \( M \) is a structure for a language \( L \) and each constant \( c \) in \( C \) has the interpretation \( a \) in \( M \). The mapping \( c \to a \) is an assignment of \( C \) in \( M \). We say that \( (M,a) \in C \) is canonical model for a presentation \( P \) on language \( L \), iff the assignment \( c \to a \) maps \( C \) onto \( M \), i.e. \( M=(a :c \in C) \).

**Definition 5.4** A signature is intelligent iff it has intelligent function symbols. We say that a language has intelligent syntax if the syntax is defined on an intelligent signature.

**Definition 5.5** A language \( L \) is said to be an intelligent language iff \( L \) is defined from an intelligent syntax.

### 5.2 Intelligent Syntax

It is essential to the formulation of computations on intelligent trees and the notion of congruence that we define tree intelligence content. A reason is that there could be loss of tree intelligence content when tree rewriting because not all intelligent functions are required to be mutually informable. Theories are presented by axioms that define them and it is difficult to keep track of what equations not to apply when proving properties. The author has presented the mathematics for intelligent tree computing and the abstract model theory at (Nourani 1994-96). Intelligent languages with models are presented at (Nourani 1998) and abbreviated here. What has to be defined, however, is some computational formulation of
intelligence content such that it applies to the present method of computability on trees. Once that formulation is presented, we could start decorating the trees with it and define computation on intelligent trees. It would be nice to view the problem from the standpoint of an example. The example of intelligent languages we could present have <O,A,R> triples as control structures. The A's have operations that also consist of agent message passing. The functions in AFS are the agent functions capable of message passing. The O refers to the set of objects and R the relations defining the effect of A's on objects. Amongst the functions in AFS only some interact by message passing. The difficult part is the functions could affect objects in ways that affect the intelligence content of a tree. Thus tree congruences are more complex for intelligent languages than those of ordinary syntax trees. Let us define tree intelligence content for the present formulation.

Definition 5.7 Say that a function f is a string function, iff f is an intelligent function symbol and f is 1-1. Otherwise, an intelligent function f is said to be a splurge function.

Remark: Nullary functions are defined to string functions. □

Definition 5.8 A function f is accessible by an agent iff either f or the agent function symbol is defined as proper language signature terms. A subtree t has access to an agent g iff there is a function symbol on t accessible by g. □

Definition 5.9 The tree intelligence degree, TID, is defined by induction on tree structures:
(0) a constant function symbol f has TID f;
(i) for a string function f, and tree f(t1,...,tn) the TID is defined by

U TID (ti::f), where (ti::f) refers to a subtree of ti with access to f via an agent function symbols on ti;
(ii) for a splurge function f, TID is defined by U TID (f:ti), where f:ti is the updated tree ti since a single access to ti from f. □

Let us commence with the preliminary logical basis as far as proofs and models.

Definition 5.10 We say that a logical theory T on intelligent syntax is an intelligent theory iff for every proof step preserves TID. We state T<IST> |- φ when T is an Intelligent Syntax theory. At equational logic φ is an equational formula. □

Theorem 2 Let T be a intelligent syntax theory. Then T is (a) A Sound logical theory iff every axiom or proof rule in T is TID preserving;
(b) A Complete logical theory iff there is a generic diagram G where the assignment of constants to M is a pair <A,G>, where A is an infinite set of constant symbols and G: A → M, such that \{G[a]: a in A\} the set of generators for M with G definable with the generic diagram, where M is a structure for the intelligent syntax language on which T is defined.

Proof By Definition of TID, definitions 5.1, 5.2., 5.3, completeness theorems for the first order logic, and completeness of induction for algebraic structures (Nourani 1991a, 1996a). □

6. Conclusions

The projects accomplishments on the Fall Symposium interest areas are: Game Tree Competitive Model Planning, Predictive Game Models, Specific Intent Inference and Adversarial Games Logic, Multiagent Planning, Predictive Modeling, Agent decision tree applications to forecasting and predictive risk analysis; Intelligent trees and intelligent gam tree applications to business cooperative computations; Game Tree Combative Business Model Planning; ERP with Predictive Game Models; Sound and complete agent logic] Predictive business modeling, Game Tree Planning; agent decision tree applications to forecasting, and predictive risk analysis.

A novel basis to decision-theoretic planning is presented classical and non-classical planning techniques, as, for example, (Hedeler et.al. 1990, Wilkins 1984) from artificial intelligence with games and decision trees providing a agent expressive planning model. We use a broad definition of decision-theoretic planning that includes planning techniques that deal with all types of uncertainty and plan evaluation. Planning with predictive model diagrams represented with keyed KR to knowledge bases is presented. Techniques for representing uncertainty, plan generation, plan evaluation, plan improvement, and are accommodate with agents, predictive diagrams, and competitive model learning. Modeling with effector and sensor uncertainty, incomplete knowledge of the current state, and how the world operates is treated with agents and competitive models. Bounds on game trees are presented as a measure on the complexity of model comparison and competitive
learning. Applications to means-end analysis and decision support with goal planning is presented.

References


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