Abstract
This paper investigates the use of Multivariate State Estimation Techniques as input in predicting the remaining useful life prediction of electronic products. A prognostics approach combining the Multivariate State Estimation Technique with life cycle damage prediction is then presented, along with a case study. The challenges of the approach are also discussed.

Introduction
Many methods are proposed for prognostics and health monitoring. N. Vichare and M. Pecht [1] classified these methods into three categories: 1) using expendable prognostic cells, such as “canaries” and fuses, which fail earlier than the host product to provide advance warning of failure; 2) the use of monitoring and reasoning of precursor parameters, such as shifts in performance parameters, progression of defects, that are precursors to impending failure; and 3) modeling stress and damage utilizing exposure conditions (e.g., usage, temperature, vibration, radiation) coupled with physics–of–failure (PoF) models to compute accumulated damage and assess remaining useful life (RUL).

An efficient RUL prediction can provide information about the future operational status of products. This information can be used to determine maintenance, replacement and logistics support strategies of products [2]. As mentioned before, the RUL can be predicted by the stress-damage prognostics approach. In this approach, the life-cycle loads are monitored in-situ, and used in conjunction with PoF-based damage models to assess the extent and rate of product degradation due to cumulative load exposures. The degradation is the combination of the product’s physical degradation (e.g., cracks, corrosion, delaminating), electrical degradation (e.g., an increase in resistance, and an increase in threshold voltage), and performance degradation (e.g., shifts of the product’s operating parameters from expected values). The life-cycle loads of a product can be generated from manufacturing, shipment, storage, handling, operating and non-operating conditions. The life-cycle loads (thermal, mechanical, chemical, electrical, and so on), either individually or in various combinations, may lead to performance or physical degradation of the product and reduce its service life [3].

Studies show that the Multivariate State Estimation Technique (MSET) can be used to monitor multiple parameters of the product, such as the temperature, humidity and vibration, and calculate the residuals between the actual and expected values of these parameters based on the healthy historic data. MSET can detect the incipient of the fault of a product by comparing the residuals with the threshold. [1][5][6]. Current research on MSET focuses on its function in fault detection, while ignoring its ability to provide degradation information of a product. S.Wegerich.[7] noted that MSET may be used to highlight degradation of a monitored product, but there have been no specific models to show this application. So far, no literature describes cases using MSET to do prognostics (the prediction of remaining useful life).

In this paper, the relationship between the degradation of a product and the residuals calculated by MSET is investigated and a degradation model using the residuals is presented. This model was used to monitor the degradation of a product and regress the trends of the degradations. The failure criteria of the product are created by combining the PoF analysis and the information from historic data about failed products. Based on the degradation trends and the failure criteria, the RUL of the product can be predicted.

MSET
MSET uses pattern recognition from healthy product data to generate an estimate of current health. The historic data is assumed to cover and provide data for the entire healthy range of the system. The results of MSET are residuals that describe the actual monitored data in terms of the expected healthy values.

Some conceptions involved in MSET include the data matrix, the system state or observation $X_{obs}$, the training data $T$, the memory matrix $D$, the remaining training data $L$, and the estimate $X_{est}$. The data matrix defined by MSET,
shown in the Figure 1, has \( n \) parameters, and each parameter has \( m \) values. Each row of the matrix is the time series values from \( t_1 \) to \( t_m \) of one parameter \( x_i \). Each column of the matrix lists the values of all the parameters from \( x_1 \) to \( x_n \) at one time \( t_i \). We call each column an observation or state of the product at the corresponding time because it contains all of the monitored parameters of the product.

The state or observation \( X_{\text{obs}} \) of the product at time \( t_i \) is represented by a vector \( X(t_i) \) or \( X_{\text{obs}}(t_i) \) of length \( n \), where \( n \) is the number of monitored parameters of the product [5].

\[
X(t_i) = [x_{i1}, x_{i2}, \ldots, x_{in}]^T
\]  
(1)

where \( x_{ij} \) is the measurement of parameter \( j (j=1,2,\ldots,n) \) at time \( t_i \).

Training data \( T \) is a matrix that consists of many healthy historic states. It can be defined by the matrix format:

\[
T = [X(t_1), \ldots, X(t_l), \ldots]
\]  
(2)

where \( l \) is the number of states selected for the training data. It is decided by users.

Memory matrix \( D \) consists of special states selected by algorithms from training data \( T \). If \( m \) amount of states from \( T \) are selected, memory matrix \( D \) can be defined as

\[
D = [X_1, X_2, \ldots, X_n]
\]  
(3)

where \( X_i \) is one state selected from the training data \( T \), \( m \) is the number of states. States in the training data that are not selected for memory matrix \( D \) form the remaining training data \( L \).

\[
T = D \cup L
\]  
(4)

The estimate of observation, \( X_{\text{est}} \), is the expected value calculated from the healthy data. This estimate has the same data format as the observation.

Figure 2 shows the MSET process. When parameters for monitoring the product are selected, new observations (\( X_{\text{obs}} \)) are acquired. Healthy data from historic or current acquired data are also chosen as training data (\( T \)). Special data from the training data are picked to create memory matrix \( D \). When memory matrix \( D \) is created, MSET will go through two processes. One, shown in the bold boxes, is to calculate estimates (\( L_{\text{est}} \)) of all the remaining training data \( L \), which is not chosen by the memory matrix though it is training data. MSET then calculates the residuals between the estimates and remaining training data \( L \). Because all these remaining training data are healthy, the residuals present the features of healthy states of the system and are called healthy residuals. MSET also calculate the estimates (\( X_{\text{est}} \)) of the new observation (\( X_{\text{obs}} \)), the residuals between the estimates and the corresponding observations. These residuals show the actual states of the product and are called actual residuals.

The following fault-detection process will compare actual residuals with healthy residuals to decide whether the current product is healthy or not. The common method is to employ a hypothesis test, such as the Sequential Probability Ratio Test (SPRT), to produce alerting patterns. The SPRT is a well-known statistical likelihood test that can be used to make a choice between two statistical hypotheses. SPRT is described in detail in literature [8], [9], [10], thus it is not reiterated here.
There are three key procedures in MSET process, selecting the training data, choosing some featured data from the training data to create the memory matrix, which is the baseline of estimate calculation, then calculating the estimates.

Two prerequisites are required for the training data. First, the data should contain all of the healthy operational states of the monitored system. Second, the data should not contain any operating anomalies, sensor failures or equipment failures that would be considered unhealthy operations of the system [5].

After the training data have been selected, the MSET will select some special states from the training data to create memory matrix D. First, the states that contain the minimum or maximum value of each parameter are selected because they contain the extreme features of the system. Then, the algorithm continues to select additional states from the remaining data in the following two steps. First, the algorithm orders the left states based on their Euclidean norms. The order from minimum or maximum is unlimited. Second, the algorithm selects the additional states with equally spaced intervals from the ordered states until the amount in column D reaches the user-specified number of states. Those non-selected states in the training data form a new matrix called remaining training data L.

The estimate of the actual observation, defined as $X_{est}$, is an n-element vector that is thought to be the weighted (linear) combination of states in memory matrix D [12]:

$$X_{est} = D \bullet W$$  \hspace{1cm} (5)

$W$ is a weight vector that decides the contribution of each state in memory matrix D for the calculation of the estimate. The formula for this vector is derived from the least square method by minimizing the error vector

$$\epsilon = X_{obs} - X_{est}$$  \hspace{1cm} (6)

when the error is minimized, the $W$ is

$$W = (D^T \otimes D)^{-1} \bullet (D^T \otimes X_{obs})$$  \hspace{1cm} (7)

So the estimate for the observation $X_{obs}$ is

$$X_{est} = D \bullet (D^T \otimes D)^{-1} \bullet (D^T \otimes X_{obs})$$  \hspace{1cm} (8)

where, $\otimes$ is a nonlinear operator. Its properties have been described in detail in literature [5].

**RUL Prediction Model Using MSET**

The residuals between the actual data and their expected estimate values, based on the healthy historic data, are monitored by MSET algorithms. These residuals show the current state of the product, and more importantly, have some relationship with the degradation of the product [7]. If this relationship is figured out, and the criterion of the failure of the product is obtained, predicting the RUL of the product is feasible. The combination of MSET with databases of historic data and PoF analysis can potentially create effective degradation and RUL prediction model, as shown as Figure 3.

![Figure 3 Process of RUL Prediction Model](image)

This model has two procedures. First, the MSET is used to calculate the residuals of actual data and the degradation model is used to calculate the actual degradations, then, the trends of degradation are regressed into the future. Second, the criteria of failure is created, which is the degradation when the product fails. In this procedure, the actual product will be analyzed to find the PoF; then, the MSET residuals of historic failed products, which are similar to the actual monitored product and have a similar PoF, are selected to calculate the historic degradation. Because the situation of these historic products is known, the degradation when these products failed will be identified. This can be considered the criteria of failure for the prediction of RUL of the monitored product. Based on the degradation regression and the criteria of failure, the RUL of the monitored product can be predicted. The key steps in this process are to create the degradation model and the criteria of failure.
Degradation model

The residual between the actual data and its expected value when the product works normally indicates its degradation. If the degradation is an accumulated process, the accumulated degradation at time \( t_k \) can be modeled as the sum of all the Euclidean norms of residual vectors from the beginning to time \( t_k \):

\[
\text{Accumulated Degradation} = Ac\_De(t_k) = \sum_{i=1}^{k} ||R(t_i)||
\]

(9)

where, \( R(t_i) \) is a vector consisting of the residuals of all parameters at time \( t_i \):

\[
R(t_i) = [r_{i1}, r_{i2}, \ldots, r_{in}]^T
\]

where, \( r_{ij} \) (\( j=1,2,\ldots,n \)) is the residual between the actual data and the estimate of parameter \( j \) at time \( t_i \). The Euclidean norm of the residual vector \( R(t_i) \) is:

\[
||R(t_i)|| = \sqrt{r_{i1}^2 + r_{i2}^2 + \cdots + r_{in}^2}
\]

(10)

The accumulated degradation is strongly affected by the amount of the sample points for the addition. In order to remove this effect, the degradation is modified as:

\[
\text{Degradation} = De(t_k) = \frac{1}{k} \sum_{i=1}^{k} ||R(t_i)||
\]

(11)

Criteria of failure

The criteria of failure are defined as the degradation at which the monitored product fails. This degradation can be calculated from the historic failed products. Here, the assumption is that the same products will fail at the same degradations if the PoF of them are similar. After analyzing the actual monitored product, the PoF of the system is identified. MSET residuals of the historic failed products that have the similar situations and PoF of the actual monitored product are selected. Also, the degradations of these failed products are calculated by formulas (10) and (11). From these historic degradations, the estimated degradation when the actual product failed can be figured out by statistical methods.

The criteria of failure can also be created from accelerated tests if the historic data of failed products are not available. The accelerated tests allow the proper test stresses (e.g., temperature, relative humidity, temperature cycling) and the levels of these stresses to be selected so as to cause wear-out failure in the shortest time without changing the failure mechanism. The assumption here is that though the time of failure is shorter, the degradation when the system fails during the accelerated tests is unchanged. Statistical methods can be used to find this degradation. During the accelerated tests, residuals are calculated by MSET, and the degradations are computed by formulas (10) and (11).

Case Study

In this case, models were used to predict the RUL of an electrical component that presented its state using three parameters. The same six components were used to conduct the same accelerated test to create the criteria of the failure. In 27 days, five components failed. The same memory matrix D was selected by MSET to calculate the residuals for these six components. Then their degradations were calculated by formulas (10) and (11). The degradation plots are shown in Figure 4. The failure point is marked by a black dot, which identifies the degradation when the component failed. The degradations and the corresponding failure times of the five failed components are shown in table 1. The mean and standard deviation of these degradations were computed. In this paper, the mean \( \pm \sigma \) of the degradations of the failed components was chosen as the criteria of the failure for RUL prediction for actual tests.

The 7th component, called the test component, was used normally. Assuming this component has the same PoF as the former 5 failed components. When the residuals were calculated by MSET, the degradations were computed by the degradation model. Then, the degradation was regressed to show the future trend, as shown in Figure 5. In this paper, linear regression was selected to make it easy to figure out equations though it is possible to obtain more accurate trends using more complex regression models. The prediction of the failure time of the test component was calculated when the regressed degradation broke the failure criteria. In Figure 6, the criteria of failure is in \([1.43, 2.37]\) -- the interval of Mean \( \pm \sigma \) of the degradations of the former 5 failed components. The predicted failure time of the test component was the interval \([154, 268]\) days.

### Table 1 Degradation When the Components Failed

<table>
<thead>
<tr>
<th>Test</th>
<th>Failure time (day)</th>
<th>Degradation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>22</td>
<td>1.46</td>
</tr>
<tr>
<td>Test 2</td>
<td>22</td>
<td>1.63</td>
</tr>
<tr>
<td>Test 3</td>
<td>11</td>
<td>1.94</td>
</tr>
<tr>
<td>Test 4</td>
<td>23</td>
<td>2.68</td>
</tr>
<tr>
<td>Test 5</td>
<td>10</td>
<td>1.77</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>1.9</td>
</tr>
<tr>
<td>Stdev (( \sigma ))</td>
<td></td>
<td>0.47</td>
</tr>
</tbody>
</table>
Figure 4 Degradation Plots of Components Conducted During Accelerated Tests (Black Dots Represent the Components failed at Those Points)

Figure 5 Regression of the Degradation of the Test Component

Figure 6 RUL Prediction of the Test Product
Challenges in RUL prediction of electronic products using MSET

Although, it is possible to use MSET in RUL prediction for electronic products, there are still challenges. First, electronic products have characteristics that make RUL prediction more difficult. For example, the components of electronics are usually so small that there are no sensors to monitor them; and the fault modes for electronics are multiple, which makes it difficult to find the fault sites, fault reasons and the physics of failure.

Second, the RUL model presented in this paper assumes that the PoF of the historic failed product is the same or similar to the actual monitored product. Otherwise, the criteria of failure obtained from historic data or accelerated tests are not suitable for the actual product. The analysis of the potential PoF of the actual product is not easy.

Third, the main function of MSET is calculating residuals, which are the most important values in the MSET RUL prediction model. The accuracy of residuals affects RUL prediction directly. However, there are still some challenges in calculating residuals.

There are two prerequisites for MSET. One is that the training data should include all of the normal operational states. But it is difficult if there is not sufficient historic data. Furthermore, the states of a monitored product are changing, and it is possible that some normal operation states exceed the range used to train the MSET model. In this situation, MSET will give a false alarm. MSET should identify this new normal condition and add it to the training data. Some researchers are focused on resolving this problem. Newer techniques, which incorporate expert system rules to determine if the product is faulted or only in a normal operation condition, have been devised by SmartSignal Corporation and are reported by Wegerich [13].

The other prerequisite is that training data should not contain any abnormal states. In some real applications, it is unknown whether abnormal states are included when the training data is selected. These abnormal states, used as the baseline to evaluate real operational states, will lead to missed alarms and false alarms.

Additionally, the proper selection of a memory matrix influences the performance of the model. Too few data on the memory matrix starves the model of information and causes poor performance. Too many vectors in the memory matrix result in slow recall performance and poor predictive performance. There is a trade-off between the predictive accuracy and the consumption of time. No common methods can determine how many vectors should be selected for the memory matrix. The principle of the vectors selection in D is that the number of them is minimized while their data span the full dynamic range of the monitored product. A general rule of thumb for the selection of the number of vectors is that there should be at least two times the number of data sources [6].

System changes of the product, such as hardware replacement and maintenance should be conducted during operations. The MSET model must be retrained. According to the specific changes, some new vectors should be added in the memory matrix, while others should be deleted. Retraining may be implemented by an operator. If the operator neglects to do it, MSET system will detect it and alert the operator to retrain the model. Currently, MSET can not do this automatically.

Another challenge has to do with the monitoring sensor. MSET depends on the output of monitoring sensors. When sensors do not work normally, their output may become abnormal causing MSET to confuse sensor abnormal signals with product abnormal to false alarms. SmartSignal Corporation [13] provides strategies to solve this problem. After detection and identification of a failed sensor, several methods are used to continue the operation: one is to use the existing process model but set the faulted sensor signals to zero; another is to regenerate a model that inferentially estimates what the failed sensor signals should be, based on the remaining operational sensors. But the question here is how to identify whether the degradation or fault is from a sensor or from the product. The signal features of the faulty sensors may be determined by experiments. One possible method is to use a working sensor and a faulty one to measure a fine parameter at the same time, compare the results, find the difference, figure out the features of the faulty sensor’s signals, and repeat the experiment on a faulty parameter. Finally, a judgment method that identifies faulty sensors should be provided.

The last challenge is the difficulty in figuring out the relationship between MSET residuals and the actual degradation of the monitored product. The model introduced in this paper is based on one of the possible relationships and may be suitable for specific situations. It needs more simulation and experiments to identify a proper model in actual application.

Conclusions

The MSET method can be used to monitor the current state of a product and provide the necessary information required to make a remaining useful life (RUL) prediction. Based on the residuals between the actual values and the expected estimates of the parameters calculated by MSET, the degradations of the product can be modeled as formula 11 and the trends of these degradations can be regressed by proper models (linear regression is used in this paper). The RUL can be predicted when the trended degradation reaches the failure criteria of the product. The failure criteria are the degradations when the product fails. They can be estimated by statistical methods from the same failed products, when they have the similar PoF with the actual
product, from history or accelerated tests. The assumption here is that the same products will fail at the same degradation values if they have similar PoFs. The case study in this paper shows the whole procedures of RUL predictions. The regression model of degradation trends may be more complex and accurate, based on the specific applications. Also, the degradation may have models other than the one in this paper.

There are challenges in the MSET type of prognostics approach. These are caused mainly by the complexity of electronic products, the need for good PoF models, and the factors used to calculate the MSET residuals. In order to overcome these challenges, a new generation of sensors, such as wireless and miniature sensors, is needed. Research of the PoF of products needs to be much deeper to figure out the actual relationship between the degradation of products and the residuals calculated from MSET. The effort to obtain proper factors for MSET, such as memory matrix D, should be continued to calculate more accurate residuals.

References


