Diagrammatical Aspects of Qualitative Representations of Space


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Abstract
In this paper we demonstrate the usefulness of the diagrammatical aspects of a qualitative representation of positions in 2-D space [6]. Qualitative representations make only as many distinctions as necessary to identify objects, events, situations, etc. in a given context (identification task) as opposed to those needed to fully reconstruct a situation (reconstruction task). While the distinctions made are expressed propositionally in form of relations, we use data structures that analogically reflect the structure of the relational domain on a higher level of abstraction. This representation allows to perform operations such as a change in point of view or the composition of relations efficiently.

As a result of the extended “imagery” debate in cognitive science [8; 13] approaches to the representation of spatial knowledge tend to fall into one of the categories “propositional” or “pictorial”. There are several problems with these two extreme positions:
The propositional approaches focus primarily on formal properties of the representation such as soundness and completeness [14]. While doing so, however, they are forced to explicitly express the rich structural properties of space using propositions. Thus, basic structural properties of physical space such as:

- homogeneity and continuity of physical space
- objects have only positive extension
- two objects cannot fill the same space at the same time
- each object exists only once (identity)

must be stated explicitly leading to an enormous computational overhead even for moderately realistic applications.

On the other hand, current pictorial representations [4; 7], while preserving many of the spatial properties mentioned above, model some aspects of space on too low a level. For example, by defining a fixed grid of a given granularity (each field of which can be either filled or not) to represent occupancy, an arbitrary discretization of the domain is introduced. This may lead to wrong results, because binary decisions are forced at an inadequate stage of processing.

However, the desirable properties of both propositional and pictorial representations can be combined. This has been recognized by several authors [12; 2], who nevertheless propose hybrid models that combine them by interfacing them as separate representations. The qualitative representation of space we propose here integrates them by exploiting the generalization capabilities of abstract relational representations while inherently reflecting domain constraints such as the neighborhood of the positional relations.

This is possible, because pictorial representations are just a special case of the more general class of analogical representations [15], i.e. those, where important properties of the represented domain are intrinsically represented by the inherent properties of the representing domain. The structural similarity between the represented and the representing domains doesn’t need to be an isomorphism and can be given at an abstract level.

1 Qualitative representation of positions in 2-D space
We focus on 2-D projections of 3-D scenes. In order to represent the relative position of two objects in 2-D space qualitatively we define a small set of spatial relations from the two relevant position of the objects. The qualitative representation of space integrates them by exploiting the generalization capabilities of abstract relational representations while inherently reflecting domain constraints such as the neighborhood of the positional relations.

A complete set of topological relations can be derived from the combinatorial variations of the point set intersection of boundaries and interiors of the involved objects by imposing the constraints of physical space on them [3]. The projection relations are disjoint (d), tangent (t), overlaps (o), contains-at-border (c@b),

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1 The sense in which “explicit” and “implicit” are used in this paper is different from the one in the “Call for Participation” for this workshop. While the latter states for example that “The power of diagrammatic representations stems from the property that they allow the explicit representation and direct retrieval of information that can be represented only implicitly in other types of representations and then to be computed, sometimes at great cost, to make it explicit for use.”, our use refers to the way the represented and the representing domains are related. Thus, while the structural properties of space must be explicitly stated in a propositional representation, they are implicitly given by the corresponding properties of the pictorial representations.
included-at-border (i¢b), contains (c), included (i), and equal (=). Figure 1 shows the structure of the projection dimension and some of the properties of the relations involved. The relations \{d, t, o, =\} are symmetric. The containment relations \{c¢b, c, i¢b, i\} are pairwise converses, e.g. A [c] B iff B [i] A. The relations \{d, t, o, c¢b, i¢b\} are oriented, i.e. if two objects are related by one of these projections it is possible to establish their relative orientations as well. This is not the case for \{c, i, =\}, which preclude orientation. Finally the containment and equality relations make a statement about relative size and shape, e.g. only a a larger object can contain a smaller one, and only objects of the same size and shape can be equal.

The structure of the relational domain results from the structure of physical space. Two relations are directly linked by an arc in the figure (neighbors) if the corresponding physical situations can result from each other without an intervening situation that could be described by a third relation of the same dimension. Thus, disjoint and tangent are neighboring relations, because two disjoint objects can come together and touch each other (tangency) without an intervening situation (distance is not a topological concept and cannot be determined qualitatively, i.e. by comparing just the two objects involved). Overlaps and contains are not neighbors, because, if one object can contain another one (in the projection) and they overlap, there is a situation in the change from overlaps to contains in which one object contains-at-border the other one. Neighboring relations behave similarly, which allows us to define hierarchically organized levels of granularity by considering groups of neighboring relations as a single “coarse” relation.

The orientation dimension results from the transfer of distinguished reference axes from an observer to the reference object. The orientation relations are (at their finest level) front (f), back (b), left (l), right (r), left-back (lb), right-back (rb), left-front (lf), and right-front (rf). As figure 2 shows, they form a uni-

Figure 1: Structure of the projection dimension

Figure 2: Structure of the orientation dimension

form circular neighboring structure. Relative orientations must be given w.r.t. a reference frame, which can be intrinsic (orientation given by some inherent property of the reference object), extrinsic (orientation imposed by external factors), or deictic (orientation imposed by point of view). When reasoning about orientations, the reference frame is implicitly assumed to be the intrinsic orientation of the parent object (i.e. the one containing the objects involved), unless explicitly stated otherwise. Converting among reference frames can be done diagrammatically as will be shown below.

These “structured relational domains” are related to the concept of “quantity spaces” introduced by Hayes. A quantity space is a small, discrete set of values a physical variable may take and is usually totally ordered. Our structured domains are also small, discrete sets of spatial relations between two objects, but have a much richer structure than just total order.

2 Reasoning with abstract maps: diagrammatical aspects

While the long term storage of spatial information is done propositionally, the spatial reasoning process (envisionment) is done diagrammatically through the use of an interim internal representation that inherently reflects the properties of the represented domain. These “abstract maps” contain for each object in a scene a data structure called “rpon” (for relative projection and orientation node), which has the same neighborhood structure of the projection and orientation relations. Figure 3 shows a simplified visualization in which the containment and equality projection relations have been merged in a single node. As in figure 1 arcs between nodes denote neighboring projection/orientation pairs. A spatial relation between two objects is modeled by creating a bidirectional pointer from the projection slot of the given (implicit) orientation of the reference object to the corresponding slot of the “inverse relation” in the primary object.

A change in point of view, which affects relations with an explicit deictic type of reference frame, can be easily accomplished diagrammatically by “rotating” the labels of the orientation with respect to the intrinsic one. In
Figure 3: Relative Projection and Orientation Node

Figure 4: Original point of view

Figure 5: Rotation

An essential operation in every relational representation is the composition of relations: Given the relation between A and B, on the one hand, and between B and C, on the other, we want to know the relation between A and C. For simplicity we consider here only the special case of composition of orientations for disjoint objects, which can be nicely computed diagrammatically as follows: Given A \([x, z]\) B and B \([y, z]\) C, the composition A \([z, C]\) contains all orientations that are “in-between” and including z and y (see Fig. 2) on the “shortest path”. The composition is symmetric, i.e. \(t(x, y) = t(y, x)\). The following examples illustrate this rule:

1. If \(x = y\) then \(t = x = y\), since there are no orientations in-between (“distance” between orientations = 0).
2. If A is to the back of B and B is to the left of C, A can be to the back, left-back or left of C as the three square configurations on the right side of figure 6 show.
3. For \(x = lb\) and \(y = r\) we obtain \(t = \{lb, b, rb, r\}\) (and not \(\{lb, l, rf, f, rf, r\}\) which are the orientations between x and y on the longer path “the other way around”). Here, \(0 < \text{distance} < 4\).
4. If x and y are opposites (e.g. b/f, l/r, lb/rf) then \(t = \{?\}\), that is any of the orientations apply, since both paths between opposites have the same length (distance = 4).

While the table-lookup method is certainly more efficient for single compositions, it requires \(n \times m\) lookups and a set union operation for the composition of underdetermined relations consisting of disjunctions of \(n\) and \(m\) primitive relations each. Given that all resulting compositions as well as most typical complex relations are connected (neighbors), the diagrammatical method does all the required operations by virtue of its built-in structure.

Spatial reasoning, both changing the point of view and computing the composition of relations, can be done at coarser or finer levels depending on the kind of information available. In particular, if only coarse information is available, the reasoning process is less involved than if more details are known. This is a very intuitive aspect of our approach, which distinguishes it from other frameworks (e.g. using value ranges or confidence intervals) in which less information means more computation.

3 Conclusion

There is another sense in which qualitative representations relate to diagrammatical ones. In the narrow sense of sketch-like representations [4; 9] diagrammatical representations are qualitative in nature, since they

- make only as many commitments as necessary
are underdetermined (i.e. stand for a whole class of possible instances) and context dependent

allow reasoning at various levels of granularity (coarse vs. fine reasoning).

Thus in this paper we have argued, as Palmer [10] does, against the “typical claims [which are] that analog representations are quantitative while propositional representations are qualitative” both by showing the diagrammatic aspects of qualitative representations, and by hinting at the qualitative aspects of diagrammatical representations.

References


