Arrangement: An Aspect-Graph like Qualitative Relation for Medical Tomographic Images

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Abstract

In this paper we propose a qualitative spatial relation called "arrangement" which serves to identify medical images obtained from similar tomographic sections. "Arrangement" describes the sequence in which neighbors of each part are situated around it in the image. "Arrangement" is closely related to the Voronoi diagram of the parts. Based on the Voronoi diagram a metric for comparing "arrangements" is proposed. The metric enables robust use of "arrangement" in real-world situations.

The mechanism for comparing tomographic sections is useful in medical image databases.

1 Introduction

Medical tomographic images are formed by the intersection of the imaging plane and the imaged object. Figure 1 illustrates this. It shows a heart and imaging planes (la). The image in a plane contains the intersection of the plane with the heart chambers. It is common practice to image the heart in a number of planes oriented along the same direction and displaced along the normal. The entire set of images thus obtained are said to arise from the same "view." Any particular image in the set is said to have a particular "tomographic section" or simply a "section." The ability to compare two images and determine whether their sections are similar is an important part of any tomographic image database system that operates in a query-by-pictorial-example manner. When an example image is presented to such a database, the database is expected to retrieve images that have similar section. Since sections do not necessarily have distinct names, it is desirable that the similarity of section be inferred from the image content.

Techniques, such as aspect graphs, which are used to estimate the viewing angle of a camera image cannot be directly applied to tomographic images since they are based on the analysis of singularities of projection rather than an analysis of intersection. Singularities of intersection occur when the number of connected components of parts in the image changes (the tomographic section becomes tangential to any one part). These visual events are not strong enough to constrain the interpretation of section. This is particularly so for cardiac images.

A stronger constraint can be obtained by the considering the change in the embedding of the cardiac chambers within the tomographic image of a heart as the section is changed by a small amount. For small changes, the shape, size, and position of the chambers change. However, qualitatively speaking, the relative position of the chambers in the image is expected to remain the same, i.e., each chamber remains surrounded by the same neighbors in the same sequence. Further we may expect that similar sections of two different hearts also display this sequence. Specifically, if we denote the left ventricle, the right ventricle, the left atrium, and the right atrium by "lv," "rv," "la," and "ra" respectively, then in figure 1b lv has rv and la for neighbors and when viewed in a counter clockwise manner they occur around lv in the sequence rv la. We represent this fact as the string "lv: rv la" where the initial "lv:" denotes that the rest of the string are neighbors of lv in
a counter clockwise manner (any circular shift of the neighbor substring is considered equivalent). We call this string the "local arrangement" around lv.

The local arrangements around each of the chambers can be obtained similarly. The set of all local arrangement strings is defined to be the entire "arrangement" of chambers. In figure 1b the arrangement is:

\[
\begin{align*}
\text{lv: } & \text{rv la } \\
\text{rv: } & \text{ra la lv } \\
\text{ra: } & \text{la rv } \\
\text{la: } & \text{lv rv ra }
\end{align*}
\]

If we define two sections as being equivalent if both images have the same arrangement, then this partitions the set of sections into equivalence classes. We expect that for most organs there would be finite number of such equivalence classes and by comparing the arrangement in a given image with each of the equivalence classes we could determine the class it belongs to. This is the key intuition behind this paper.

2 Voronoi Diagrams

More formally, suppose we have \( N - 1 \) disjoint "parts," denoted by \( P_i, i = 1, \ldots, N - 1 \), embedded in a region of interest \( O \) (figure 2a). The parts are assumed to be either points or regions. The part \( P_N \) is defined as the closure of the complement of \( O \).

The Voronoi domain of a part is the set of points closer to that part than to any other part. The Voronoi diagram of the entire set of parts is the set of points equidistant from two or more parts[1][2]. It is well known that the Voronoi diagram is a plane graph [2] (see figure 2). It can also be shown that the only structurally stable configurations for Voronoi diagrams are those with every vertex having exactly three edges incident on it. In general a Voronoi diagram may have more than one connected component, but in the rest of this paper, it is assumed that the Voronoi diagram has a single component with at least one vertex (the main ideas of the paper can be easily extended to the multiple components).

The boundary of the Voronoi domain of \( P_i \) can be traversed by keeping its Voronoi domain to the left. In doing so, if \( P_{j_1}, P_{j_2}, \ldots, P_{j_n} \) is the sequence of neighbors that form the edges of the boundary, then the local arrangement around \( P_i \) is

\[
P_i : P_{j_1}, P_{j_2}, \ldots, P_{j_n},
\]

where any circular shift of \( P_{j_1}, P_{j_2}, \ldots, P_{j_n} \) is considered equivalent. Repeating the procedure for the entire set of parts we get the set of strings \( A(P_1, \ldots, P_N) \)

\[
A(P_1, \ldots, P_N) = \left( \begin{array}{c}
P_1 : \\
P_2 : \\
\vdots \\
P_N :
\end{array} \right),
\]

where, each string gives the local arrangement around a part.

The set \( A(P_1, \ldots, P_N) \) defines the arrangement of \( P_1, \ldots, P_N \). It can be shown that the arrangement of parts fully specifies the embedding of the Voronoi diagram in the image when the Voronoi diagram is considered as a graph[3]. We shall only be concerned with the graph structure of the Voronoi diagram.

Finally, two arrangements \( A_1(P_1, \ldots, P_N) \) and \( A_2(P_1, \ldots, P_N) \) are declared identical if and only if the sets \( A_1() \) and \( A_2() \) are equal (any circular shift of the substring following "\( P_i : " \) is considered equivalent).

3 The Diagonal Exchange

So far we have defined arrangements and conditions under which two arrangements are identical. Since the arrangement of parts is just the specification of the embedding of the graph structure of the Voronoi diagram, it is possible to investigate the change in arrangement by investigating the change in the embedding of the graph structure of the Voronoi diagram.

An operation called the diagonal exchange can be repeatedly applied to a given Voronoi diagram to transform it to any other. It arises because the only structurally stable vertices of the Voronoi diagram have three edges incident on them. The diagonal exchange is illustrated in figure 3. Consider the edge \( e \) in figure 3a. If the four Voronoi domains shown in the figure arise from four distinct parts \( P_1, P_2, P_3, \) and \( P_4 \), the diagonal exchange is the transformation that changes the configuration of figure 3a to the configuration of figure 3b. Figure 3 also shows the change in the dual graph of the Voronoi diagram. Before the exchange, the dual edge of \( e \) forms the diagonal of the quadrilateral shown in the figure. After the exchange, this diagonal is replaced by the other diagonal of the quadrilateral. The replacement of the diagonal is the reason for calling this operation the diagonal exchange.

We do not present detailed properties of the diagonal exchange. They are available in [3]. The key property of the diagonal exchange for further development is: there always exists a sequence of diagonal exchanges that converts a Voronoi diagram to any other Voronoi diagram caused by (a possibly different) embedding of the same parts. The sequence of exchanges is not
unique. There are infinitely many sequences of diagonal exchanges that transform one Voronoi diagram into another [3].

If \( d(A_1, A_2) \) is the minimum number of diagonal exchanges required to transform the Voronoi diagram of arrangement \( A_1 \) to the diagram of arrangement \( A_2 \), then

1. \( d(A_1, A_2) \geq 0 \),

2. \( d(A_1, A_2) = d(A_2, A_1) \). This is because \( d(A_1, A_2) \) is the minimum number of diagonal exchanges and because diagonal exchanges are reversible.

3. \( d(A_1, A_2) = 0 \) iff the Voronoi diagrams of \( A_1 \) and \( A_2 \) are the same plane graph.

4. \( d(A_1, A_2) \leq d(A_1, A_3) + d(A_3, A_2) \), with equality iff \( A_3 \) occurs in the minimum exchange sequence from \( A_1 \) and \( A_2 \).

Therefore, \( d(A_1, A_2) \) is a metric.

If \( \{P_i\} \) and \( \{P_i^*\} \) are two different sets of parts that have some parts in common, the embeddings of \( \{P_i\} \) and \( \{P_i^*\} \) can be compared by computing the metric between arrangements of the parts that are common to both. Computational aspects of the arrangement metric can be found in [3].

4 Experiments

As described in section 1, we expect two cardiac images whose arrangements are similar to have the same or similar sections. This is the basis of a query-by-pictorial-example image database system that we are currently developing. In this database, as each image is entered, the outlines of the chambers are manually traced, labeled, and stored along with the image. The Voronoi diagrams of the outlined parts are computed and saved along with the image.

At run time, the user selects an example image. Every image in the database is compared with the example image and the arrangement metric between the two is computed. Since the metric is integer valued, the resulting image is binned according to whether the numerical value of the metric is 0, 1, \( \geq 2 \). After all the images have been compared and binned, the bins are presented to the user in the sequence 0, 1, \( \geq 2 \). It is expected that images in the low metric bins have the same or similar section as the example image.

Figure 4a shows an example image with the outlined and labeled chambers and the region of interest. Figure 4b shows a typical retrieved image whose arrangement metric is 0 when compared to the example image, figure 4c shows an image with metric 1, and figure 4d shows a typical image with metric 2. Our results indicate that almost all images with metric 0 or 1 have similar sections to the example image, whereas most of the images with metric \( \geq 2 \) do not.

Besides subjective evaluations, two further experiments were conducted to statistically compare the performance of the method with that of 3 expert cardiac radiologists using a database in which 900 queries were made. We do not report the details of the experiments here for lack of space, the interested reader may refer to [3]. Briefly, these experiments validate the technique by showing that (1) most of the information regarding tomographic section is contained in the arrangement, i.e., masking out all of the information except the arrangement of chambers does not result in a significant loss of an expert’s ability to infer the tomographic section of the image, and (2) that the performance of the arrangements technique in retrieving similar tomographic sections is comparable to that of expert radiologists.

5 Conclusions

A qualitative spatial relation called “arrangement” that describes how different parts are embedded in an image is proposed. It is also proposed that two tomographic sections of an organ be considered equivalent if they have identical arrangement. It is further shown that a metric can be defined between arrangements by considering changes in the Voronoi diagram. Experiments demonstrating the utility of the metric in retrieving images with similar section from a cardiac MRI image database are reported.

The relation is similar to aspect-graphs in that it also (1) partitions the viewing range (i.e. the set of sections) into finite equivalence classes, and (2) it is based on considerations of spatial stability (of neighboring parts).

References


Fig. 1 Formation of a Tomographic Image

Fig. 2 The Voronoi Diagram

Fig. 3 The Diagonal Exchange

Fig 4. Retrieval