Abstract
In usual game theory, it is normally assumed that "all the players see the same game", i.e., they are aware of each other's strategies and preferences. This assumption is very strong for real life where differences in perception affecting the decision making process seem to be the rule rather than the exception. In this paper, we present a hypergame approach as an analyze tool that allows us to analyze such differences in perceptions. In particular, we explain how agents can interact through a third party when they have different views and particularly misperceptions on others' game. After that, we show how agents can take advantage of misperceptions. Finally, we conclude and present some future work.

Introduction
In classical game theory, it is normally assumed that "all the players see the same game", i.e., they are aware of each other's strategies and preferences. This assumption is very strong for real life where differences in perception affecting the decision made seem to be the rule rather than the exception. Attempts have been made to incorporate misperceptions of various types from at least as early as 1956 (Luce 1956). Perhaps the most notable theoretical development is the work of Harsanyi (Harsanyi 1968) on game with incomplete information played by "Bayesian" players, i.e., players having a subjective probability distribution over all the alternative possibilities.

Recently, uncertainty in game theory has been addressed under risk control by Wu and Soo (Wu 1999). In this context, Wu and his colleague have shown how the risk control can be carried out by a negotiation protocol using communication actions of asking guarantee and offering compensation via a trusted third party.

In this work, we have taken the same road but rather than introducing uncertainty into the model, we have considered that the players are trying to play "different games". This approach suggested by Bennett (Bennett 1977) allows for all types of differences of perception while still allowing the model to remain reasonably simple. In its first and simplest form, this approach takes as a structure not a single game, but a set of perceptual games, each expressing a particular player's perspective of the situation in question. Such a set of games was termed a hypergame.

A Brief Formal Introduction to Hypergames
We can specify completely a hypergame by the following elements:

- **Players**: They are the parties (individual agents, groups, coalitions, etc.) that may affect the multiagent situation that we want to study using the hypergame.
- **Strategies**: Each player may see a number of combinations of actions available to herself and to each of the other players. Notice that all players may not recognize the same actions as being available for each given player since they do not perceive the same actions as relevant.
- **Preferences**: For each player, her various strategies define a set of perceived outcomes. Usually, she prefers some outcomes to others and has some beliefs about other players' preferences.

**Definition 1** An *n-person hypergame* is a system consisting of the following:
1. a set $\mathcal{P}_n$ on $n$ players,
2. for each $p, q \in \mathcal{P}_n$, a non-empty finite set $S^q_p$ which reflects the set of strategies for player $p$ as perceived by player $q$.
3. for each $p, q \in \mathcal{P}_n$, an ordering relationship $O^q_p$, defined over the product space $S^1_p, \ldots, S^n_p$ and which reflects the $p$'s preference ordering, as perceived by $q$.

Thus, $S^q_p$ and $O^q_p$ express $q$'s perception of $p$'s options and aims. The set $S^1_p, \ldots, S^n_p$ makes up $q$'s strategy matrix and together with $\mathcal{P}_n$ and the ordering $O^1_p, \ldots, O^n_p$, reflect player $q$'s game $G^q$ within the hypergame $G$. Thus, an hypergame $G$ can be considered as a set of $n$ game, $G^1, \ldots, G^n$, one for each player. We assume that each player $i$ makes her strategy choice with full knowledge of her own game $G^i$. Obviously, a player may realize that others may perceive the situation differently: if so, she may have more or less an idea as to what games they are trying to play. Or she may see only her own game, which she assumes to represent her perception shared by all.

Having defined our hypergame, the final step is of course to analyze it using general principles, and hence to draw...
some conclusions about the multiagent situation one has to be modeled. To achieve that, we have introduced (in the complete version) some set of decision rules for the players. Such rules are based on the notion of a “dominant” strategy (Rasmussen 1989).

One could hope to define a uniquely rational course of action for each agent-player. Thus, if used in a normative way, the hypergame approach would thus provide a very definite prescription for the decision-maker to follow; if used descriptively—under an assumption that agents will act rationally—it would give a prediction of the outcome to be expected.

**Coordination with a Third Party**

Suppose that the two players p and q are two agents representing two companies, each desiring “not be aggressive about the other (in the sense of market)” but suspicious of the other. We can give a hypergame model of this situation by assuming that each player has a choice between a cooperative ( ) strategy and an aggressive one ( ). Player p, we suppose, places the four possible outcomes in the following order of decreasing preference: (1) ( ) Co-existence; (2) ( , ) Attack without q retaliating; (3) ( , ) Attack without q retaliating; (4) ( , ) Mutual aggression; (5) ( , ) Attack by q without reply.

In fact, these preferences are not correctly perceived by q. More precisely, q believes p to have the following preference order: (1) ( , ); (2) ( , ); (3) ( , ); (4) ( , ); (5) ( , ).

On the other hand, q has the same preferences as p and these preferences are also not correctly perceived by p which perceives them as q perceived those of p. This situation can be represented by the following 2-person hypergame.

<table>
<thead>
<tr>
<th>Agent p’s Game $G^p$</th>
<th>Agent q’s Game $G^q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S^p_p$</td>
<td>$S^q_p$</td>
</tr>
<tr>
<td>4,3</td>
<td>3,1</td>
</tr>
<tr>
<td>1,4</td>
<td>2,2</td>
</tr>
</tbody>
</table>

Considering the situation from p’s point of view by looking at the game $G^p$. Since p does not have a dominant strategy it cannot apply rule 1 introduced in the previous section. However, it can use rule 2 since q has a dominant strategy which is . In these conditions, p assumes that q will adopt this aggressive strategy and consequently he is faced with outcomes ( , ) and ( , ). According to rule 2, it chooses to be aggressive also, that is, it chooses ( , ) which seems to be for her a Nash equilibrium. q reasons similarly on $G^q$.

With classical game, we cannot see the players’ differing perceptions and consequently we cannot understand exactly why players deviate from cooperation. In fact, if each player had not mistaken each other’s preferences, both would converge on the cooperation option.

Now suppose that p and q want to verify their misperceptions by consulting a third party. As external observer views the “exact” perceptions of p and q represented by the following matrix:

\[ \begin{array}{c|c|c}
S^p_p & S^q_p & S^p_q \\
\hline
4,4 & 3,1 & 4,4 \\
1,3 & 2,2 & 1,3 \\
\end{array} \]

In the case where p or q is trusted by p and not by q, the matrices reflecting p’s perception and q’s perception might be the following:

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Thus, p’s perception is the right perception and it is identical to perception. Looking at the situation from p’s point of view, it will be seen that neither p nor q have a dominant strategy and as the game is considered by p as non-conflict game, this player applies rule 3 and chooses ( , ) which is Nash equilibrium which dominates ( , ). Looking now at the situation from q’s point of view, it will be seen that this player has not been convinced by and consequently she maintains her misperception on p. His reasoning is: p has a dominant strategy and she must act on the assumption that p will adopt this strategy (according to rule 2). In this situation, q is faced with two choices ( , ) and ( , ). As she is rational, she will opt for ( , ). From ’s point of view, p and q have opted for ( , ), that is that p will cooperate and q attack. This is a very bad choice for p.

To avoid this problem, we suggest the following rule:

**Rule 4:** An agent accepts to revise her perceptions on another agent on the basis of what a third party suggests if she is ensured by that q has the correct perception of her.

According to this rule, if q does not trust and she does not want change his misperception of p, informs p and this latter persists with her former preferences. In this case, p and q will opt for ( , ) as explained previously. We suppose here for simplicity that all players are sincere and in this case, we exclude the case where for instance q might say to he will change her misperception of p and she does not do it. As we see this case reflects some reality where agents can be insincere, a case which seems to be very difficult to deal with and which needs further work.

In the case where p and q both trust, their respective perceptions are the following:

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Now each agent supposes she is in cooperative-game and applies rule 3 that leads her to the dominant strategy ( , ), a new equilibrium which dominates ( , ).

We have assumed here that the third party have convinced p and q on p’s order of preference ( , ), ( , ), ( , ) and ( , ). If he have assumed that have convinced p and q on q’s order of preference, i.e., ( , ), ( , ), ( , ) and ( , ), we obtain as final perceptions of p and
q (after they have correct their misperception by trusting ):

\[
\begin{array}{c|c|c}
\text{Agent } p \text{'s Game } G^p & \text{Agent } q \text{'s Game } G^q \\
\hline
S^p_p & S^q_p & 3,3 & 1,4 \\
S^p_q & S^q_q & 4,1 & 2,2 \\
\end{array}
\]

Game now turns out to be the famous “Prisoner’s Dilemma” (DP) for which the dominant strategy equilibrium is ( , ). which is worse than the strategy ( , ). To force p and q to adopt both the strategy ( , ), we add a new rule.

Rule 5: If two players and agree to choose an outcome under the supervision of a third party , then as soon as one of them deviates from this outcome, informs the other.

If our players p and q follow this rule, they adopt the dominant strategy “forced equilibrium” ( , ) since they know if one of them deviates from this “forced equilibrium”, the other knows it (informed by ) and both switch to ( , ).

Our rule 5 reduces in fact the DP matrix to only two outcomes ( , ) and ( , ) and where the first one dominates the second one. In this case, choices of players p and q are facilitated.

Thus, the DP usually used to model many different situations, including oligopoly pricing, auction bidding, political bargaining, etc. does give a rationale for some behaviors. But without an hypergame representation, the essential element of the story -misunderstanding- is left out.

Gaining Advantage from Differences

Suppose a 2-player hypergame for which p perceives two options and which are not available for q. In p’s point of view, option is an option for p and is an option for q.

\[
\begin{array}{c|c|c|c|c}
\text{p’s Game } G^p & \text{q’s Game } G^q \\
\hline
S^p_p & S^q_p & 1,3 & 2,3 \\
S^p_q & S^q_q & 4,1 & 3,2 \\
\end{array}
\]

From q’s point of view, it can be seen that q believes that p will play strategy and she will play (in order to obtain the stable outcome ( , ). The player p is far from this point of view since she perceives two additional strategies that q does not see. From her point of view, q has a dominant strategy which is ( ) and as she assumes that q is rational, she believes that q will opt for that strategy. Knowing that, p will opt for so that she gains the best payoff. We are faced with two points of views, according to p, the stable outcome is ( , ) whereas according to q, the stable outcome is ( , ).

Suppose now that p is curious and wants to know if q has or not the same perceptions. In this case, she could ask a third party which knows p and q for instance and this third party informed her that q has a limited view and she does not view options and . Knowing that, p might let q opting for with the intention to choose in order to obtain a more preferable outcome ( , ) than ( , ).

Notice that this case is similar to the case where q sees two options that p does not perceive and which can be represented by the following matrices.

\[
\begin{array}{c|c|c|c|c}
\text{p’s Game } G^p & \text{q’s Game } G^q \\
\hline
S^p_p & S^q_p & 1,3 & 2,3 \\
S^p_q & S^q_q & 4,1 & 3,2 \\
\end{array}
\]

Notice that the reasoning is similar for the following cases: (1) p (or q) perceives one option (or ) for her but which is not available for q (or p); (2) p (or q) perceives one option (or ) for the other agent but which is not available for herself; (3) etc.

Suppose now that the points of view p and q are the following:

\[
\begin{array}{c|c|c|c|c}
\text{p’s Game } G^p & \text{q’s Game } G^q \\
\hline
S^p_p & S^q_p & 1,3 & 2,3 & 2,3 \\
S^p_q & S^q_q & 4,1 & 3,2 & 4,3 \\
\end{array}
\]

In this case, q’s reasoning is the same as previously and she believes that stable outcome is ( ). p believes that q has a dominant strategy which is ( ) and consequently, she will opt for the outcome . However, as she is uncertain about what q perceives as outcomes, she communicates with her in order to tell her the different options that she perceives: , and . Once q is convinced, both agents perceive the same options and the same preferences and in this case, p and q opt for ( , ).

Future Work

There are many extensions to this work. Among these extensions, we see a lookahead-based exploration strategy for a model-based learning agent that enables exploration of the opponent’s behaviour during interaction in a multiagent system. By adopting such strategy, an agent might correct her misperceptions in our hypergame approach. Another extension consists in seeing how we can be selective about the nested perceptions that we use in the recursive hypergame in order to have a tractable approach.


References


