Modeling context-aware distributed knowledge

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Abstract

This paper presents a multi-agent model to support decision-making in organizations. The model is characterized by being interactive, distributed, and incremental and by the use of cognitive maps to represent the knowledge of decision-making actors. The main proposition is to consider the context of concepts belonging to cognitive maps in a way that it represents agent's mental states, allowing some kind of inference. To do so, context is conceptualized in cognitive maps, defining agent's mental states from concepts being causally related to their context.

I. Introduction

This paper proposes a multi-dimensional reasoning process in a multi-agent system. Software agents are used to support decision making in organizations, representing the knowledge of the actors that participate in the decision making process. In the model, cognitive maps are used as instruments to represent dispersed knowledge sources. This kind of cognitive model is used to compose a collective solution to a goal through a distributed and incremental process, based on agent's interactions. Rational relations between agent's mental states are mapped during agent's internal reasoning processes. Finally, the emergence of collective knowledge, where interactions give rise to some kind of organizational culture, is represented in the artificial agent's cognitive maps.

The main proposition of this paper is to consider the context of concepts belonging to cognitive maps in a way that they can represent mental states, allowing some kind of inference. To do that, the philosophical positioning of functionalism is here assumed, aiming to model relationships (some kind of "functional roles") between agent's mental states.

Philosophy and artificial intelligence both try to understand, in a physical world, all kinds of perception, action, intelligence and consciousness phenomena. In particular, artificial intelligence is a domain where mental experiments have been conducted with a main goal: starting from a given conception of what can be the mind, controlled mental experiments simulate reasoning and they use it in software programs. Their main advantage remains on the possibility of refutation during the experimentation process. This approach is different from the one generally followed in philosophy,

where a given reality is studied - in this case there is no artificial model to experiment. Artificial intelligence is inspired by a functionalist conception of the mind, characterized by the recognition, by a physical system, of a given functional organization [Block,95]. If this functional idea of the mind is fair, then experiments conducted by artificial intelligence can help us to understand the knowledge representation systems used in our minds. On other side, artificial intelligence can improve upon the understanding of folk psychological constructs such as mental states. Actually, the notions of intelligence and mental states are strongly connected. Intelligence can be seen as the capacity of problem solving and decision. Mental states are representations of the world. There is no problem solving and decision capacity without some representations of the world. These representations can be, in folk psychology terms, intentional states such as intentions, beliefs and desires. According to this point of view, artificial intelligence can be seen as a laboratory where new software architectures, starting from a specified idea of what is the mind, are conceived and tested [Miguens,99]. Therefore, artificial intelligence experiments can be seen as a way of really doing philosophy, because they search the conditions that make possible cognition in general.

I believe that this position is according to the computational ideas that support multi-agent systems. In a multi-agent system we consider the hardware as the brain of an artificial agent, the software as its mind, and following this kind of parallel, intentional agent's mental states can be defined by the roles they play in the system. A software agent is characterized by its autonomy regarding the user, by its proactiveness - it acts to achieve its goals - and by its intentionality [Kampis, 99]. To represent agent's intentionality, I make use of some mentalistic notions found in folk psychology, such as beliefs and desires, as they are described for human behavior. Folk psychology allows us to make conclusions from mental states using assumptions. An intentional agent has beliefs, desires and, in a generic way, different kinds of mental states. The folk understanding of mental states has been presented as a theory of mind with an interesting operational content, to study the role of our own mental states in our behavior [SEP,02]. According to this approach, the understanding of internal mental states, and their internal cognitive mechanisms gives individuals the

capacity to predict and explain their own behavior. Folk Psychology permits also the manipulation of knowledge some kind of data structures representing mental states, which mediates between our observation and our predictions or explanations. Actually this point of view acts as a functionalist theory, identifying mental states in terms of their causal-functional relations.

In a recent research [Louçã,00 & 02a] I have proposed an inter-disciplinary approach concerning decision-making in human organizations, cognitive mapping and interaction between intelligent artificial agents. Multi-dimensional reasoning processes were modeled as multi-agent systems. I aimed to process automatically some mental faculties of individuals and groups. To do so, causal cognitive maps [Axelrod,76] were used as instruments to support collective reasoning. Those kinds of cognitive models were used to represent agent's mental states and to compose a collective solution to a goal through a distributed and incremental process, based on agent's interactions.

I propose now to map rational relations among an agent's mental states and to use this mapping to study the emergence of collective knowledge, where interactions give raise to some kind of organizational culture.

This document is organized as follows. The next section presents the functionalist idea of contextual mental states, from which is based this research. After that, the third section presents the domain of cognitive mapping. The fourth section concerns the main proposition of the article, e.g. to consider the context of concepts in a way that they can represent mental states, allowing some kind of inference. The document concludes with the discussion of some perspectives of research in the domain.

II. The functionalist idea of contextual mental states

The philosophical positioning assumed in here is that of functionalism. The philosophical doctrine of functionalism holds that ordinary people understand each common mental state descriptor to pick out a distinctive "functional role", i.e., a set of causal – functional relations as stimulus inputs, behavioral outputs, and other mental states [Goldman,93].

Two main subjects have been studied regarding functionalism: intentionality and consciousness. I will focus on the first one. To explain where intentionally come from, the base idea of functionalists stands that mental content is identified with causal-functional roles. In general terms, functional analysis decomposes mind in parts each time smaller until they are uniquely reactive. These components belong to a net of mechanical capacities, where the intelligence of the system is in the interaction of its

components [Block,95]. Several theories were proposed based on this model, as for instance the Functional Role Theories of Content.

According to these theories, a network of inference relationships among states defines mental states. An example of this approach is the Global Workspace Theory [Baars, 88], where mental states are defined in terms of their functions. The main characteristic of the Global Workspace Theory is that a mental state is activated (e.g., it is a conscious representation) when its message is broadcasted to the whole system. Then, according to the nature of the mental state, some specific receptors will process the message. These receptors are working memories acting in parallel, composing a distributed control structure with their interactions [Franklin,01]. This picture of the mind, a collection of intercommunicating subsystems where reasoning is done through a set of messages posted in a large blackboard for all cognitive subsystems to read, it is far from being intuitive for most people [Goldman,93]. Another problem of this approach is its holistic dimension - the content of every belief depends on the content of every other belief in the blackboard.

To reduce this holistic dimension to some pertinent and applicable dimension – to distinguish mental states from the hole – the Causal Covariance Theory of Content [Allen,02] proposes that mental states get their content by being causally related only to what they are about (e.g., to those mental states belonging to its own specific context). This idea is in here adopted in a general way, to define and operate mental states. To understand this propositions let's previously present cognitive maps, descriptive cognitive structures used by psychologists to represent the decision making process in organizations.

III. Cognitive Mapping

From an epistemological point of view, the decision making process can be studied through an individual perspective, concerning *methodological individualism*, or through an organizational perspective, relating to *holism*. The analysis of a behavior or situation in its context is the holistic approach used in social sciences [Cossette,94]. Cognitive mapping facilitates the adoption of this kind of approach, allowing the representation of knowledge in a holistic perspective, taking into account individual knowledge of deciders in the context of collective decision making in organizations. A cognitive map is a graphical representation of the behavior of an individual or a group of individuals, concerning a particular domain. This kind of graphical representation can help the understanding of different points of view and can evidence conflicts between deciders [Axelrod,76].

A cognitive map is composed by *concepts* (representing things, attitudes, actions or ideas) and *links* between concepts.

Figure 1 exemplifies a cognitive map composed by three different kinds of concepts – *tasks*, *goals* and *states of the world*.

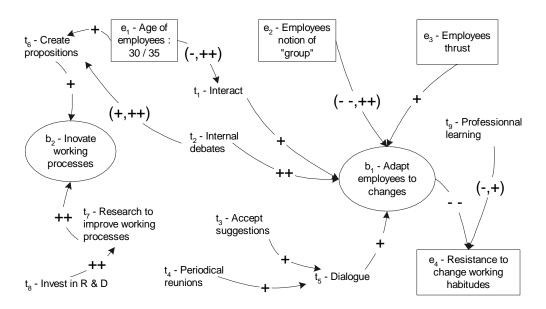


Figure 1: Example of a cognitive map [Louçã,00]

Those links can represent different kinds of connections between concepts, such as causality or influence.

The main interest of cognitive maps it's their reflexive character, allowing people to became conscious of their implicit knowledge, through the visualization of all direct and indirect links between concepts. We each construct our private versions of reality and deal only with those constructions, which may or may not correspond to some real world [Lissack & Ross,99]. Cognitive maps are used, mainly by psychologists, as data structures to represent knowledge and to make behavioral analysis in what concerns decisionmaking in organizations. According to Karl Weick and others, organizations can be seen, at some abstraction level, as systems of construction and interpretation of reality [Weick,95] [Lissack & Gunz,99]. Following this approach, cognitive maps can be employed at an individual level, to represent individual viewpoints, and at an institutional level through the use of collective cognitive maps. Generally, this kind of cognitive models facilitate the analysis of the graphically represented ideas and lines of thought, facilitating communication inside a group supporting discussion and negotiation between the elements of the group having different points of view. Cognitive maps can also be used to detect conflict situations between deciders.

This cognitive map implements a concept typology similar to the one proposed in [Carlsson & Walden,96], regarding knowledge representation in an organization. According to this typology, tasks are concepts that describe actions or attitudes (for example, to interact, to make reunions, to dialogue, to invest in R&D). Goals must be achieved by doing tasks (for instance, to achieve the innovation of working processes or to accomplish the adaptation of employees to changes). States of the world represent things or qualities of the environment. All those concepts, tasks, goals and states of the world, are connected by influence links. Influences can be very negative (--), negative (-), positive (+) or very positive (++). For instance, in the example above the link between tasks t8 - Invest in R&D and t7 – Research to improve working processes is (++), meaning that t8 has a very positive influence in t7. On another hand, t7 has a very positive influence on the achievement of the goal b2 - Inovate working processes. A particular case is represented by links representing simultaneously different qualities, as the case between the task t9 - Professional learning, connected to e4 - Resistance to change working habitudes by a (-,+) link. This kind of double links represent the existence of different opinions concerning the nature of the link. In this example, the cognitive map evidences two points of view: one standing for a negative influence between t9 and e4 and another saying that there is a positive influence between those two concepts.

Several methodologies in psychology are used to compose cognitive maps, including two main alternatives to extract and represent knowledge from individuals: the *phenomenological* and the *normative* ways. The first alternative considers the subjective dimension of behavior - it must be the individual by itself to compose its own cognitive map. The normative methodology stands for the use of observers, specialized in extracting concepts and links between concepts from written texts and oral interviews. These observers are normally psychologists [Cossette & Audet,94].

IV. Cognitive maps standing for mental states

In a previous research I have proposed a multi-agent model based on multi-dimensional reasoning processes [Louçã,00 & 02a]. In this proposition each artificial agent supports the decision of an individual participating in the collective decision-making process in the organization. Causal cognitive maps are used to represent knowledge of those deciders. Artificial agent's knowledge is used to compose a collective solution to a goal, through a distributed and incremental process based on agent's interactions. This distributed and incremental process is represented in Fig.2.

When an actor requests its agent to propose a solution to a goal, this one uses the set of concepts represented in its cognitive map to compose the solution. Throw a reasoning process, which is an extension of the *Negative-Positive-Neutral Logic* [Zhang,96], named *NPN*^e *Methodology* [Louçã,00], the agent become aware of the tasks, goals and states of the world that influence, directly or indirectly, the achievement of the goal. Then the agent composes, with those concepts, its individual solution to the goal, represented by a partial cognitive map.

The second step concerns the allocation of sub-goals (those goals that belong to the previous solution) to other agents in the system. As represented in Figure 2, each agent that receives an allocation message including a goal, starts its own reasoning process to the sub-goal and, in return, responds with a solution to the sub-goal. This distributed reasoning process allows representing several points of view concerning the sub-goals.

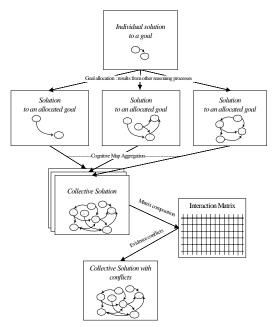


Figure 2: An interactive, distributed and incremental resolution process [Louçã,00]

Next, agents aggregate their partial solutions in a collective solution, throw the NPNe Methodology of Aggregating Cognitive Maps detailed in [Louçã,02a]. This is done, mainly, throw de composition of the interaction matrix, where are represented links between all concepts that belong to the different partial solutions. The matrix represents all links between to given concepts, including conflicting points of view. Then, according to the NPN Logic [Zhang,96], only the most acute opinions are considered to compose the collective solution. For instance, lets consider the concepts referred in the previous section, t9 - Professional learning and e4 – Resistance to change working habitudes. Supposing that, after goal allocation, the system obtains three different viewpoints: +, - and --. In this case the link between those two concepts would be (+,--), evidencing a conflict in the organization. This way, the collective reasoning mechanism will detect and evidence conflicts in a collective solution, graphically represented in the form of a cognitive map, allowing a clear discussion and negotiation, in the organization, about those conflicts.

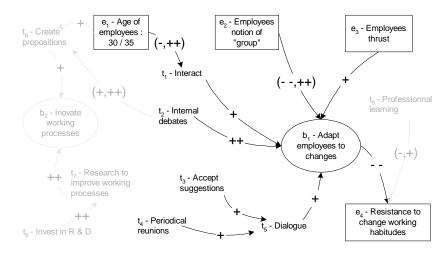


Figure 3: Example of a *scheme* included in a cognitive map [Louçã,02a]

An extension of the research previously described is the proposition to consider cognitive maps composed, on one hand, by concepts and by causal links between those concepts, in a strictu senso way [Weik,79], and on the other hand to consider the context from where we can take the assumptions allowing some kind of inference. The idea of context is fundamental to clarify the collective meaning of a concept. I stand for a pragmatic constructivist approach [Lissack,99] that allows us to understand context following a circular cognitive process that departs from several contextual hypotheses to, interacting with the user, arrive to contexts defining the agent mental states¹. This way, we can define mental states from cognitive maps by getting its content (e.g. sub-maps) from the concepts being causally related to their context. More precisely, a mental state is represented by a concept and its context. According to Krippendorf, meaning connects the features of an object and features of his context into a coherent unity [Krippendorf,89]. So, we can say that text unifies concepts in its mental states. In cognitive mapping terms we have that: (2) concepts can't be understood without context, (3) the context of a concept is composed by concepts that influence and that are influenced by the concept, and (3) each concept is coupled to its context, which can be called a *scheme* [Bougon & Komocar,94].

Given this notion of scheme as a concept and its context, and the notion of context as the linked concepts that influence and that are influenced by the named concept, the mapping of mental states can take the graphical form represented in Figure 4.

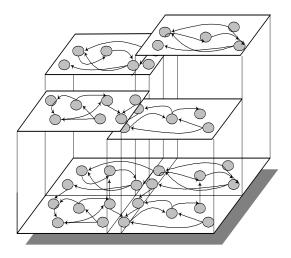


Figure 4: Mapping mental states that participate in the internal resolution process of an agent

During the internal resolution process of an artificial agent, those mental states that have some concept matching a goal allocated to the agent or matching a task to be achieved by the agent are activated and participate in the aggregation of the local solution. This is a relational perspective of the

¹ A pragmatic adoption of this approach is detailed in [Louçã,02a].

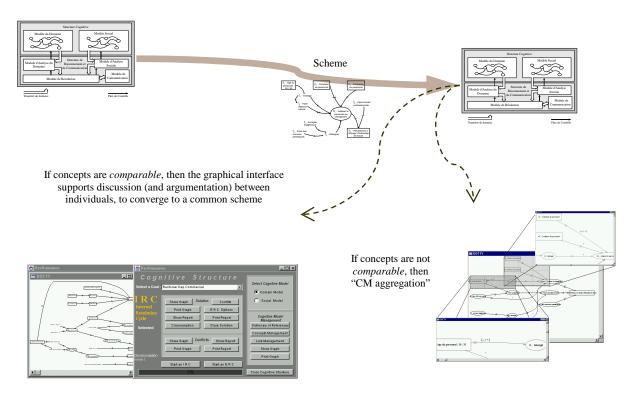


Figure 5: Convergence to a common scheme in STRAGENT

reasoning process, where the relevance of a mental state is a function of its relation with the other mental states of the system. The agent's local solution, in the form of a cognitive map, is like a higher-order mental state, representing the result of the conjunction of the activated mental states.

On another hand, agents interact by communicating schemes concerning a given situation. As a scheme represents the meaning of a given concept, communicating schemes influence agent's mental states. This influence represents the spreading of collective knowledge, characterized by schemes shared by the elements of the organization.

The mechanism that generates collective knowledge is as follows (see figure 5): when an agent communicates a scheme to another one, the receiver verifies if the concepts belonging to the scheme are comparable (e.g., if they are the same or similar) to the concepts belonging to its own cognitive map. If the concepts are *comparable*, it means that both agents have points of view about the subject of the scheme. These opinions can be alike or distinct (they can represent a conflict). In this last case, the graphical interface of the software system proposes a discussion between the users that correspond to the artificial agents.

The aim of this discussion is to converge to a common scheme. Finally both agents actualize their cognitive maps according to the result of the discussion, leading to the emergence of a collective point of view about the subject. On the other hand, if the concepts belonging to the communicated scheme are *not comparable* to those existing in the cognitive map of the receiver, this means that this agent don't have an opinion about the given subject. In this case the receiver improves its knowledge through the aggregation of the scheme to its own cognitive map. In conclusion, collective knowledge is mainly the result of a dynamic process of interaction (argumentation, negotiation) in the interior of the group. The quality of the results and the cohesion of a group depend largely on the quantity and quality of its collective knowledge [Langfield-Smith,92].

This approach has several advantages: (1) the expressability in verbal behavior of a plan of actions, (2) the transmission of the content of some mental state to some other mental state in the system, and (3) a higher-order state which reflects on the target state [Louçã,02b].

These propositions above were tested in STRAGENT, a distributed software system to support decision-making in human organizations (Figure 5). This prototype was applied

in an industrial enterprise in the domain of telecommunications and electronics, to support the collective decision making process. Cognitive maps were design from documents and interviews. The main goal of this application was to model collective discussions, to represent actor's knowledge, to support conflict resolution and to identify and understand the interactions between agent's mental states during the decision making process.

V. Related work

This research can be compared with other propositions about intentional systems and mental states concerning artificial agents, as well as bayesian and semantic networks concerning knowledge representation. Finally a reference is made to previous systems that have already contributed with some notions used in this research, such as the aggregation of cognitive maps or the use of an interaction matrix.

The notion of *intentional system* was originally proposed by Daniel Dennet, to whom those systems are defined by *beliefs* and *intentions* [Dennet,87]. The notion of intentional agent is largely used in multi-agent systems literature. Mental states such as *beliefs*, *desires* and *intentions* are studied by what is named the *BDI approach* [Rao & Georgeff,95], [Georgeff et al.,99], proposing and operational model of agents, founded on a logical formalism. The notion of artificial agent presented in this research is also driven by *beliefs* (concepts and links between those concepts) and *intentions* (schemes to attain goals) – e.g., agents have an intentional attitude. Nevertheless, this proposition has the advantage of representing mental states using cognitive maps, a non-formal tool *really* used in organizations and by psychologists to represent knowledge.

The use of cognitive maps to represent knowledge can be put side by side with an artificial intelligence approach the uses a graphical notation, named *semantic networks* [Sowa,91]. Such as cognitive maps, semantic networks represent knowledge through nodes connected by arcs. Nevertheless, in those networks, nodes are hierarchically typed, with derivation, according to the generality level of the nodes. Those systems are mainly used to classify or to group knowledge in natural language systems. On another side, cognitive mapping concerns less restraint notions, which do not need some particular typing - it's a general methodology, and one of its strengths is precisely its ability to be adapted to a large variety of domains.

The same argument can be used when comparing cognitive maps with another graphical knowledge representation: Bayesian networks. Actually, those two tools have already been associated to define the *qualitative probabilistic networks* [Wellman,94], a sort of cognitive mapping with causal probabilistic links, allowing bayesian reasoning in

cognitive maps. However, the use of the original version of cognitive maps has the advantage of simplicity – cognitive maps can represent a larger domain of situations, it's a tool used by psychologists and allows qualitative reasoning.

The POOL2 system, proposed by [Zhang et al.,92], composes collective maps through the aggregation of individual cognitive maps. This system is, as STRAGENT, based on the NPN Logic. POOL2 doesn't incorporate the notion of interaction between artificial agents. In A-POOL - Agent-Oriented Open System Shell [Zhang et al.,94] the same authors use cognitive maps to represent artificial agents knowledge. The communication is done through the exchange of partial cognitive maps and the purpose of interactions is to compose an organisational map. The most recent evolution of this system includes the propagation of numerical values [Zhang,96]. However, the use of quantitative inference is far from the qualitative spirit of cognitive mapping. In the line of thought of A-POOL, [Chaib-draa,98] proposes a method of causal reasoning adapted to multi-agent negotiation. Chaib-draa introduces the notion of interaction matrix to represent several points of view concerning the same subjects. Nevertheless, the conflict detection is not dynamic along interactions, it's done at a given moment – this model isn't adapted to artificial agents that dynamically and continuously adjust their knowledge to a changing environment.

VI. Conclusion

I propose a model of multi-dimensional reasoning in a multi-agent system. In this model, cognitive maps were used as instruments to represent agent's mental states. These cognitive maps are used to compose a collective solution to a goal through a distributed and incremental process, based on agent's interactions. Rational relations between agent's mental states are mapped during agent's internal reasoning processes. Finally, the emergence of collective knowledge, where interactions give rise to some kind of organizational culture, are represented in the cognitive maps of the artificial agents

References

Allen, C., 2002, *Lectures on the nature of mind*, http://www-phil.tamu.edu/~colin/Phil251/.

Axelrod, R., 1976, Structure of decision: the cognitive maps of political elites, Princeton University Press

Baars, B., 1988, A Cognitive Theory of Consciousness, Cambridge: Cambridge University Press.

- Block, N., 1995, *The Mind as the Software of the Brain*, in *An Invitation to Cognitive Science*, Osher, D. and Sternberg, S. eds., MIT Press.
- Bougon, M., and Komocar, J., 1994, Façonner et diriger la stratégie. Approche holistique et dynamique, in Cossette, P., ed., Cartes Cognitives et Organisations, pp. 57-81, Laval, Canada: Les presses de l'Université Laval.
- Carlsson, C., and Walden, P., 1996, Cognitive maps and hyperknowledge support system in strategic management, Group Decision and Negociation, n° 6, pp.7-36.
- Chaib-draa, B., 1998, A relational modeling of cognitive maps, in Proceedings of the Canadian Society for Computational Studies of Intelligence on Advances in Artificial Intelligence (AI-98), LNAI, Vol.1418, pp.322-333, Ed.Springer.
- Cossette, P., 1994, La carte cognitive idiosyncrasique. Etude exploratoire des schèmes personels de propriétaires-dirigeants de PME, in Cossette, P., ed., Cartes Cognitives et Organisations, pp. 113-115, Laval, Canada: Les Presses de l'Université Laval.
- Dennett, D., 1987, The intentional stance, MIT Press.
- Franklin, S., 2001, Conscious Software: A Computational View of Mind, in Soft Computing Agents: New Trends for Designing Autonomous Systems, Loia, V. and Sessa, S. eds., Berlin: Springer (Physica-Verlag), pp.1-46.
- Georgeff, M., Pell, B., Pollack, M., Tambe, M., Wooldridge, M., 1999, The belief-desire-intention model of agency, in Proceedings of Agents, Theories, Architectures and Languages ATAL'99.
- Goldman, A., 1993, Consciousness, Folk Psychology, and Cognitive Science, in Consciousness and Cognition, n°2, pp.364-382.
- Miguens, S., 1999, *Dennett e a IA* (in portuguese), in *Intelectu*, nº 9, Lisboa, Portugal.
- Kampis, G., 1999, *The Natural History of Agent*, in *Agents Everywhere*, Gulyás, L., Tatai, G. and Váncza, J. eds., Springer, Budapest, pp. 24-48.
- Krippendorf, K., 1989, On the Essential Contexts, of Artifacts or on the Proposition that Design Is Making Sense (of Things), in Design Issues, vol.5, n°2, pp.9-39.
- Langfield-Smith, K., 1992, Exploring the need for a shared cognitive map, in Journal of Management Studies, Vol.29, Num.3, pp.349-368.
- Lissack, M., 1999, Complexity: the Science, its Vocabulary, and its Relation to Organizations, in Emergence, Vol. 1, No. 1.
- Lissack, M. and Gunz, H., 1999, Managing Complexity in Organizations: A View in Many Directions Quorum Books,

- Westport.
- Lissack, M. and Ross, M., 1999, *The Next Common Sense*, London: Nicholas Brealey Publishers.
- Louçã, J., 2000, Cartographie Cognitive, Réflexion Stratégique et Interaction Distribuée : une Approche Multi-Agent (in french), PhD Thesis, Université Paris IX Dauphine and Faculdade de Ciências da Universidade de Lisboa.
- Louçã, J., 2002, Cognitive Mapping in a Multi-Agent System for Strategic Planning, in 12th Euro Conference on Decision Support Systems, Bruxels, Belgium.
- Louçã, J., 2002, Autonomous agents as systems of emerging consciousness, in Sixt Conference of the Association of the Scientific Study of Consciousness Consciousness and language: reportability and representation in humans and animals, Barcelone, Spain.
- Louçã, J., 2002, Modeling intentional agents: contextual cognitive maps standing for mental states, in Conference on Intentionality: Past and Future, Miskolc, Hungary.
- Rao, A., Georgeff, M., 1995, BDI agents: from theory to practice, ICMAS-95.
- Sowa, J.F. (Ed.), 1991, Principles of semantic networks-explorations in the representation of knowledge, Morgan Kaufmann Publishers.
- Stanford Encyclopedia of Philosophy, 2002, Folk Psychology as a Theory, http://plato.stanford.edu/entries/folkpsych-theory/, 2002.
- Wellman, M., 1994, Inference in cognitive maps, in Mathematics and Computers in Simulation, n°36, pp.137-148.
- Weick, K., 1979, The Social Psychology of Organizing, 2nd Edition, Reading, MA: Addison-Wesley.
- Weick, K., 1995, Sensemaking in Organizations, Thousand Oaks, CA: Sage Publications.
- Zhang, W., Chen, S., Wang, W., King, R., 1992, A cognitive-map-based approach to the coordination of distributed cooperative agents, in IEEE Transactions on Systems, Man, and Cybernetics, Vol.22, Num.1, January/February.
- Zhang, W., Wang, W., King, R., 1994, A-POOL: an agent-oriented open system shell for distributed decision process modelling, in Journal of organizational computing, Vol.4, Num.2, pp.127-154.
- Zhang, W., 1996, NPN fuzzy sets and NPN qualitative algebra: a computational framework for bipolar cognitive modeling and multiagent decision analisys, in IEEE Transactions on Systems, Man, and Cybernetics Part B: Cybernetics, Vol.26, Num.4.