How Representations and Strategies Influence Design Spatial Problem Solving

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Abstract

Spatial ability is used to predict success in certain domains, including design. However many ambiguities in spatial ability need to be clarified if education in design is to become more effective. Ambiguities in spatial ability include the overlapping of spatial sub-factors, strategy effects in spatial problem solving, and the ambiguity between 2D and 3D representations. Since it is hard to identify the sub-factors of spatial ability especially when design problems are much more complex than spatial aptitude tests, problem solving strategy might be a better way to approach what students learn that contributes to the improvement of their spatial ability.

Introduction

Spatial ability is a fundamental human ability and there is an enormous literature about it. Those studies demonstrated that spatial ability is not a single factor but has lots of subdivisions (Lohman 1988). Although spatial tests used with factor analysis have been shown to correlate strongly with success in many fields, including architecture and industrial design, this can only give us a general understanding that spatial ability is a critical ability for design students. Owing to the complexity of spatial ability, it is usually hard to discriminate the subdivisions. For example, the complexity of a *general visualization* test also relies on a general *reasoning* factor (Guilford and Lacey 1947, cited in Lohman 1988), which is not a subdivision of spatial ability at all; moreover, *spatial orientation* as a sub-factor is hard to separate from *general visualization* (Barratt 1953, cited in Lohman 1988). In addition, Cooper and Mumaw (1985) pointed out that this factor analysis approach might become problematic when the tasks become more complex, because the sub-factors and problem-solving strategies might be hard to identify and distinguish. Since spatial problems in design are usually much more complex than spatial tests, it is very hard to know how and when a particular subdivision of spatial ability is required. More fundamentally, will general reasoning ability, i.e. *reasoning factor*, be a better assessment for design achievement than *general visualization* sub-factor? In addition, studies indicated that spatial relations tests can be solved with other strategies than mental rotations (Lohman 1988). That is to say, do we really need "spatial ability" to become a good designer? Or can a good designer just rely on better problem solving strategies? In this paper, we review some important work in spatial problem solving, including the factor analysis approach, the ambiguity between 2D and 3D tasks, and processing strategy issues. At the end, we propose new research directions that will address the issues mentioned above.

Factor Analysis Approach

According to Lohman (1988), the sub-factors of spatial ability include: *general visualization, spatial orientation, flexibility of closure, closure speed, serial imagination, speeded rotation, spatial scanning, perceptual speed, visual memory*, and *kinesthetic*. These sub-factors can be summarized as following: *General visualization* is the most common sub-factor and it refers to the ability of doing several kinds of spatial transformations, such as rotation and reflection, mentally. Researchers usually use paper folding test and mental rotation test (Shepard and Metzler 1971) to define it. However, this sub-factor is usually hard to distinguish from *general reasoning* sub-factor, because test tasks inevitably involve additional reasoning and makes most tests also highly loaded on the *general reasoning* sub-factor. *Spatial orientation* usually requires the ability to imagine an object or scene from other perspectives. Perspective taking test (Piaget and Inhelder 1967) is the most classic example for this sub-factor. However, the difficulty of testing it is that participants might solve the test by rotating the object rather than reorienting themselves to the new location. If they solve the problem by rotating the object, then the test would become a *general visualization* test. The *flexibility of closure* sub-factor is usually tested by embedded figures that participants are asked to find inside a more complex image. However, when the test figure becomes too complex and then the *general visualization* sub-factors or

general reasoning sub-factor will inevitably become involved. The close-ups game is usually employed to test the *closure speed* sub-factor, in which participants are required to figure out as soon as possible what is the object, given various clues. The clues could be part of the object or a distorted image. However, again, it is hard to avoid participants using *spatial visualization* ability to solve the problem especially, when the picture becomes too complex. *Serial imagination* refers to the ability in integrating partial spatial images into a whole. However, this kind of test, such as successive perception, might also load on *closure speed* or even visual working memory. *Speeded rotation*, also known as *spatial relations*, refers to the ability to mentally rotate stimuli and then perform variance judgments. 2D and 3D mental rotation tests and cube comparison tests are examples that load on this sub-factor. However, depending on the complexity of the test, the *spatial visualization* might become the primary sub-factor for this kind of tests. The maze tracking task is one of the *spatial scanning* tests, in which participants usually scan possible paths quickly before they draw the correct one onto the paper. Lohman noticed that the *perceptual speed* sub-factor is also strongly correlated with it. The *perceptual speed* sub-factor usually uses the Identical Forms test as stimuli. Participants are asked to compare visual stimuli in a very short time. Although several sub-factor tests discussed previously, such as *speeded rotation* and *general visualization*, require the experimenter to control the complexity in order to avoid using *general reasoning* ability, over simplifying the task will instead load more on *perceptual speed* sub-factor than the desired ones. In terms of *visual memory* sub-factor, participants are usually asked to memorize figures and then recall or recognize them later. This type of task should avoid transformations otherwise it will then become hard to discriminate from other sub-factors. Lastly, the *kinesthetic* sub-factor refers to the ability that is able to differentiate right or left promptly. This ability is an essential part for certain tasks like Shepard and Metzler's spatial rotation tasks. Among all of the sub-factors, even though there is no general agreement on the subdivisions, *general visualization*, *spatial orientation* and *spatial relations* (also called *speeded rotation*) are the most well known and distinguishable subdivisions of spatial sub-factors (Lohman 1988; Carroll 1993).

From the short summary above, we can observe that complexity is a big issue because complex tasks usually load mainly on general reasoning ability rather than spatial sub-factors. This limitation can prevent us from studying design spatial ability via factor analysis approach, since design objects are usually much more complex than the stimuli mentioned above; and needless to mention there are various complex transformations applied in design that dramatically increase the complexity that designers need to handle mentally. Even though we agree that several spatial sub-factors, such as spatial orientation, speeded rotation and spatial visualization, can often be observed during the design processes, Lohman's argument about general reasoning ability strongly suggests that we reconsider the importance of general problem solving ability in design. Additionally, even though the spatial aptitude test is a kind of well-defined problem solving task and it usually has only one correct answer, there are obviously different ways for a person taking the test to approach its tasks. One example is that a perspective taking task could be solved either by spatial orientation approach or by speeded rotation approach. These issues encouraged us to investigate design spatial ability from other directions, like a spatial problem solving strategy perspective.

Design Problem

In the field of design, spatial ability is usually regarded as a crucial ability in order to succeed in the area. In addition, many studies have shown that spatial ability is a good predictor for success in the design area (Smith 1964; Ghiselli 1973, cited in Lohman 1988), For example, spatial ability predicts the performance in engineering drawing (Holliday 1943, cited in Lohman 1988). However, we also know that taking an engineering drawing course will improve students' spatial aptitude scores (Field 1999; Olkun 2003), but the research was not detailed enough to understand how student performance was affected. Additionally, we know that design spatial ability involves the ability to apply spatial transformations with different kind of 2D and 3D drawings, but we know little about how those skills can be developed. One of the biggest but implicit assumptions in architectural design education is that we are able to increase students' spatial ability through academic training (Ho, Eastman and Catrambone 2005). However, if this assumption turns out to be false, then we might be wasting students' time in college. Moreover, although studies suggest that engineering drawing can help improve spatial abilities, it is not clear whether the improvement comes from spatial reasoning ability development or just because the participants became more familiar with the graphical conventions. Perhaps people just become more familiar with the stimuli instead of really increasing their spatial ability.

In addition, studies have shown that spatial ability will be affected by using different problem solving strategies, such as analytic and constructive strategy (Cooper and Mumaw 1985), piecemeal strategy (Hegarty and Steinhoff 1997) and the strategy to offload information externally. Those strategies described how people would interact with external representations while solving spatial problems, since our mental imagery ability appears to be limited by working memory (Just and Carpenter 1992). Since designers use different types of external representations, e.g. sketch, engineering drawing, 3D mockup and computer model, what kind of representations might be most proper for design and what kind will be most proper for skill development? As Ho, Eastman and Catrambone (2005) noted, architecture students usually learn design starting from mapping real objects onto 2D drawings and also learn to represent their design concepts in 3D in

charcoal and watercolor drawings, but there is no study investigating the impact of these kind of courses on students' spatial reasoning skills. Since architects heavily rely on both 2D and 3D representations, Ho et al. (2005) conducted a study to clarify the relationship between 2D and 3D strategies. Based on the framework Shown in Figure 1, even though the line version of Tic-Tac-Toe and the number version of Tic-Tac-Toe game (named Tic-Tac-Toe and Book respectively in Figure 2) are isomorphs, people treat them differently. The difference not only came from the different representations but also from the differences in strategy that people used to approach the tasks (Zhang 1997). More importantly, it seemed that there are certain people with high mathematical ability can see the symmetry of numerical representation which is very different from the spatial one.

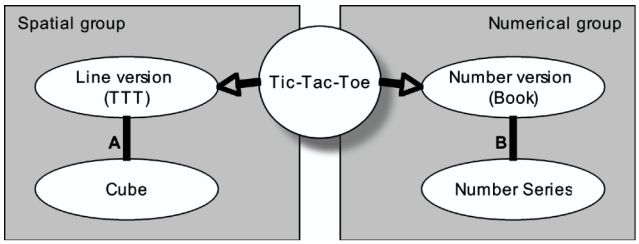

Figure 1. The experiment framework Ho et al. (2005) study

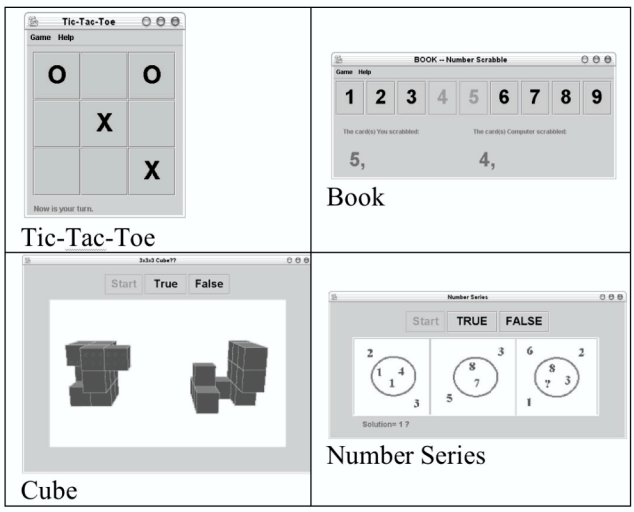

Figure 2. Examples of the tasks in Ho et al. (2005) study

In advance, the same distinction between spatial and mathematical abilities can also be observed between a more complex spatial task, i.e. 3D Cube, and a more complex numerical task, i.e. Number Series (Figure 2). The research study showed that the Cube task is only significantly correlated to line version of Tic-Tac-Toe, while the Number Series is only significantly correlated to the number version of Tic-Tac-Toe (Table 1). Ho et al. (2005) confirmed that numerical and spatial representations involve two different abilities. However, although the correlation between Cube and Number Series tasks is not significant, the result shows that they are loosely correlated. The reason might be that they are both complex problem solving tasks and general reasoning factor is loaded on both of the tasks.

Table 1. Correlation matrix of stepwise regression analysis about accuracies of four puzzles (Ho et al. 2005)

	TTT	Book	Cube	Series
TTT	1.00			
Book	0.25	1.00		
Cube	0.33	0.21	1.00	
Series	0.19	0.39	0.34	1.00

Within the spatial group, we can see that one of the tasks is 2D while the other is 3D. Ho et al. (2005) concluded that the performances, in terms of accuracy, in 2D and 3D are dependent. This finding is consistent with a fMRI study (Jordan et al. 2001) that the brain activation areas are the same when participants are solving mental rotation tasks, no matter they were 2D shapes or 3D cubes (Figure 3). However, it is not clear what kind of spatial sub-factors the tasks were loaded on. That is, we cannot know for sure whether the consistency between 2D and 3D problem solving is resulted from spatial rotation sub-factor, spatial visualization sub-factor, or even from general reasoning factor which is not a spatial ability factor at all. It turns out that we cannot simply assume the Shepard and Metzler kind of mental rotation task is a 3D rotation ability test. The reason we gave previously about the uncertainty of the sub-factors is one type of counter argument, and the ambiguity about 2D and 3D tasks, which was discussed in Ho et al. (2005), is the second one. Let's consider this in more depth.

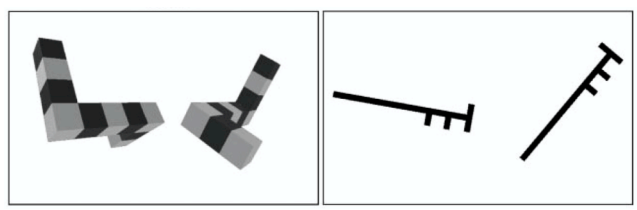
Figure 3. Examples of the 3D and 2D rotation tasks (Jordan et al. 2001)

2D and 3D Controversy

The cube rotation task introduced by Shepard and Metzler (1971) is one of the most popular spatial aptitude tasks and it has evolved through many test versions. However, Ho et al. (2005) mentioned that other researches had identified there are sub-processes involved in mental rotation tasks (Just and Carpenter 1976; Pelligrino and Goldman 1983). 2D feature mapping strategy seems inevitably involved in 3D rotation tasks. For example, Just and Carpenter pointed out from an eye movement study (Figure 4) that there are three stages for solving a cube rotation task, i.e. search stage, transform-and-compare stage and confirmation stage. In Figure 4, from fixations 1 to 4 and 8 to 12, the participant was simply scanning the features between

stimuli. He or She was not performing 3D transformations during this time.

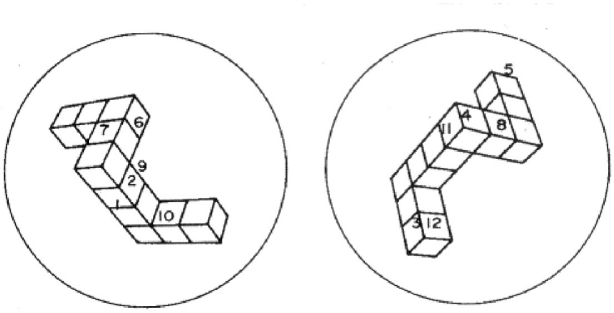

Figure 4. Eye movement fixations (Just and Carpenter 1976)

Furthermore, Gittler and Glück (1998) had shown us how it is possible that we can solve a task which is traditionally categorized as a 3D rotation task without actually using 3D ability (Figure 5). These two examples (Figure 4 and 5) tell us that we need to be extra careful in designing 3D tasks, not just "3D-like" tasks, if we are going to study pure 3D abilities. However, we still cannot successfully generate a pure 3D task. As Ho et al. (2005) claimed, such a test should allow us to compare 2D and 3D abilities in detail. This is the second difficulty for studying spatial ability.

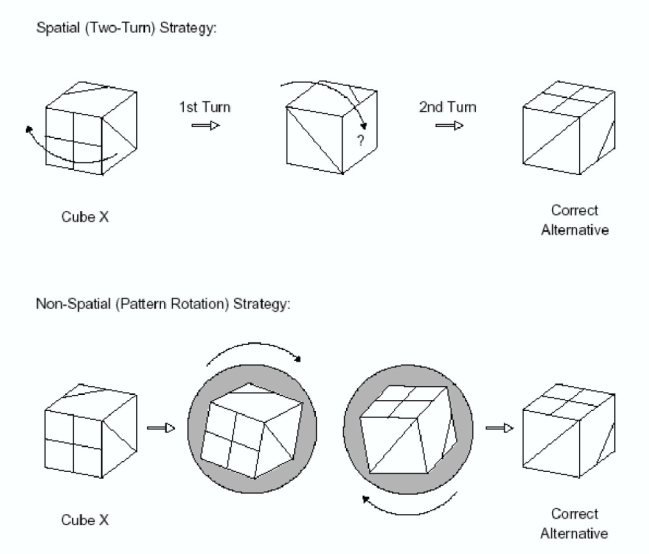

Figure 5. An example of three-dimensional cube test (Gittler and Glück 1998)

Based on the 3D-like tasks, it is reasonable to infer that 3D reasoning requires more working memory than 2D reasoning. It is because we need to carry information about one more dimension which could dramatically increase the complexity of shapes, segments and features from the geometry point of view. Referring back to architecture design, it is clear that architects can answer without difficulty the question about from where the photo was taken, as presented in Figure 6. In order to answer the question, on the one hand we can construct a 3D mental model from the floor plan and then imagine the perspective view from it; on the other hand we can scan the features on the photo and floor plan to allocate the spot. The former strategy in Cooper and Mumaw's (1985) term is a constructive strategy while the latter one is an analytic strategy.

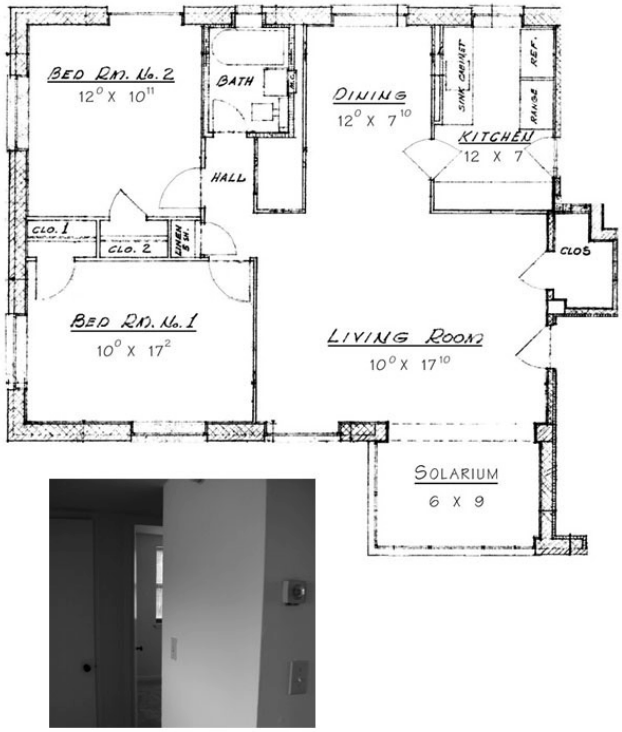

Figure 6. An example of architecture related question: from where the photo was taken? (the answer is from living room targeting bed room no. 2)

In Cooper and Mumaw's (1985) study, they argued that factor analysis approach used to assume implicitly that people would follow the instruction to solve the test and the individual differences would basically result from the efficiency of that particular sub-factor loaded on the test. However, their empirical study revealed that different problem solving strategies could lead to different performances, especially for those who were categorized as low spatial ability participants by traditional spatial aptitude tests. One of their experiments used engineering drawing as stimuli and participants were required to identify the third view in a given drawing (Figure 7). In the experiment, participants were categorized into high and low aptitude groups based on their spatial aptitude test scores. Then, Cooper and Mumaw defined two different strategies, named constructive strategy and analytic strategy. The former refers to the strategy that participants will construct a 3D mental model first and then figure out which is the missing side view; the latter refers to the

strategy that participants will scan features within the stimuli and the possible answers to decide which side view is correct. They observed that the difference in using analytic strategy was not significant between high and low aptitude groups. However, there is a significant difference between two groups using constructive strategy, both in terms of time and accuracy. The finding is surprising in that even though they did not instruct participants which strategy to use, more low aptitude participants (9 out of 13) used the constructive strategy, which is the one in which all participants were less capable. This implies that if we can explicitly teach participants to think in terms of analytic strategy then we would have a chance to improve their spatial aptitude test scores.

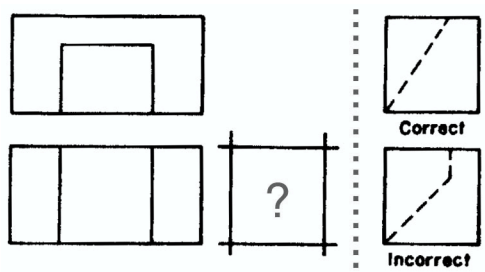

Figure 7. Example of an engineering drawing problem

Strategies in Design

2D and 3D Strategies

As we discussed in the beginning of this paper, there might be a big difference between spatial aptitude test's tasks and much more complex real world spatial problem tasks. The former might load mainly on spatial sub-factors while the latter could primarily load on general reasoning ability. That is, we should take other issues into account in order to look at more complex spatial tasks like architecture design and how 2D and 3D strategies are used in real world situation. Can we teach students to become good designers who just rely on better problem solving strategies and analytic strategy without being sophisticated in constructive spatial strategies?

There are several phenomena and questions we can observe from design activities. First of all, architecture designers usually use multiple representations in their sketches (Goel 1995). However, since they are designing 3D objects, why do they need to spend efforts in switching between 2D and 3D views? Secondly, why do architecture designers often work on 2D drawings at the very beginning? Is it simply because 2D reasoning is more economical cognitively? To what degree should we offload complicated 3D manipulations onto CAD tools, and in advance, how do these visualized 3D manipulations in CAD system could help students to develop 3D abilities? Clarifying these questions could allow us to have a better understanding about how complex spatial skills are developed. Moreover, it can also help us to develop a better design curriculum especially for those who are less capable in spatial abilities. Although there is no clear answer to those questions yet, investigating how students learned to manipulate 2D and 3D external representations might give us some clues.

In the field of design, we believe spatial ability, as for cognitive ability, can be trained in terms of the zone of proximal development (Vygotsky 1980) within a school's set up. As noted in Ho et al. (2005), students are assumed to be able to develop, with certain experiences, complex spatial skills. Students learn how to objectively represent 3D objects in 2D drawings in engineering drawing courses and measured drawing exercises. In addition, they learn how to subjectively represent textures and artifacts in perspective and in advance to represent mental images (design concepts) in realistic 3D renderings. Moreover, they also learn how to use CAD software in handling both 2D and 3D drawings. However, among those design tools, 2D representations are still regarded as a primary tool in most architecture training courses even though more and more sophisticated CAD tools are available nowadays. In addition, studies show that high spatial ability persons can construct better mental image with well-structured figures than low spatial persons, but not random pattern (Lohman 2000). Lohman (1988) also found that low spatial ability participants seemed to be weak at constructing mental representation when the representation becomes more complex. That is, it seems high spatial ability persons have better declarative knowledge and procedural knowledge that allow them to recognize meaningful patterns to construct better mental representations. Hence, we can hypothesize that with the increase of their experience or with the help provided by 3D CAD tools, they can have better spatial ability because they are able to construct better mental imagery. That is to say, the meaning of spatial ability could become how well designers can resolve spatial issues in terms of figural or diagrammatical material (representations) with their extensive procedural knowledge. Let's explore the skill development issue in terms of declarative and procedure knowledge in more depth.

Declarative and Procedure Knowledge

Based on Lohman (1988), the improvement of spatial ability could be discussed in terms of declarative knowledge and procedural knowledge, which are defined by Anderson (1983). With training, participants will gain more "episodic memories" about the experimental settings and stimuli that increases their spatial declarative knowledge. Lohman argued that this is the major improvement that allows participants to obtain better spatial aptitude test scores. We assume this improvement occurs by allowing participants to match new complex shapes and shape features with a larger repertoire of known shapes. In terms of procedural knowledge, participants will create automatized chunking of processes after repeatedly solving similar tasks. For example, they may use symmetry to identify axes, then identify asymmetrical features.

Based on these two types of knowledge, Lohman conducted a study (1988) showed the correlation between the improvement on spatial ability and extensive practice. In his study, the stimuli included the cards and figures test and the paper folding test. The participants were divided into two groups, one received feedback in terms of right or wrong answers while the other received no feedback. He observed that the no feedback group increased their speed of response but no improvement in terms of accuracy. This result implies that this type of improvements mainly resulted from the familiarity of the tasks and procedural learning. In other words, participants acquired procedural knowledge from extensive practice but no declarative knowledge. On the contrary, the feedback group was observed that the participants improved their performances in both speed and accuracy. That is to say, those participants with feedback acquired not only declarative knowledge but also procedural knowledge. Lohman confirmed the skill acquisition model proposed by Anderson (1983). Moreover, he reported this kind of learning effect could be observed mostly from low spatial ability participants. The findings can lead to some hypothesis in terms of how design students might improve their spatial ability. First of all, students at the very beginning will learn the mapping between notations and physical objects. After repeatedly training, the use of notations should become automatized. So, they can get rid of the situation that lacking of declarative knowledge while reasoning through the notations and drawings. That is, their reasoning speed would improve but the quality of the reasoning would not. Secondly, they would gain lot more episodic memories and facts with repeated exercising. This knowledge can provide a foundation that allows students to increase their declarative knowledge even though studies have shown that there existed other issues, like visual analogy (Casakin and Goldschmidt 1999) and problem solving strategy (Ho 2001), needed to be involved at the same time to ensure the design quality. In other words, there are qualitative and quantitative issues interweaved in design training. Students acquire large amount of episodic memories from design history class and accumulate their procedural knowledge; and in addition they also learn qualitative knowledge in studio that allows them to better apply their prior knowledge. That is, we think developing a large repertoire of declarative shape knowledge through extensive practices is an important means to further enhance their spatial ability. This hypothesis will allow us to see how design students learned from exercises with 2D and 3D drawings and how these two representations would help to scaffold the development of spatial skills and strategies.

Summary

Spatial ability is usually regarded as a core issue of design training. However, it is surprising that we know little about how students develop these kinds of complex reasoning skills in school. Even though it is well documented that spatial aptitude is used to assess likely abilities in design fields, it is not clear that there have been any studies that ex post facto assess if the tests did successfully predict success. One counterexample is SAT used to predict the success in college, but in fact the correlation between SAT and first year's GPA is only around 0.45 (National Research Council et al. 1999). Moreover, Lohman (1988) also suggested that complex spatial ability tasks would become figural reasoning tasks that have spatial components instead of being the tasks which we can use to assess some combination of spatial sub-factors. Design tasks are usually very complex, and the complexity could prevent us from using factor analysis approach to study spatial ability in design.

Another problem we pointed out is that traditional spatial aptitude tests failed to make a clear distinction between 2D and 3D representations and strategies involved in the tasks. That is, we need to have a clearer way to define pure 3D tasks before we can study the importance of 2D and 3D skills in spatial problem solving domains. In spite of the difficulties, some research suggests that some spatial problem solving approaches might be promising. Based on those, we generate some hypothesis about how to improve students' spatial abilities. First of all, in terms of design spatial problems, spatial abilities are tightly embedded in general reasoning tasks and these should be taught together. Secondly, we believe investigating what rules of declarative and procedural knowledge played in design, in terms of spatial issues, is a promising research direction since this is how general reasoning abilities appear to be developed. Thirdly, since spatial abilities and general problem solving abilities are tied together in design and studies (see Cross 2004 for an overview) suggested expert and novice have a qualitative difference in strategies, we believe processing strategies can have a big performance impact on design spatial reasoning, especially for those who are less able. It is the first author's intentions to verify these hypotheses in future studies.

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