Ambiguous Landmark Problems in Cognitive Robotics: 
A Benchmark for Qualitative Position Calculi

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Abstract
In this paper we introduce a task which can serve as a benchmark for qualitative relative position calculi. In this task ambiguous local landmark observations have to be integrated into survey knowledge. We show that the most prominent relative position calculus, Freksa’s Double Cross Calculus can solve a specific instance of this task. The observations can be represented in a constraint network and standard constraint propagation solves the ambiguity problem.

However, more general instances of the ambiguous landmark problem cannot be solved using the Double Cross Calculus. Therefore we present an extension to relative position ternary point configuration calculi which uses an adaptable level of granularity. This family of calculi is capable to solve general instances of the proposed benchmark. Thereby robot applications including reasoning about ambiguous perceptions will be made possible.

Introduction
A qualitative representation provides mechanisms which characterize central essential properties of objects or configurations. A quantitative representation establishes a measure in relation to a unit of measurement which has to be generally available. Qualitative spatial calculi usually deal with elementary objects (e.g., positions, directions, regions) and qualitative relations between them (e.g., "adjacent", "on the left of", "included in").

The constant general availability of common measures is now self evident. However, one needs only remember the example of the history of technologies of measurement of length to see that the more local relative measures, which are qualitatively represented, (for example, "one piece of material is longer than another" versus "this thing is two meters long") can be managed by biological/epigenetic computational models for projective relations in relative reference systems (Moratz and Tenbrink 2006). The results point to three different options to give a qualitative description of spatial arrangements of objects labelled by Levinson (Levinson 1996) as intrinsic, relative, and absolute.

We can find examples of all three options of reference systems in the QSR literature. For instance, an intrinsic reference system is used in the dipole calculus (Schlieder 1995b), (Moratz, Renz, and Wolter 2000), a relative reference system in QSR was introduced by Freksa (Freksa 1992b), and finally Andrew Frank’s cardinal direction calculus is suitable for an absolute reference system (Frank 1991), (Ligozat 1998).

Qualitative relative position calculi can be viewed as computational models for projective relations in relative reference systems. To model projective relations (like “left”, “right”, “front”, “back”) in relative reference systems, all objects are mapped onto the plane. The centers of projected objects can be used as point-like representation of the objects.

Figure 1 shows a simple model for the left/right-dichotomy in a relative reference system, which is given by origin and relatum (corresponding to Levinson’s termi-
Figure 1: The left/right-dichotomy in a relative reference system

Figure 2: Adding relations for referents on the reference axis

Figure 3: Examples of point configurations and their expressions in the flip-flop calculus.

Figure 4: The single cross calculus
Figure 5: Acceptance regions of the extended Double-Cross calculus

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<thead>
<tr>
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<th>left</th>
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<th>left-back</th>
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<tbody>
<tr>
<td>Flip Flop</td>
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<td>impossible</td>
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<td>Double Cross</td>
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Figure 6: Relative reference by projective predicates for the different calculi

Ambiguous Landmark Problems

In this section we introduce a task which can serve as a benchmark for reasoning with qualitative relative position calculi. In this task ambiguous local landmark observations have to be integrated into survey knowledge. We show that the most prominent relative position calculus, Freksa’s Double Cross Calculus can solve a specific instance of this task. The observations can be represented in a constraint network and standard constraint propagation solves the ambiguity problem.

Figure 7: Double Cross reference system/partition

We use for our demonstration the QSR toolbox SparQ (Wallgrün et al. 2007). In this system the original version of the Double Cross Calculus without Thales’s circle is used (the relation symbols used in this system can be found on figure 7).

We can use the Double Cross Calculus to represent our local observation based underdetermined spatial knowledge of the robotics example depicted in figure 8. The robot’s observation at time point 1 (the red landmarks are close and can be distinguished, the green ones are to far away to be distinguished):  

\[ R1, R2 \ (2.5, 3.6) \ G1 \] (1)

The robot’s observation at time point 2 (the green landmarks are close and can be distinguished, the red ones are to far away to be distinguished):

\[ G1, G2 \ (5.2, 6.3) \ R1 \] (3)

\[ G1, G2 \ (5.2, 6.3) \ R2 \] (4)

The observation corresponding to equation (4) can be reformulated:

\[ G1, R2 \ (3.5, 3.6) \ G2 \] (5)

It follows:

\[ R2, G1 \ \text{INV} \ (3.5, 3.6) \ G2 \] (6)

\[ R1, R2 \ (2.5, 3.6) \ \text{INV} \ (3.5, 3.6) \ G2 \] (7)

\[ R1, R2 \ (3.5, 2.5, 1.5) \ G2 \] (8)

The conjunction (intersection) of equation (2) and equation (8) yields:

\[ R1, R2 \ 2.5 \ G2 \] (9)

This manual deduction shows how the ambiguity is resolved in this landmark configuration. In general the observations can be represented in a constraint network and standard constraint propagation solves the ambiguity problem.

However, since the Double Cross calculus is coarse only special configurations of landmarks can be solved with this formalism. In the configuration which we used for our demonstration the landmarks are arranged as corner points of a rectangle. This rectangular shape corresponds to the structure of the double cross. Landmark configurations which do not follow this structure cannot be disambiguated based on constraint-propagation reasoning with the Double Cross Calculus.

More fine grained calculi like the GPC$m_c$ calculi described in the next section are capable of solving much more general problems. This approach is ongoing work, first results are promising.

Generalizing ternary point configuration calculi

Applications exist in which finer qualitative acceptance areas are helpful. The possibility to use finer qualitative distinctions can be viewed as a stepwise transition to quantitative knowledge. The idea of using context dependant direction and distance intervals for the representation of spatial knowledge can be traced back to Clementini, di Felice, and Hernandez (Clementini, Di Felice, and Hernandez 1989).
Relative distance: how far is \( C \) compared to \( A \)? In other words, how does the distance from \( C \) to \( A \) compare with the distance from \( A \) to \( B \)?

Relative direction: what is the direction of \( C \) relative to \( A \) and \( B \)?

These two relative localisations will then be combined to lead to relative position.

The newly proposed calculus is called granular point configuration calculus GPCC. In this calculus two points are the basis for a reference system. The reference system can be interpreted as a partition of the plane into acceptance regions for a third point. All options for places of the third point which are in the same part of the partition are considered to be in an equivalence class and are treated in the same way in categorization and reasoning tasks by subsequent modules. One variant of the GPCC calculus and its partition on the plane is shown in figure 9.

For the cases with \( A \neq B \) we define a relative radius \( r_{A,B,C} \):

\[
r_{A,B,C} := \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} \quad \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}
\]

\( A, B \text{ same } C \) : \( r_{A,B,C} = 0 \)

and for \( A \neq B \neq C \) a relative angle \( \phi_{A,B,C} \):

\[
\phi_{A,B,C} := \tan^{-1} \frac{y_C - y_B}{x_C - x_B} - \tan^{-1} \frac{y_B - y_A}{x_B - x_A}
\]

The further base relations have an acceptance area depending on the granularity of the calculus to be applied. The calculus shown in figure 9, GPCC\(_3\), has a level of granularity of 3 and 267 relations. A calculus of the granularity level \( m \), described below as GPCC\(_m\), has \((4m - 1)(8m + 3)\) base relations. The base relations of GPCC\(_3\) are thus defined:

\[
A, B \perp_0 C := 0 < r_{A,B,C} \leq 1/3 \land \phi_{A,B,C} = 0
\]

\[
A, B \perp_1 C := 0 < r_{A,B,C} \leq 1/3 \land 0 \leq \phi_{A,B,C} \leq 1/6 \pi
\]

\[
A, B \perp_2 C := 0 < r_{A,B,C} \leq 1/3 \land \phi_{A,B,C} = 1/6 \pi
\]

\[
A, B \perp_3 C := 0 < r_{A,B,C} \leq 1/3 \land 1/6 \pi \leq \phi_{A,B,C} \leq 2/6 \pi
\]

\[
A, B \perp_{23} C := 0 < r_{A,B,C} \leq 1/3 \land 11/6 \pi \leq \phi_{A,B,C} \leq 12/6 \pi
\]

\[
A, B \perp_0 C := r_{A,B,C} = 1/3 \land \phi_{A,B,C} = 0
\]

\[
A, B \perp_1 C := 1/3 \leq r_{A,B,C} \leq 2/3 \land \phi_{A,B,C} = 0
\]

\[
A, B \perp_2 C := 3/2 \leq r_{A,B,C} \leq 3/1 \land \phi_{A,B,C} = 0
\]

\[
A, B \perp_{23} C := 3/1 \leq r_{A,B,C} \land 11/6 \pi \leq \phi_{A,B,C} \leq 12/6 \pi
\]

To give a precise, geometric definition of the GPCC-relations we describe the corresponding geometric configurations in an analogue way to the TPCC calculus (Moratz and Ragni 2008) on the basis of a Cartesian coordinate system represented by \( \mathbb{R}^2 \). First we define the special cases for \( A = (x_A, y_A) \), \( B = (x_B, y_B) \) and \( C = (x_C, y_C) \).

\[
A, B \text{ dou } C := x_A = x_B \land y_A = y_B \land (x_C \neq x_A \lor y_C \neq y_A)
\]

\[
A, B \text{ tri } C := x_A = x_B = x_C \land y_A = y_B = y_C
\]
compose them in a way such that we can use it for enforcing
local consistency (Scivos and Nebel 2001). In trying to
generalize the path-consistency algorithm (Montanari 1974), we
would like to enforce 4-consistency (Isli and Cohn 2000).
We then had to use the following (strong) composition operation:

$$\forall A, B, D : A, B \cup r_1 \cup r_2 \dashv \exists C : A, B \cup r_1 \cup C \cup B, C \cup r_2 \cup D$$

Unfortunately, the \( GPCC_m \) calculi are not closed under
strong composition. For that reason we can not directly en-
force 4-consistency. But we can define a weak composition
operation \( r_1 \cup r_2 \cup \) of two relations \( r_1 \) and \( r_2 \). It is the most
specific relation such that:

$$\forall A, B, D : A, B \cup r_1 \cup r_2 \cup D \dashv \exists C : A, B \cup r_1 \cup C \cup B, C \cup r_2 \cup D$$

While using the weak composition we can not enforce 4-
consistency we still get useful inferences.

The problem is calculating the permutation and composi-
tion results for such structures by machine. The operation
tables can be approximated with the aid of a composition of
distance orientation intervals (DOI) (Moratz and Wallgrün
2003). Thereby areal segments and their borders are sum-
marized. Thus one obtains thereby a quasi-partition in which
only linear overlappings occur.

The calculi are, with respect to the transformation \( \text{HMI} \),
closed:

$$\text{HMI} ( \downarrow_\downarrow) = \downarrow_\downarrow$$

In robotic applications the relevant areal base relations
with their borders are summarized into general relations.
Out of this, one obtains a closed region in a plane (with the
exception of its exterior segments which continue infinitely)
as acceptance area for the third point of a ternary relational
proposition. The bounded line segment acceptance areas be-
ting to both neighboring segments and border points typi-
cally belong to four segments. All inner segments contain
the point which corresponds to the relation \( \text{sam} \).

The areal measure of these ambiguous acceptance areas
is however 0. In the event that a corresponding border point
triple is to be represented qualitatively, a disjunction of all
bordering base relations must be used. As a result one ob-
tains then a fine grained quasi-partition for the representa-
tion of the relative position of a point with respect to a refer-
ce system build by two points.

Obviously, the calculi \( GPCC_3, \ GPCC_4, \) and \( GPCC_5 \) can
solve more natural instances of the ambiguous landmark
problem than the Double Cross Calculus. Which granular-
ity is needed to solve reasonably designed random instances
of the ambiguous landmark benchmark is subject to future
investigations.

**Conclusion**

We showed a robotics problem about the disambiguation
of landmarks. This disambiguation of landmarks can be
achieved by constraint-propagation only, since the underde-
termined spatial knowledge about the landmark position
can be expressed as constraint networks. The Double Cross Cal-
culus is capable to solve a simple instance of this problem.
For more general tasks one needs a finer granularity of the
position calculus. We presented a first draft of such a calculus which in principle can solve general instances of the landmark disambiguation problem.

With the ambiguous landmark benchmark we have a test case which puts an emphasis on a qualitative decision as output of qualitative spatial reasoning based on observed data. From my point of view this is a more natural task than abstraction constraint satisfaction problems which try to find spatial instances based on purely abstract input constraints.

References


