The Geometry of Social Anticipation

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Abstract

Our group is engaged in the design of some potentially disruptive systems that can competently capture social intent and related anticipated futures. Novel logic and geometric analogs are used.

Introduction

Our group, Earl Research, is developing next generation systems for intelligence analysis. These are based on prior experiments on classified programs and promise to be disruptive in several ways. Three components are being coded: one that provides a next generation collaborative environment for human analysts, employing ontological federation via mobile agents; an extremely innovative second component that supports self-organization of information elements into narratives of analytical value; and a third that provides real-time collaboration of automatic recognition programs from sensors. All integrate, using a next generation integrating infrastructure. This paper deals with processes associated with the latter component.

All three work at the most fundamental levels of novel logic, exploiting the edge of what is newly known, the self-organizing mobile agent system being the most radical. The novel logics used are not discussed in this paper, which is restricted merely to a reporting on a poster. They involve newly explored geometric logics in a two-sort; the second sort handles situations, intuitions, socially-rooted causes, anticipated futures and other "soft" information.

The general approach at the logical level is not discussed here, but one can interpret the way we handle the sensor collaboration in simple terms as taking probabilistic artifacts from sensors that will have logically rooted semantics associated with them (by the external interpreters), and managing them by geometric means. That level is what this paper describes. The work includes the ability to do anticipative reasoning over social and culturally motivated causal dynamics.

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Softness: Social Dynamics and Anticipated Futures

One goal of these projects is to reason properly over so-called "soft" elements, as described below (Goranson 2006). These are a diverse group of phenomena that have been poorly handled by current methods. We address three soft dynamics here.

The first concerns the topic of this workshop: capturing the elusive dynamics of social, cultural and emotional behavior. This can only be adequately handled by modifying the base logic; we present some algebraic support for that here. The second area of concern is the well-defined problem of reasoning about contexts that are essential to interpretation but are not explicitly representable, whether due to cost, accessibility or inscrutability. As with the socio-cultural models, this is a well understood problem in both the humanities and computer science, with indications that only a radical approach can suitably address the problem in a formal context. We frame the difficulty in terms of the ability to reason consistently about contexts that are unknown or unrepresentable.

The third "soft" condition is less well understood as a problem: the ability to reason about futures that have complex, perhaps apparently non-linear and/or causal consequences.

We consider all three cases to be addressable in a single framework. Our approach is to invest in a two-sorted logic where all the ordinary elements (read: facts) are handleable in a well characterized (but novel) logical framework and all the others (the three we note here) are dealt with in the second sort. This work is founded on situation theory (Barwise, Perry 1982), which we extend to support the lightweight instream processing applications necessary for wide-ranging intelligence. One of those applications is the near real-time semantic enrichment of multiple streaming media types for ingestion into a knowledgebase. We call this project our "universal Watcher," and the work described herein is part of the architectural work for building such a watcher for the intelligence community.

Here, "situations" serve as contexts that subsume the three soft categories of concern. How they handle contexts is obvious; that is what situation theory was designed for.

The original intent was to solve a problem in logic applied to language where you needed the ability to reason about information related to a communication that is not explicitly represented in the utterance (Perry 1999). The second sort in the logic and operational calculus was originally worked out to deal with this case.

Others (Devlin 1995) have pushed the notion of situation to support general softness as context. We extend it further in this direction, applying the concepts of constructed narrative to situation (Goranson, Cardier 2007). Principles of narrative and quantum interaction will manage the intermediary metric of form, in order to systematize the composition, blending, and prediction of situations. This approach allows the ability to reason about future states — see as narratives — dealing with them as anticipated situations.

The mechanisms for narrative construction are outside the scope of this paper; we do use linear, dynamic logic (Goranson 2008; Lehmann 2007) to construct the narratives. Indeed, the approach is to build infrastructure to allow information to self-organize into stories that are handled as the "situations" discussed here. The organizational mechanics are based on symmetries (Wigner 1960) in the emergent system. An advantage of this approach is that it supports on-the-fly semantic federation where knowledge comes from heterogeneous sources.

Finally, we use these rich narrative situations in the same way that humans do, to reason about human dynamics that surely have some sense to them but which are not cleanly modelable in a simple logical system.

These projects manage abstractions in a translation sequence roughly as follows: probabilistic notions, geometric notions and logical ones. This supports mechanics that address these three soft conditions, thereby allowing us the ability to claim – as we do – anticipative sociocultural reasoning.

The Use of Geometry

Our approach is based on careful attention to abstractions, with the most important aspect being an ability to shift among the three paradigms (probability, geometry. logic), using each where best suited. The probabilistic paradigm is best for a class of input and automated enrichment processes that the Watcher must support (Goranson 2009). These include, for instance, object recognition, text interpretation and sound identification.

Figure 1 shows the notional design of the Watcher, indicating the serial flow through the three abstraction domains.

The benefits of the logical abstractions that the Watcher produces in step 3 are for ordinary reasoning and our experimental emergent behavior. They provide near real-time enrichment; provision for adaptive learning; sensor control; and unified knowledge context.

The geometric or algebraic abstractions of step 2 are optimal for uniting the three tasks discussed in this paper. In each case, we extract a (group) "form" from the basic representation, be it logical or probabilistic. We then take advantage of the handy computability of those forms to deal with scalability and performance problems that would otherwise be impossible.

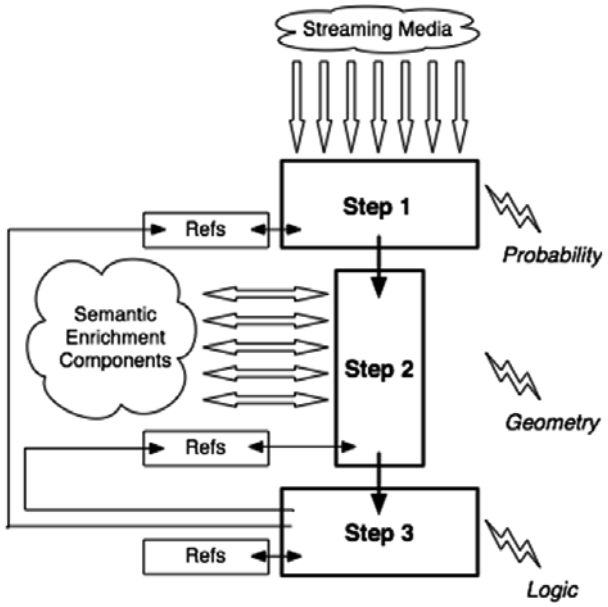

Figure 1: Notional design of Watcher.

Quantum Interaction

Underlying the congruence of the three abstraction paradigms are results from the quantum logic community, collected around new, category-theoretic tools for "quantum interaction" (Coecke 2007; Sadrzadeh 2007).

There is a long tradition of attempts to develop tools based on the logic behind quantum mechanics, an agenda first envisioned 70 years ago (von Neumann 1932; Birkhoff; von Neumann 1936). Only recently with work in category theory (Abramsky and Coecke 2004) do we have theories that support a possibility of implementable systems that can merge the three abstraction paradigms. Without these unifying concepts, one is locked in a single abstraction. The unification function is independent of whether we employed quantum-inspired methods in each of the abstraction domains or not. In step 2, we do not.

To be specific, for the first time, we now see the possibility of moving among semantic, geometric and probabilistic abstractions within a common formal framework.

Ontological Dispatch

The first and simplest of the three distinct problems we handle geometrically has to do with ontological dispatch. We have a pool of constantly re-enriched primitive features extracted by probabilistic means from source media. Examples would be n-grams words and phrases from text;

phonemes and modulations from speech; and edges, patterns and motion from video. We have — as mentioned — a method for assigning symmetry features to these extracted elements.

We also have a significant number of special purpose applications from others that interpret based on these features. For example, we will have a module that takes video and performs face recognition, another that interprets speech and yet a third that teases out emotional intent from body language.

Some features are obviously inappropriate for specific modules, for instance speech phonemes are not used for face recognition. The "dispatch" function sends the right primitives to the appropriate module. One could conceivably accomplish this by a hardwired table: certain video features would always be sent to the face recognition module, for instance.

But we want to additionally support two difficult capabilities: we want the ability for the system to learn, based on semantic reasoning and audits further down the food chain. This is indicated in figure 1 by the feedback loops to what we have schematically shown as "references." A variation on this is that we want the ability to modulate what is sent and how based on dynamic notions of the quality and scale of the inputs, their quality, and the processing latency.

The second capability is that by means described below, we will have compound, aggregated (in the sense of composed) features. An example is that we might have compound face recognition results that identify a person as someone who has lied before. We might separately have voice analysis that indicates he is probably lying. And we also have speech recognition and interpretation software that provides key facts about what he is saying. Wouldn't it make sense to have the "facts" be colored by the probable fact that they are lies?

This means that we need geometrically-informed semantic interpretations for dispatch. By looking at the form characteristics of features, feature composites and higher level structures, we should be able to quickly determine which of our external applications should see it, and in what order, pending further relevant enrichment.

For this and the other geometric reasoning tasks, we start with the theory and mechanics of what Leyton calls "recovered history" from shape-based process grammar (Leyton 1992; 2001).

Lightweight Narrative Aggregation

Our second problem is far more challenging; it integrates with what we just described above, creating the higher level facts mentioned. In this case, we want to associate elements to build small semantic assemblies. This is a primitive sort of reasoning, performed using this geometric method because of timeliness and scalability requirements.

We have our shape-characterized primitives, and we also have some that are newly enriched by external applications. We want to associate certain of these together so that the external applications will be better informed. This process is a shape-based narrative construction process that emulates the more robust (and much slower) logic-driven narrative construction.

Here is a practical example, a bit more complex than what we used above. Suppose we have acoustic sensors, and we have an external processor with very strong abilities to recognize that tanks have moved into the area and that some significant repair is underway.

We also have a satellite, miles above looking at the tree canopy and detecting minute variations in the leaf movement. Coupled with that are analytical tools that can tease out which are caused by vehicles and which natural.

Externally we have human intelligence that comes to us fully interpreted that someone has intercepted a radio message that one tank is disabled for a week. All three of these are characterized according to the process grammar we have described.

We need the ability to determine a process order, which in this case determines that:

- the external intelligence advise the acoustic interpreter to spend more time than usual and to weigh the probability that tanks are present.
- the resulting semantic information be characterized in a way that advises the leaf movement interpreter.
- the results from the leaf interpreter, having indicated tentative paths and locations, advise the satellite tasker where to look further and with what fidelity and sensor type to ensure a better analysis.

It should be mentioned that this is not the final analysis. That comes later in step 3, when downstream tools take all the automatically enriched results and use genuine logical methods, conventional or novel as we intend. All that happens here in step 2 is speedy, geometrically informed semantic aggregation and reshaping of the geometric "indices" to support collaborative, ordered dispatch.

Fittedness Measure

The third thing we do with form is in the semantic space, the third abstraction step late in the process. At this point, we are performing sophisticated semantic reasoning, based on either conventional reasoning mechanisms or our novel linear logic based narrative construction techniques. To support that narrative construction, we employ a form-based fittedness measure extended from the previously described foundations.

We take a collection of semantic elements; these may be from different sources and use different ontologies. They may be anywhere on a spectrum from simple data elements, to facts and predicates, to partially assembled semantic structures.

We produce many tentative narrative constructions; these are logically represented. The problem is that we have to determine which of the very large number of narrative candidates we want to keep. We compare the candidates to certain maintained patterns ("target stories") and applying a "fittedness measure," discard all the

candidates outside a range of acceptability. It should be mentioned that this alone is a major improvement; current techniques would set that range purely based on induction over past cases. This method anticipates unique futures based on causal projection.

Metaphorically, what we are doing is assembling stories, using a process of tentative completion, and then testing for which of these are "good" stories. The tests are a combination of truth (in the contexts), relevance to a designated user, and "elegance." In this context, elegance refers to a class containing potentially inscrutable factors that just "seem right," and emerge from repeated experience.

In a sense, this is also dispatch, merely dispatch of the stories that look good to the "keep" bin. It is also like the lightweight aggregation in that is evaluating good assemblies even though it is not directing the assembly. (In a role not here described we will allow the agents to use this measure in incremental assembly to prevent obviously bad candidates.)

In the tactical intelligence scenario we have been using for our examples, we are presumed to be working in a very closed world. That means that there are only a few stories we are interested in, those that pose some tactical threat, for instance. We want to know those stories; we want to know what is likely to happen and more, what *could* happen if we take certain actions. Most of these stories deal with human motives, intents and a variety of other behaviors we reason over as "urges."

We have certain stories — in this case stories about threats — that we use as exemplars. Our many constructed narrative candidates are compared to these exemplars using this geometric fittedness measure. A companion paper (Cardier 2009) provides some principles of the narrative construction metrics.

Note that we have cleverly swept into one concept the three soft notions mentioned earlier: human motives (as urges), anticipated futures (as completed narratives) and implicit or unknown information (as story context). And we evaluate them all in the same mechanism, by geometric fittedness.

The Methods

All three of the following methodologies hinge on the same geometric notions, each successive method expanding the previous one. To explore the formalism we use, consider the processing of video data, though these methods will work on different types of data streams, such as streaming text or audio.

Ontological Dispatch

After a video is probabilistically analyzed we extract and append a geometric form that has a shape analog.

The mathematical structure which describes the generative history of a shape is the wreath product (Leyton 2001). The wreath product constructs a new group from two given groups, being a mixture of semidirect and direct products of groups, denoted:

$$G_1 \wr G_2 = \prod_{i=1}^{n} G_1 \rtimes G_2.$$

In the above equation G_1 is referred to as the *fiber group*, while G_2 is the *control group*. Here both groups are taken to be of finite order with G_2 having order n, but this is only a notational convenience and in general finiteness is not required. The semidirect product is taken with respect to the map

$$\Theta : G_2 \to Aut(\prod G_1)$$
$$g_2 \mapsto g_2 - g_2^{-1}$$

Note that since each element of the fiber group product has as many entries as there are elements of G_2, the conjugation described above is realized as permuting the entries of an element of the fiber group product. That is to say, components of the group $\prod G_1$ are indexed by elements of G_2, and conjugation by g_2 happens via indices. From this map we have the multiplication that turns the wreath product described thus far into a group

$$(\bar{g_1}, g_2) * (\bar{g_1}', g_2') = (\bar{g_1}\theta(g_2)\bar{g_1}', g_2 g_2'),$$
$$= (\bar{g_1}(g_2 \bar{g_1}' g_2'^{-1}), g_2 g_2')$$

with $\bar{g_1}, \bar{g_1}' \in \prod G_1$ and $g_2, g_2' \in G_2$. One can easily make wreathing into an n-ary operation, allowing a control nested hierarchy with arbitrarily many levels.

Leyton proposes that any form, physical or abstract (as we employ), can be expressed in terms of its generative history, which is given by its wreath product structure (Leyton 2001). The simplest and most well behaved examples involve only discrete symmetry groups and 1-parameter Lie groups (which behave as a kind of continuous symmetry group). Under this restriction a vast library of simple shapes can be expressed in terms of this type of product.

One can easily give some primitive-based characterization of (physical) geometry in terms of wreath products, following the conventions Leyton defines for the standard primitives of CAD (Leyton 2001). However, our input is a *stream* of data, not a discrete type and we extract characteristics to reason in ways that may have no physical analog. To support the dispatch in a way that exploits the geometry of the situation we construct a method to see the wreath products that represent primitives that are collected along some parameter (in this case *geometric*) as one object through whatever transitions that are captured in the time-dependent feature. Some greater aggregation is achieved with geometric construction and the narrative construction (Cardier 2009).

"Dispatch" is the process whereby cogent features are sent to appropriate external applications for semantic enrichment. For this, one defines a reference set of feature characteristics that are of interest to the external applications. The reference set is expressed as wreath products. The features extracted from the video can be

compared to the, relatively small, prototype reference for every category.

The wreath product allows for the form to be described entirely in terms of processes. In Leyton's application there is no need for primitives (Leyton 2001); their function here allows for efficient comparison of observed and prototypical groups, for example whether they share wreathed components and how many groups control a shared wreathed component. Several group theoretical methods of comparison will be employed simultaneously: counting (or classifying) elements of specific orders or computing relevant quotients. Whatever categorical prototype an observed wreath product matches best determines to which types of recognition engines it is dispatched.

For example, a wreath structure that "matches" what a face recognition application needs will dispatch the features to the applications that do that task. What is reported back is a result that then gets incorporated as new, higher level features: "This is Ted's face," as one way we have enriched the primitive. We then produce a new geometric object which might dispatch aggregated features again. This feature is the basis of what we next describe.

Lightweight Narrative Aggregation

We must efficiently manage the combination of the process described above happening over various types and numbers of streams and across many modules. An example: "the person speaking in this video has lied before, his, words are phrased, his face moves and he intones in a manner that collectively indicate lying now." This is accomplished by considering recognized wreath products and their constituents as a new object in the following way.

Two predecessor components are, respectively, represented by $\mathbb{R} \wr \mathbb{Z}_4 \wr \mathbb{R}$ and $SO(2) \wr \mathbb{R}$, where $SO(2)$ is the relevant characteristic. To represent a video aggregation that represents an object we take their Cartesian product and wreath it with a new control group that can move the objects relative to one another in 3-space. Namely, our new object has wreath structure given by:

$$[(\mathbb{R} \wr \mathbb{Z}_4 \wr \mathbb{R}) \times (SO(2) \wr \mathbb{R})] \wr AGL_3(\mathbb{R}),$$

where $AGL_3(\mathbb{R})$ is the general affine group of linear transformations in 3-space plus translations. This group defines one object component. To make the action of this group explicit we identify the configuration of the two example objects with the most symmetry with the identity element of the top level control group. The process of combining relevant features is not limited to a particular type of input type and can span types because of our generic abstraction space.

In cross media aggregation like the "lying" example, we use a reference vocabulary of products and control groups "learned" from downstream processes, initially human knowledge. For instance, our emotional state recognition module would enrich our detected video features with "facts" about how the person in the video is moving and using his body. The geometric nature of this type of data is measured as the asymmetry in the feature's form relative to a reference that indicates deceitful intent, then is incorporated in the aggregated feature and its new form.

The interpreted geometric structure of a collection of streams usually does not have a physical interpretation directly related to any physical space involved. The space in which different data streams are aggregated is abstract and has reference to causal intent. Though there is a geometric interpretation of this space for user interface purposes, there are challenges in showing details in context that we have not quite resolved.

It is precisely the generality of this abstraction which yields great advantages in the processing of many data streams of different types at once in an intelligent way. In particular, it helps with the following, vexing problem.

Fittedness Measure

The benefits of soft geometry are not limited to the aggregation of extracted features. In step 3 we create many complex, candidate semantic structures as "narratives." Once created, these are characterized by form in the same way as before, and the problem is to decide by the characteristics of that form which are more true, coherent and cogent.

We have a reference of "good stories," that is similar in nature to the references used for determining something like "fittedness" between the candidates and the entries in the reference for dispatch to semantic enrichers. This reference is ontological in nature.

The difference between this operation and the one before is a matter of causal complexity. In the first operation, a feature is essentially a discrete object. We have some complexity in these objects so far as sequence is concerned, and indeed we take advantage of the streaming nature of the source media to create "change features." In the video case, these are b-frames: patterns that persist from frame to frame but that move or perturb in some way. But this complexity is contained internal to the object.

In the second step, we have assembled objects but the assembly is based on associative relations, for example the spatio-temporal relationship which associates emotional face indicators with emotion voice ones.

The step addressed now assembles by linear relations that include cause and that have a logical grammar. We employ the "and-then" connective as described by (Lehmann 2007) and associated with quantum geometry. The assembly thus has an extractable "shape" in the linear assembly. At the same time, constructive methods allow assemblies that only produce narratives, stories.

The fittedness has to refer to two interrelated ontologies. One is of the type already used which essentially maps the "history" in terms of its apparent features. This is appropriate because in essence what the Watcher does is extract the meaning from the media in historical terms. The other ontology captures intangibles that cannot be inferred by considering the constituents. (Cardier 2009) deals with

these by analogy; (Devlin) by situations, and (Bohm 2002) as “implicate” order.

The reference algebra thus has to have a quaternion foundation. Clifford algebras provide us with a strong candidate for our reference algebra. We use a similar mechanism as before, but employ Clifford algebraic means rather than the computationally cheaper Lie algebra of previous steps. There are potential alternatives rooted in the categorical formulation of quantum information theory, however, we focus on the Clifford algebraic approach.

The foundations of the Clifford algebraic presentation of quantum mechanics are discussed in (Hiley 2002) at great length. This provides an appropriate formal framework that captures the quantum nature of the soft elements described above. The challenge then becomes how to enable this framework to process geometric data in a wreath structured format. Clifford algebras can be wreathed with respect to their additive structure. In general, the wreath product of two Clifford algebras will not be a Clifford algebra, owing to the fact that the wreath product exploits the algebras simply as additive groups (the only way in general to see the entire algebra as a group). Despite this difficulty, when the Clifford algebras share a base field an algebraic structure can be imposed on their wreath product. Scalar multiplication and multiplication of elements happens component-wise.

This algebraic structure may be less telling of the nature of a situation, but it does offer the advantage of combining soft situations with recognized histories of data in an explicit fashion. One may combine recognized wreath structures with algebraic structures which represent inexact situations by again applying the wreath product. Additionally, there is notion of local recognition of soft situations. This can be achieved by carrying the multiplicative structure of a Clifford algebra lost in the wreathing as a sort of tag that the watcher can appeal to when analyzing a situation.

Each narrative assembly has an associated geometric structure. What a particular semantic object is, spans the entire range of things interpretable by the Watcher. Using the geometric mechanics described above we can associate a shape to each semantic object simply by retaining the (in this case linear) geometries associated to each of the semantic components of a desired semantic object and combining them in a relevant way.

Once all the semantic elements under consideration have an associated geometry they can be organized into all the potential narrative paths (or stories) by linear logic using narrative construction dynamics and listed in some arbitrary fashion. From this point, the geometric representative for each semantic element in a given narrative will combine in a fashion maximizing the recoverability and transfer of knowledge (i.e. using wreath products). The result here is a list of potential stories each with a shape associated to it. It is functionally impossible to try and sift through the potential narratives in a purely semantic form, searching for narratives that make sense and are relevant.

The associated geometry we use instead is enacted by comparison against a set of target stories. These are the narratives which are consistent with the system’s accumulated "experience" and restricted to the scope of user-based relevance and some notion of "elegance." In the situated tactical intelligence case, these targets could be be associated with immanent danger to friendly soldiers, or civilians in terrorist scenarios.

The successful implementation of these fittedness measure techniques has far reaching implications in the general theory of narrative construction, as well as copious applications in the intelligence community.
have a lot of work to do on the fittedness, but we believe it holds promise for radical advances in real-world scenarios.

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