Solution Consistency and Convergence in Cooperative Distributed Problem Solving

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Abstract
A negotiation strategy for resolving conflicts in cooperative distributed problem solving is presented. The strategy, which is called Progressive Negotiation, aims at minimizing backtracking to previous solutions and provably guarantees the consistency of distributed solutions and the convergence on a globally-satisfiable solution among heterogeneous cooperating agents. The progressive negotiation strategy is enforced by a task-independent agent called Facilitator, which coordinates and controls the interaction of cooperating agents. The interaction of cooperating agents includes the communication of messages, the identification of conflicts, and the negotiation of conflicts as a way to resolve them. In this paper, we formally present our conceptualization of cooperating agents and their interaction via the facilitator. We next discuss the conflict types identified by agents and then present the progressive negotiation strategy for resolving conflicts. We then present two theorems that discuss solution discuss the consistency and convergence of distributed solutions ensured by the strategy. Finally, we conclude with a summary of this paper and remarks about the strategy.

1 Introduction
Conflict resolution is an essential requirement for cooperation of autonomous, intelligent, interacting agents [Adler 1989].

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vocabularies that are related by a set of predicate logic axioms, provided in the facilitator. After a formal description of cooperating agents and their interaction via the facilitator, we formalize conflict types and introduce the strategy of progressive negotiation for resolving conflicts. Then, we present theorems that discuss the solution consistency and convergence of the progressive negotiation strategy. Finally, the paper concludes with a summary and a few remarks about the strategy.

2 Cooperating Agents

A cooperating agent \( \alpha \) has knowledge \( K^\alpha \) as a set of predicate logic axioms, a set of criteria constraints \( C^\alpha \) as predicate logic axioms, and a database \( D^\alpha \) as a set of ground predicate logic atoms, all expressed over a vocabulary consisting of a set of predicate, function, and constant symbols, \( X^\alpha \). A cooperating agent \( \alpha \) has authority to make a final decision over a vocabulary \( Y^\alpha \subseteq X^\alpha \). Final decisions are those decisions that conclude a disagreement between agents. The goal of every cooperating agent \( \alpha \) is to find a complete local solution \( G^\alpha \) that is consistent with both of its knowledge \( K^\alpha \) and constraints \( C^\alpha \). Formally, this can be expressed as follows:

\[
G^\alpha \cup K^\alpha \cup C^\alpha \text{ is consistent.} \tag{1}
\]

At time \( t \), agent \( \alpha \) maintains a partial local solution, which consists of a set of predicate-logic atoms \( D^\alpha_t \). In the process of finding a solution, agent \( \alpha \) generates a set of assertions and retractions of atoms, \( V^\alpha \) that is consistent with its knowledge \( K^\xi_t \) and constraints \( C^\alpha_t \), and updates its partial local solution to \( D^\alpha_t \). Agent \( \alpha \) also updates its solution when it receives a set of messages reflecting assertions and retractions of predicate logic sentences. Formally the solution update can be expressed as follows:

\[
\forall \text{assertion}(v) \in V^\alpha_t \mid D^\alpha_t = D^\alpha_t \cup v, \quad \forall \text{retraction}(v) \in V^\alpha_t \mid D^\alpha_t = D^\alpha_t - v. \tag{2}
\]

3 Agent Interaction

Cooperating agents interact via a task-independent agent called facilitator, which coordinates and controls the exchange of messages. The facilitator captures the interests of agents and performs various functions aimed at facilitating the exchange of assertions and retractions of predicate-logic sentences. In the event of receiving messages, the facilitator determines the appropriate recipient agents of the messages and forwards them accordingly. In addition, it translates between vocabularies used by different agents in their exchange of sentences. The translation is achieved through the a set of predicate-logic axioms \( R^\xi \) over a subset of all agents' vocabularies \( X^\xi \subseteq X \).

Consider the group of agents \( \Gamma = \{ \alpha, \beta, \ldots, \zeta \} \) that are cooperating in solving a problem defined by agents’ knowledge \( K^\alpha, K^\beta, \ldots, K^\zeta \) over a vocabulary \( X \). The group of agents interact via the facilitator \( \phi \), which captures the agents' interests expressed as sets of predicate logic axioms \( I^\alpha, I^\beta, \ldots, I^\zeta \) and a set of translation axioms \( R^\xi \) over \( X^\xi \). This can formally be expressed as follows:

\[
X^\xi \subseteq X = X^\alpha \cup X^\beta \cup \ldots \cup X^\zeta, \\
K = K^\alpha \cup K^\beta \cup \ldots \cup K^\zeta \cup R^\xi, \quad \text{and} \\
I = I^\alpha \cup I^\beta \cup \ldots \cup I^\zeta. \tag{3}
\]

At time \( t \), when an agent \( \xi \in \Gamma \) generates messages \( V^\xi_t \), it updates its current local solution to \( D^\xi_t \) and then communicates \( V^\xi_t \) to the facilitator. When the facilitator receives the set of messages \( V^\xi_t \), it first deduces additional sentences based on the axioms \( R^\xi \). The result of this translation step is the set of sentences \( U^\xi_t \) whose number is typically greater than that in \( V^\xi_t \). This translation step can formally be expressed as follows:

\[
U^\xi_t = \text{sentences}(V^\xi_t) \cup R^\xi \text{ is consistent.} \tag{4}
\]

Then the facilitator checks for the agents that are interested in \( U^\xi_t \). For every \( \delta \in \Gamma -
\( \{ \xi \} \), if \( I^d \cup U^d \) is consistent, agent \( \delta \) is interested and is added to the set of interested agents \( \Delta \). For every interested agent \( \delta \in \Delta \), the facilitator forwards an appropriate set of messages \( W_i^\delta \). When agent \( \delta \) receives the set of messages \( W_i^\delta \), it updates its current local solution to \( D_i^\delta \).

4 Conflict Types
Agent \( \delta \) checks the consistency of the updated local solution \( D_i^\delta \) with respect to its knowledge \( K_i^\delta \) and constraints \( C_i^\delta \). If

\[
D_i^\delta \cup K_i^\delta \cup C_i^\delta \text{ is consistent,} \tag{5}
\]

there is no conflict and agent \( \delta \) accepts the messages. Otherwise, if

\[
D_i^\delta \cup K_i^\delta \cup C_i^\delta \text{ is inconsistent,} \tag{6}
\]

there is a conflict and agent \( \delta \) identifies the conflict as one of three types: critical conflict, non-critical conflict with authority, and non-critical conflict without authority. In this section, the three conflict types are formally discussed.

- **Critical Conflict:** A critical conflict is a conflict in which the updated solution based on the received messages is inconsistent with the agent’s knowledge \( K_i^\delta \). Formally, a critical conflict can be expressed as follows:

\[
D_i^\delta \cup K_i^\delta \text{ is inconsistent.} \tag{7}
\]

- **Non-Critical Conflict with Authority:** A non-critical conflict with authority is a conflict in which the updated solution is consistent with the agent’s knowledge \( K_i^\delta \), but inconsistent with the agent’s constraints \( C_i^\delta \), and the vocabularies of each sentence in \( W_i^\delta \) do not belong to \( X^\delta \) over which agent \( \delta \) has authority. Formally, this can be expressed as follows:

\[
D_i^\delta \cup C_i^\delta \text{ is inconsistent, and} \forall \text{message}(w) \in W_i^\delta \mid \text{vocabulary}(w) \in X^\delta. \tag{8}
\]

- **Non-Critical Conflict without Authority:** A non-critical conflict without authority is a conflict in which the updated solution is consistent with the agent’s knowledge \( K_i^\delta \), but inconsistent with the agent’s constraints \( C_i^\delta \), and the vocabularies of each sentence in \( W_i^\delta \) do not belong to \( X^\delta \) over which agent \( \delta \) has authority. Formally, this can be expressed as follows:

\[
D_i^\delta \cup C_i^\delta \text{ is inconsistent, and} \forall \text{message}(w) \in W_i^\delta \mid \text{vocabulary}(w) \notin X^\delta. \tag{9}
\]

5 Progressive Negotiation
Our conflict resolution research has focused on developing a strategy called progressive negotiation that minimizes backtracking to previous solutions. In this strategy, conflict resolution is carried out by agents. Depending on the type of conflict, negotiation takes place in an attempt to resolve the conflict. In this strategy, critical conflicts are always resolved because they result from the agent’s knowledge, which must be achieved in order to reach a satisfiable solution. Non-critical conflicts are resolved by getting one of the agents to relax some of its violated criteria constraints in order for an agreement to be reached. The following is a formal treatment of how conflicts are resolved for the three types outlined in the previous section.

5.1 Critical Conflicts
When a critical conflict is identified, agent \( \delta \) determines a set of axioms \( Q_i^\delta \subseteq K_i^\delta \) that caused the conflict and sends it to the facilitator. The facilitator in turn forwards appropriate axioms to the sending agent \( \xi \) and other agents in \( \Delta \) interested in \( Q_i^\delta \). Formally, the violated axioms can be expressed as follows:

\[
Q_i^\delta = \{ q \mid \forall q \in K_i^\delta \text{ such that} \}
\]
\[ D_i^g \cup q \text{ is inconsistent}. \] (10)

Once the sending agent \( \xi \) receives the set of axioms \( Q_i^g \), at time \( t' \), forwarded by the facilitator, it checks its consistency of the axioms \( Q_i^g \) with its knowledge \( K_i^g \). If \( Q_i^g \cup K_i^g \) is inconsistent, there is no solution that is consistent with the knowledge of both agents. If \( Q_i^g \cup K_i^g \) is consistent, however, then there could be a solution that is consistent with the knowledge of agents. In this case, the agent’s constraints are updated to \( C_i^g \) ensuring that \( C_i^g \cup Q_i^g \) is consistent. This update may involve relaxation of some constraints in \( C_i^g \). After updating its constraints, agent \( \xi \) generates new messages \( V_i^g \) and updates its local solution to \( D_i^g \), in a way that ensures the consistency of the updated solution with its knowledge and updated constraints. Formally, we can write:

\[ D_i^g \cup K_i^g \cup C_i^g \text{ is consistent.} \] (11)

The messages \( V_i^g \) are then sent to the facilitator, which forwards appropriate messages to all interested agents. At time \( t'' \), agent \( \delta \) receives the new messages \( W_{\delta}^g \), updates its local solution to \( D_{\delta}^g \). The new local solution, at time \( t'' \), for agent \( \delta \) will provably be consistent with the agent’s knowledge since \( K_{\delta}^g = K_{\delta}^g \). Formally, we can write:

\[ D_{\delta}^g \cup K_{\delta}^g \text{ is consistent.} \] (12)

5.2 Non-Critical Conflicts with Authority

When a critical conflict is identified, agent \( \delta \) determines a set of axioms \( P_{\delta}^g \subseteq K_{\delta}^g \) that caused the conflict and sends it to the facilitator. The facilitator in turn forwards appropriate axioms to the sending agent \( \xi \) and other agents in \( \Delta \) interested in \( P_{\delta}^g \). Formally, the violated axioms can be expressed as follows:

\[ P_{\delta}^g = \{ p \mid \forall p \in C_{\delta}^g \text{ such that } D_{\delta}^g \cup p \text{ is inconsistent} \}. \] (13)

Once the sending agent \( \xi \) receives the set of axioms \( P_{\delta}^g \), at time \( t' \) forwarded by the facilitator, it checks the consistency of the axioms \( P_{\delta}^g \) with its knowledge \( K_{\delta}^g \) knowing that \( K_{\delta}^g = K_{\delta}^g \). If \( P_{\delta}^g \cup K_{\delta}^g \) is inconsistent (i.e., the sets of violated axioms and the agent’s knowledge do not lead to a solution), agent \( \xi \) rejects the received set of axioms \( P_{\delta}^g \), and they are sent back to agent \( \delta \), which relaxes its set of constraints to \( C_{\delta}^g \) such that \( D_{\delta}^g \cup C_{\delta}^g \) is consistent since \( D_{\delta}^g = D_{\delta}^g \).

If \( P_{\delta}^g \cup K_{\delta}^g \) is consistent, however, then agent \( \xi \) relaxes some of its constraints to \( C_{\delta}^g \), if necessary, such that \( P_{\delta}^g \cup C_{\delta}^g \) is consistent. Then agent \( \xi \) generates new messages \( V_{\delta}^g \) and updates its local solution to \( D_{\delta}^g \), in a way that ensures the consistency of the updated solution with the agent’s knowledge and constraints,

\[ D_{\delta}^g \cup K_{\delta}^g \cup C_{\delta}^g \text{ is consistent.} \] (14)

The messages \( V_{\delta}^g \) are then sent to the facilitator, which forwards it to all interested agents. Agent \( \delta \) receives the new messages \( W_{\delta}^g \) at time \( t'' \), and updates its local solution to \( D_{\delta}^g \). The new local partial solution for agent \( \delta \) will provably be consistent with the agent’s knowledge and constraints since \( K_{\delta}^g = K_{\delta}^g \) and \( C_{\delta}^g = C_{\delta}^g \),

\[ D_{\delta}^g \cup K_{\delta}^g \cup C_{\delta}^g \text{ is consistent.} \] (15)

5.3 Non-Critical Conflicts without Authority

This case is similar to the case of non-critical conflicts with authority. The only difference is that when agent \( \xi \) receives the constraints \( P_{\delta}^g \) forwarded by the facilitator, it checks \( P_{\delta}^g \cup K_{\delta}^g \cup C_{\delta}^g \) knowing that \( K_{\delta}^g = K_{\delta}^g \) and \( C_{\delta}^g = C_{\delta}^g \). If \( P_{\delta}^g \cup K_{\delta}^g \cup C_{\delta}^g \) is inconsistent, agent \( \xi \) rejects the set of constraints, and they are sent back to agent \( \delta \), which relaxes its set of constraints to \( C_{\delta}^g \) such that \( D_{\delta}^g \cup C_{\delta}^g \) is consistent since \( D_{\delta}^g = D_{\delta}^g \).

If \( P_{\delta}^g \cup K_{\delta}^g \cup C_{\delta}^g \) is consistent, however, then agent \( \xi \) generates new messages \( V_{\delta}^g \)
and updates its local solution to $D_\delta^t$ in a way that ensures the consistency of its knowledge and constraints,

$$D_\delta^t \cup P_\delta^t \cup K_\delta^t \cup C_\delta^t \text{ is consistent.}$$ (16)

Again the messages $V_\delta^t$ are then sent to the facilitator, which forwards it to all interested agents. For agent $\delta$, the new messages $W_\delta^t$ are received at time $t''$ and the local solution is updated to $D'_\delta$. The new local solution for agent $\delta$ will provably satisfy the agent's knowledge since $K'_\delta \cup K_\delta \cup C'_\delta \cup C_\delta$ is consistent. (17)

6 Consistency and Convergence of Distributed Solutions

In this section, we present two theorems for proving the consistency of distributed local solutions and convergence on a common globally satisfiable solution under certain conditions for the progressive negotiation strategy. Before presenting these two theorems, we give a number of definitions that are used in the proofs.

Definition 6.1: Let $\xi$ denote an agent from the group of agents, $\xi \in \Gamma$. The interests of agent $\xi$ are said to be complete if and only if:

$$\forall x \in X_\xi, \text{ interest in } x \text{ is expressed in the set of axioms, } x \in I_\xi.$$ (18)

Definition 6.2: Let $K^a$, $K^b$, ..., $K^\zeta$, denote the knowledge for the group of agents $\Gamma = \{\alpha, \beta, ..., \zeta\}$ and $R^\phi$ denote the translation axioms captured in the facilitator $\phi$. The initial sets of axioms expressing agents' knowledge are said to be consistent if and only if:

$$K = K^a \cup K^b \cup ... \cup K^\zeta \cup R^\phi$$ is consistent. (19)

Definition 6.3: Let $C^a$, $C^b$, ..., $C^\zeta$ denote the initial sets of constraints for the group of agents $\Gamma = \{\alpha, \beta, ..., \zeta\}$. The initial sets of constraints are said to be consistent if and only if:

$$C = C^a \cup C^b \cup ... \cup C^\zeta \text{ is consistent.}$$ (20)

Definition 6.4: Let $Y^a$, $Y^b$, ..., $Y^\zeta$ respectively denote the authorities of the group of agents $\Gamma = \{\alpha, \beta, ..., \zeta\}$. The authorities of the group of agents $\Gamma$ are said to be disjoint (i.e., authorities over sets of interrelated vocabularies do not overlap) if and only if:

$$\forall \xi, \psi \in \Gamma \mid Y^\xi \subseteq X^\xi \land Y^\psi \subseteq X^\psi \Rightarrow Y^\xi \cap Y^\psi = \emptyset, \text{ and}$$

$$\forall r \in R^\phi \text{ defined over } Z^\phi \subseteq X^\phi \land$$

$$\forall \xi, \psi \in \Gamma \mid Y^\xi \cap Z^\psi \neq \emptyset \land$$

$$Y^\psi \cap Z^\psi = \emptyset.$$ (22)

6.1 Consistency of Distributed Solutions

In this section, a theorem that states the consistency of distributed local solutions for the progressive negotiation strategy is presented.

Theorem 6.1: For the group of agents $\Gamma$, if the interests of the agents are complete (which is usually the case), the progressive negotiation strategy guarantees the consistency of the distributed local solutions generated after every exchange among agents. Formally, the distributed local solutions after an exchange at time $tn$ can be expressed as follows:

$$D_{in} = D_{in}^a \cup D_{in}^b \cup ... \cup D_{in}^\zeta \text{ is consistent.}$$ (23)

Proof: Suppose that $D_{in} = D_{in}^a \cup D_{in}^b \cup ... \cup D_{in}^\zeta$ is inconsistent, then there must exist at least two local solutions $D_{in}^a$ and $D_{in}^b$ such that
$D^a \cup D^\beta$ is inconsistent. \hspace{1cm} (24)

This means that there are at least two ground atoms, $d^a$ and $d^\beta$, in the solutions $D^a$ and $D^\beta$ respectively such that $d^a = \neg d^\beta$, which in turn, means that one of the agents $\alpha$ or $\beta$ did not receive a message about it or update its solution. This is not possible because it contradicts the solution update step of agent interaction (2). Therefore, there cannot be any two partial solutions such that $D^a \cup D^\beta$ is inconsistent. Thus, $D = D^a \cup D^\beta \cup ... \cup D^\xi$ is always consistent according to the progressive negotiation strategy.

6.2 Convergence on a Common Solution

This section presents a theorem that discusses the convergence of the progressive negotiation strategy.

**Theorem 6.2:** For the group of agents $\Gamma$, if the interests are complete (which is usually the case), and the sets of axioms expressing agents’ knowledge are consistent, whereas the set of initial axioms expressing agents’ constraints is not necessarily consistent, and the authorities are exhaustive and disjoint, then the progressive negotiation strategy guarantees convergence on a common solution that satisfies all agents’ knowledge, and a relaxed version of the initial constraints. Formally, the following must be proven:

\[ D \cup K^\alpha \cup K^\beta \cup ... \cup K^\xi \cup R^\phi \]

is consistent. \hspace{1cm} (25)

**Proof:** Suppose that $D \cup K^\alpha \cup K^\beta \cup ... \cup K^\xi \cup R^\phi$ is inconsistent. Thus, one can write:

\[ D^a \cup D^\beta \cup ... \cup D^\xi \cup K^\alpha \cup K^\beta \cup ... \cup K^\xi \cup R^\phi \]

is inconsistent. \hspace{1cm} (26)

Given that $K^\alpha \cup K^\beta \cup ... \cup K^\xi \cup R^\phi$ is consistent as stated in Definition 6.2 and having proved that $D^a \cup D^\beta \cup ... \cup D^\xi$ is always consistent in Theorem 6.1, one can conclude that equation (26) can hold if and only if either of two possibilities holds:

- There exists at least one agent $\xi \in \Gamma$ such that its local solution is inconsistent with the facilitator’s translation axioms: $D^\xi \cup R^\phi$, or
- There exist at least two agents $\xi, \psi \in \Gamma$ such that their local solutions are inconsistent with their knowledge: $D^\xi \cup D^\psi \cup K^\xi \cup K^\psi$.

Case 1: In order for $D^\xi \cup R^\phi$ to be inconsistent, there must be sent or received messages (respectively $V^\xi$ or $W^\xi$) that are inconsistent with $R^\phi$ since the local solution atoms are typically updates of atoms in $V^\xi$ or $W^\xi$. Formally, one of the following must be true:

\[ V^\xi \cup R^\phi \text{ is inconsistent, or} \]
\[ W^\xi \cup R^\phi \text{ is inconsistent.} \hspace{1cm} (27) \]

This is not possible because of the facilitator’s translation step expressed in equation (10) and the completeness of agents’ interests. Therefore, $D^\xi \cup R^\phi$ is consistent and there cannot be any agent whose solution does not satisfy the facilitator’s translation axioms.

Case 2: $D^\xi \cup D^\psi \cup K^\xi \cup K^\psi$ can never be inconsistent because $D^\xi \cup D^\psi$ is consistent from Theorem 6.1, $K^\xi \cup K^\psi$ is consistent from Definition 6.2, and $D^\xi \cup K^\xi$ is consistent and $D^\psi \cup K^\psi$ is consistent from equations (11, 12, 14, 15, 16, and 17) and other equations of the progressive negotiation strategy. Therefore, there cannot be any agent whose local solution is inconsistent with other agents’ knowledge. Thus, $D^\alpha \cup D^\beta \cup ... \cup D^\xi \cup K^\alpha \cup K^\beta \cup ... \cup K^\xi \cup R^\phi$ is always consistent and the solution convergence of the progressive negotiation strategy is guaranteed.

7 Summary and Concluding Remarks

In this paper, we formally presented a conflict resolution strategy called
Progressive Negotiation that guarantees the consistency of distributed solutions and convergence on a globally-consistent satisfiable solution. The strategy involves communication of predicate-logic axioms to alter distributed agents' local solutions incrementally to reach a globally-satisfiable solutions. It aims at minimizing backtracking to previous solutions by getting agents to communicate their violated axioms and thus to inform all agents involved in a conflict about those axioms, which ensures they will not commit that violation again. The strategy assumes that interacting agents exchange their messages via a task-independent agent called facilitator, which controls the exchange of messages in such a way that ensures the satisfaction of agents knowledge and constraints with respect to their current local solutions. The theorems presented for proving convergence of the progressive negotiation strategy show the conditions under which a group of distributed cooperating heterogeneous agents are guaranteed to reach an agreement in the course of negotiation. It is possible that an agreement among agents can be reached in some cases even if they do not comply with all the convergence conditions. However, if these conditions are not followed, convergence is not guaranteed in every exchange.

References