# Quantifying the Search Space for Multi-Agent System (MAS) Decision-Making Organizations

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**Abstract**

When a group comes together to pursue a goal, how should the group interact? Both theory and practice show no single organization always performs best; the best organization depends on context. Therefore, a group should adapt how it interacts to fit the situation. In a Multi-Agent System (MAS), a *Decision-Making Framework* (DMF) specifies the allocation of decision-making and action-execution responsibilities for a set of goals among agents within the MAS. *Adaptive Decision-Making Frameworks* (ADMF) is the ability to change the DMF, changing which agents are responsible for decision-making and action-execution for a set of goals. Prior research embedded ADMF capabilities within an agent to search, evaluate, select, and establish a DMF for a given situation and given goal(s) the agent sought to achieve. Using the ADMF capability, the Multi-Agent System improved system performance compared to using the same, *static* Decision-Making Framework (DMF) in all situations (Martin, 2001). While the motivation for an agent possessing ADMF has been proven and an example MAS system with agents employing ADMF has been built, interesting questions arise as one investigates the ability of an agent to find the "best," or "near optimal" or "sufficient" DMF among all the possible DMFs. This paper presents initial exploration of this investigation by asking, "How large is the DMF search space for an agent?" This paper presents tight computational bounds on the size of the search space for Decision-Making Frameworks by applying combinatorial mathematics. The DMF representation is also shown to be a factor in the size of this search space.

## Introduction

How many ways can agents collaborate to make group decisions? Prior research has qualified the Multi-Agent System (MAS) performance advantages gained when agents dynamically form groups and allocate decision-making and action-execution responsibilities in response to changing situations (Martin, 2001). If *decision-making* is the process of creating, selecting, and allocating sub-goals (or actions) to achieve a goal, then Decision-Making Frameworks (DMFs) have been defined as the organization constructed to carry out Decision-making. As an agent seeks to select a DMF, the agent may search over the possible decision-making organizations within a given Multi-Agent System. An agent may search all DMFs to find the optimal DMF or may search some DMFs to find a sufficient DMF. The tradeoff between the cost of finding a DMF and the benefit or utility of using a particular DMF is a real consideration. Thus, this paper investigates the size of the DMF search space as a factor in the cost of finding a DMF.

This paper outlines the algorithmic steps to implement the *Adaptive Decision-Making Framework* (ADMF) capability – the ability to search, evaluate, select, and establish a DMF given a situation and specific goal(s) to be achieved by the DMF. By enumerating the steps, the analysis can characterize the run-time complexity of ADMF algorithms.

To understand the computational complexity of ADMF, one must understand the qualitative size of the space (the number of candidate DMFs) that ADMF algorithms must explore. This paper examines the state space size as a function of the DMF representation.

A Decision-Making Framework (DMF) specifies how agents work together to achieve a given set of goals. A particular DMF representation has been previously defined as an assignment of variables in three sets, **({D}, {C}, {G})** (Barber and Martin, 2001b). This *Decision-Making Framework (DMF)* representation models the set of agents **{D}** deciding a set of goals for another, controlled, set of agents **{C}**, which are bound to accept sub-goals to accomplish the goal set **{G}**. This model specifies the agent's decision-making interaction style, controlling how that agent participates in the decision-making process for some goal set. For example, a "master/command-driven" DMF in which an agent, Agent1, acts as the deciding agent and other agents, Agent2 and Agent3, are controlled by Agent1 to accomplish a goal, G1, would be represented by the (D = {Agent1}, C = {Agent2, Agent3}, G = {G1}) assignment. The set of DMFs covering all goals in the system is the *Global Decision-Making Framework*, denoted by GDMF. The GDMF is the **({D}, {C}, {G})** DMF assignments such that all goals are in exactly one

DMF, but one **({D}, {C}, {G})** assignment may apply to multiple goals.

This paper addresses the questions: "How do Decision-Making Frameworks combine in Global Decision-Making Frameworks?" and "What are the bounds on the size of the space of Global Decision-Making Frameworks for a given number of goals and agents?" Next, the paper will explore the motivation for considering different factors (in addition to D and C) that may influence the performance of Decision-Making Frameworks. Finally, we consider the consequences of considering additional factors in terms of search space size and strategies for finding satisficing Decision-Making Frameworks.

## Decision-Making Framework and Multi-Agent System Definitions

A *Multi-Agent System* (MAS) is a group of distributed software programs, called *agents,* which interact to achieve a goal(s) (Weiss, 1999). An *agent* is able to sense its environment, reactively or deliberatively plan actions given inferred environmental state and a set of goals, and execute actions to change the environment. Agents in a Multi-Agent System (MAS) can interact in sensing, planning, and executing. The study of Multi-Agent Systems examines individual agent- and system-level behavior of interacting software agents.

A *MAS decision-making organization*, is an instantiated set of agents working together on a set of goals, the (explicit and implicit) decision-making protocols used to select how to accomplish the goals, and the coordination protocols used to execute the decided actions. That is, any given decision-making organization is a particular set of agents using some particular protocols to decide and enact a particular set of goals. The term *organization* will be used in this paper as shorthand for *MAS decision-making organization.*

A *Decision-Making Framework (DMF)* specifies the set of agents interacting to achieve a set of goals (Barber and Martin, 2001b). A Decision-Making Framework consists of (1) the *decision-making control set* {**D**} specifying which agents make decisions about the goals, (2) the *authority-over set* {**C**} specifying which agents must carry out the decisions made, and (3) the set of goals {**G**} under consideration. Agents form a DMF for one or more goals and an agent may participate in multiple DMFs for different goals simultaneously. Decision-Making Interaction Styles (DMIS) describe how an individual agent participates in a DMF. Four Decision-Making Interaction Styles are:

- **Command-Driven (CD)–** The agent does not make decisions about how to pursue this goal set and must obey orders from its Master agent(s).

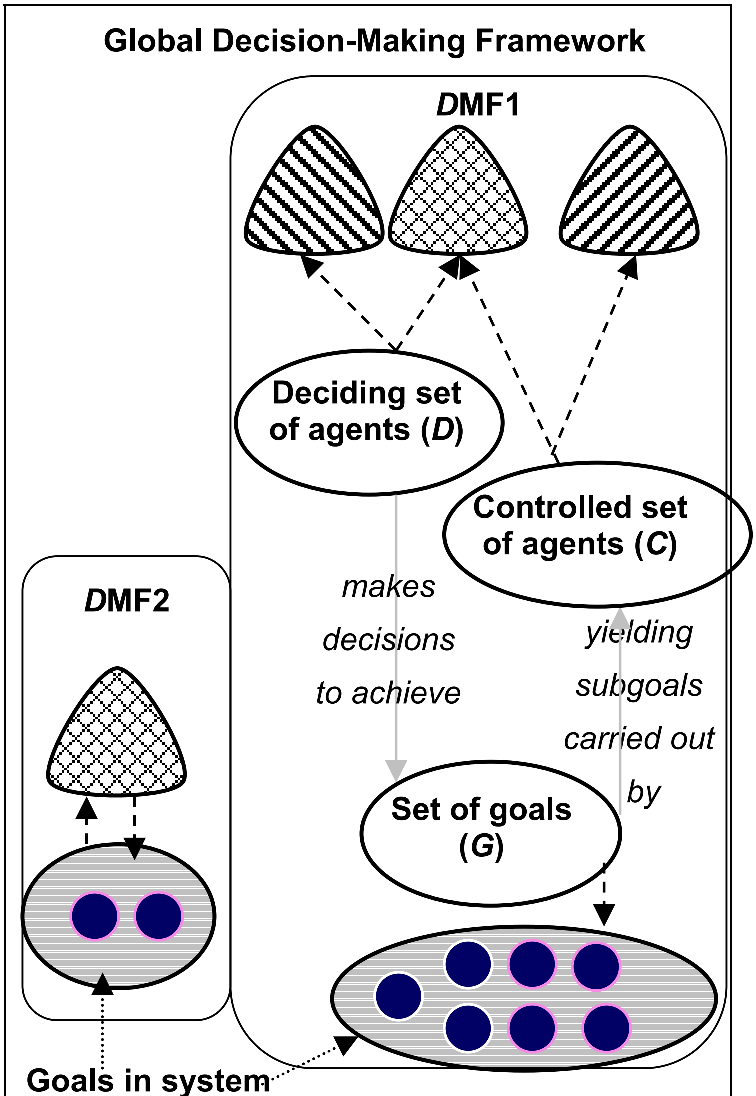


**Figure 1: Decision-Making Frameworks in a Global Decision-Making Framework**

- **Consensus (CN) –** The agent works as a team member, sharing decision-making control and acting responsibility equally with all agents in the DMF.
- **Locally Autonomous (LA) / Master (M) –** This agent alone makes decisions for these goals. Masters give other agents orders, while Locally Autonomous agents act alone.

A single Decision-Making Framework is composed of a coherent set of individual Decision-Making Interaction Styles for all participating agents (e.g. Master/ Command-Driven or all Consensus frameworks).

A *Global Decision-Making Framework (GDMF)* is a partition of the system's goal and agent set into DMFs so that, at any time, each goal actively under consideration by an agent in the system is in exactly one DMF. The degenerate case of a DMF is an agent working alone on a goal, which is a Locally Autonomous DMF.

**Error! Reference source not found.** illustrates the DMF and GDMF relations. Figure 1 shows a Global Decision-Making Framework consisting of two Decision-Making Frameworks. *DMF1* has two agents controlling the decision-making for seven goals and two agents bound to accept sub-goals. One of the agents is both in the decision-making set and the controlled set. The second DMF has one agent deciding and executing the other two intended goals in the system.

Multi-agent systems capable of using *Adaptive Decision-Making Frameworks (ADMF)* have the ability to change Decision-Making Frameworks during operation (Barber and Martin, 2001b). Multi-agent systems incapable of ADMF use a *static Decision-Making Framework* (established prior to system start-up) throughout system operation.

## Adaptive Decision-Making Framework (ADMF) Process

The ADMF process consists of five steps (Martin, 2001). This process is described from an agent's perspective as the agent works to construct a DMF for one or more of its goals.

- **Search:** The agent searches among candidate DMFs. The ADMF algorithm will determine the extent to which the agent searches among all or some of the possible DMFs.
- **Evaluation:** The agent uses some metric to evaluate the candidate DMFs.
- **Selection:** The agent selects a DMF based on whether the cost of implementing the DMF, and perhaps breaking current commitments to other DMFs, is worthwhile.
- **Negotiation:** The agent proposes the desired DMF to other agents who are proposed as members of {D} and {C} in the desired DMF. The other agents run an ADMF algorithm to decide if the DMF is attractive and evaluate alternatives. Each agent may counter-propose. The negotiation can continue until unanimity is reached. An example negotiation protocol is provided by (Barber et al., 2001).
- **Instantiation:** The agents begin to make decisions and act based on the chosen DMF.

Note that the ADMF process is itself a Multi-Agent System decision. The algorithms implementing ADMF in software systems have used two DMFs in effect, working alone at first to evaluate DMF and then in a unanimous consensus negotiation to select and form multi-agent DMFs.

Each of the ADMF steps implies agent design decisions; for instance, design decisions impact the search algorithm, the evaluation metrics, and the negotiation protocol. This research examines how some of these decisions are constrained by the nature of the ADMF problem. Specifically, how many organizations can be encoded by different representations of Decision-Making Frameworks? Again, we must limit the question to a finite number of goals and agents. In order to scope the difficulty of searching for the optimal organization, one must know how many DMFs are possible. Even for a given number of goals and agents, the class of DMFs may be at least countably infinite. As a proof, consider a DMF characterized by constituent member agents and the decision-making voting strength of each agent. Since the voting strength could be represented by a rational number (for instance, some number of voting shares out of a total number) and the set of rational numbers is countably infinite, the set of DMFs can be countably infinite in this case. Some combinatorial math provides estimates of the size of the DMF search space for two DMF representations. This research presents a computational analysis placing tight bounds on the sizes of different spaces of possible DMFs given the representational constraints. Given these bounds, we can state how much a Decision-Making Framework representation limits the search space of possible decision-making organizations.

## Size of the Decision-Making Framework Space

Under the Decision-Making Framework representation presented by Barber and Martin (Barber and Martin, 2001a), each agent has either one or no votes to influence the decision about how to solve a goal and is either entirely or not at all committed to execute the group's decision. That is, controlled agents **C** have agreed to accept any sub-goals the deciding set of agents **D** proposes. Thus, for one goal, or one arbitrary set of goals, a DMF could be represented by two bits for each agent in the MAS: one bit representing participation in the decision-making set of agents and the other representing the presence or absence of commitment to execute the decision. So the entire DMF for a goal set in a MAS with **n** agents could be represented by 2 **n** bits, **n** for whether each agent is in the deciding set, and **n** whether each agent is in the executing set. Thus, for a single DMF, there are $2^{2\mathbf{n}}$ combinations of deciding and executing sets of agents.

## Size of Related Decision-Making Organization Representations

The DMF representation can easily be extended to cover different numbers of votes per agent and different commitment levels. Instead of encoding the agents' membership in the deciding or enacting sets with one bit, one can encode arbitrary granularity of decision strength and commitment with multiple bits per agent. A simple scheme represents different levels of voting strength by counting the number **v** of different voting strengths in the system and representing **v** as a binary number. For instance, 2 bits could represent a system where each agent holds exactly 1, 5, 10, or 100 voting shares. An analogous scheme could represent levels of commitment to executing the decided goal. Thus, in the general case, for **d** possible deciding agents and **c** possible controlled agents with at most **v** gradations of voting strength and at most **u** gradations of commitment, there are $\mathbf{2^{d \log v + c \log u}}$ of voting-strength, commitment-strength DMFs. Allowing all **n** agents to each have zero or one vote and all-or-none commitment, this reduces to the $2^{2n}$ derived above for DMFs.

## Size of the Global Decision-Making Framework (GDMF) Space

How many different combinations of instantiated Decision-Making Frameworks are possible in a Global Decision-Making Framework? Note that a GDMF can be represented as a concatenation of the bit strings for the individual DMFs. Thus, a GDMF made up of **m** DMFs can be represented by a string of **m** bit strings, each of which in the simplest case is 2**n** bits long, where **n** is the number of agents. Since the entire bit string is 2**mn** bits long, there are $2^{2mn}$ combinations of DMFs in a GDMF for **m** goals and **n** agents where each goal is in its own DMF. For instance, the number of combinations in a DMF with eight DMFs over eight agents and eight goals is at least $2^{2*8*8} = 2^{128} \sim= 3.4 \times 10^{38}$.

However, more than one goal may be in a Decision-Making Framework. If some of the goals are grouped into a DMF, and each goal may only be in one DMF at a time, then any GDMF with goals grouped in a DMF will have fewer DMFs in it. The combinations within a GDMF are exponential in the size of DMFs. For example, if $GDMF_1$ contained **m** goals and **n** agents with one constituent DMF sharing two goals, $GDMF_1$ will be half the size of $GDMF_2$ if $GDMF_2$ contains a separate DMF for each goal (given the same number of goals and agents in $GDMF_2$ and $GDMF_1$). By induction and the sum of exponentials, the state space size of a GDMF is asymptotically $O(2^{2mn+1})$, where for **n** agents and **m** goals in the simple case.

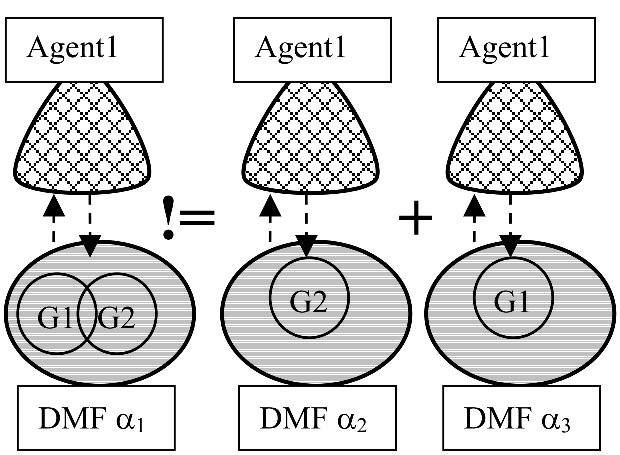


**Figure 2: A DMF is a contract, not just a collection of agents and goals.**

Now consider a representation for the Global Decision-Making Framework (GDMF) for DMFs with heterogeneous levels of voting strength and commitment level. As above, each agent has one of **v** voting strengths and one of **u** commitment levels to act to fulfill the decided sub-goals.

Let {V} be the set of possible voting strengths for each agent in {D} within a DMF and {U} be the set of possible commitment levels for each agent in {C} within a DMF. Let the *Control and Commitment Allocation (CCA)*, be the tuple ({V}, {U}). Therefore, the number of combinations of DMFs with heterogeneous voting and commitment strength in a GDMF is $\mathbf{O(2^{(d \log v + c \log u)m+1})}$, for **d** deciding agents with **v** gradations of voting strength, **c** controlled agents with **u** gradations of commitment, and **m** goals.

However, a DMF $\alpha_1$ containing two goals $G_1$ and $G_2$ is not functionally the same as two simultaneous DMFs $\alpha_2$ and $\alpha_3$ each containing one of $\alpha_1$'s two goals and sharing the same Control and Commitment Allocation as $\alpha_1$ (Figure 2). In $\alpha_1$, the two goals must be decided together, but in $\alpha_2$ and $\alpha_3$, the agents could decide on a solution for one goal without necessarily considering the other at that time. Therefore, we must distinguish between the cases when one DMF acts as a group to pursue more than one goal versus separate DMFs, each with identical Control and Commitment Allocation in ({V}, {U}) for each goal.

Additionally, the performance of a DMF in a GDMF depends on its interaction with the other DMFs, for both cooperative and competitive agents. For instance, cooperative agents may select an existing DMF as opposed to new DMF since the agents have already established knowledge about the existing DMF performance. On the other hand, as game theory shows, the performance of a problem-solving strategy depends on the strategies of competing groups (Axelrod, 1984) (Binmore, 1992). Likewise, the performance under a DMF for one group of agents depends on the DMFs of competing groups of agents for competing goals (Martin, 2001). The utility of each individual DMF depends on the other DMFs in the Global Decision-Making Framework, and of course, the utility of the GDMF in a situation is the utility of its constituent DMFs.

To calculate the exact size of the state space of Global Decision-Making Frameworks (GDMF), we must first figure out how many different ways a set of goals can be split into DMFs such that every goal is in exactly one DMF. This is equivalent to partitioning the set, i.e. breaking the set into subsets such that the set is covered. The Bell number ($B_n$) of a set with $n$ elements is the number of ways the set can be partitioned into disjoint, non-empty subsets (Dickau, 1997). A disjoint, non-empty subset is called a *block*. The nth Bell number is the sum of the first n Stirling numbers of the second kind: $\mathbf{B_n = \Sigma_{k=0}^{n} S(_n^k)}$**, where** $\mathbf{S(_n^k) = S(_{n-1}^{\ k-1}) + K\ S(_{n-1}^{\ k})}$**.** The first ten Bell numbers are {1, 2, 5, 15, 52, 203, 1877, 4140, 21147, 155975}. The total number of disjoint DMFs possible over the space of GDMFs is the Bell number $B_m$ for m goals. For instance, for 8 goals, there are 4140 different sets of partitions of goals into disjoint DMFs.

The size of the GDMF space in terms of permutations of agents and goals is the sum across the Bell number of different goal partitions is $B_{m}*(2^{2n})$. For eight agents with a total of eight goals, the formula is $4{,}140*2^{2*8} \sim= 2^{12}*2^{16} = 2^{28} \sim= 10^{8}$. For nine agents with a total of nine goals, the formula is $\mathbf{21{,}147*2^{2*9} \sim= 2^{14}*2^{18} = 2^{32} \sim= 10^{9}}$. For ten agents with a total of ten goals, the formula is $\mathbf{155{,}975*2^{2*10} \sim= 2^{17}*2^{20} = 2^{37} \sim= 10^{11}}$.

The Lovasz approximation of $B_n$ is $\mathbf{n^{-05}(\lambda(n))^{n+0.5}e^{\lambda(n)-n-1}}$, where $\mathbf{\lambda(n)\ln(\lambda(n)) = n}$ (Lovász, 1979).

Given this approximation, $B_n$, the size of the GDMF space is exponential in the number of goals. Given the bound on DMF size, the GDMF space is also exponential in the number of agents. Therefore, no brute-force algorithm can search the space in polynomial time.

Brainov and Hexmoor show that finding a group with the maximum *autonomy* is NP Complete through a reduction to the subset sum problem (Brainov and Hexmoor, 2001). By autonomy, they mean the relative comparison of group performance to the performance of each agent working alone. Thus, Brainov and Hexmoor had the same goal as this research – finding the set of agents that work well together to maximize the group's performance. Brainov and Hexmoor's proof of NP-Completeness also holds here. We can reduce our problem to theirs by taking the group of agents working together to be the disjunction of **D** and **C**. Therefore, the AMDF problem is also NP-complete.

D'Inverno, Luck, and Wooldridge show a similar problem is NP-Complete (D'Inverno et al., 1997). They define a *Structure*, which is a representation of which agents cooperate for which goals. A *Cooperation Structure* is a Structure that fulfills a set of properties, e.g. that it is non-empty, acyclic, and the goals contained are mutually consistent. A Cooperation Structure is *complete for an agent i and goal g* if recursively, all sub-goals of that goal are instantiated and successfully delegated to some agent or the agent itself is capable of achieving that goal in isolation. A complete Cooperation Structure is thus roughly equivalent to the recursive closure on sub-goals of the set of all Decision-Making Frameworks (DMFs) needed to satisfy the intended goal set. D'Inverno, Luck, and Wooldridge define *COOPSAT as* the problem of finding a complete Cooperation Structure, given the agents, goals, goal consistencies, sub-goal relations, agent willingness to cooperate with each other agent, and agent capabilities. They show COOPSAT is NP-Complete through a reduction to the Hamiltonian Cycle problem.

## Maximizing ADMF Benefits and Minimizing the Cost of ADMF

Intelligence has been defined as the ability to control or avoid combinatorial state space explosion. How can we control the combinatorial explosion of the size of the Global Decision-Making Framework space? The easiest solution, of course, is not to search at all. In this case the benefit (performance) of one single GDMF (defined by the constituent DMFs) must be adequate across all occurring situations, or the cost-benefit tradeoff of switching does not benefit the agent. Another simple solution is to search a tightly bounded subset of the space, but this subset should be chosen in a principled way if the system designer wants to prove (near) optimality.

Perhaps the simplest method of providing an agent the Adaptive Decision-Making Frameworks (ADMF) capability is through a simple rule base suggesting certain DMFs when certain aspects of the problem occur. For instance, Martin found that extremely simple rule bases performed comparatively to a comprehensive case-based approach for choosing GDMFs out of a set of five GDMFs (Martin, 2001). It should be noted that Martin derived this simple rule base after she conducted significant analysis of the domain and DMF performance against multiple and varied situations in the domain.

A rule base must be crafted with an understanding of the interaction of agents in situated problem solving for each particular domain. Creating a rule set that can be justified analytically is an open problem, and comprehensively empirically testing rule bases will be intractable in many domains. The simple rule base approach is equivalent to Gigerenzer's fast and frugal heuristics (Gigerenzer, 2000). For instance, one simple maxim is that DMFs requiring communication usually perform poorly when communication is cut off. Indeed, this rule alone provided much of the benefit of ADMF in Martin's experiments (Martin, 2001).

A case-based approach requires a distance metric to match new cases to similar previously seen cases. For provably optimal selections, the distance metric must always accurately distinguish what aspects of the situation affect the utility of a GDMF. The problem of designing a distance metric may be no easier than designing a rule base. Additionally, even given a perfect distance metric, to pick the "best" DMF, the case base must either include the optimal set of DMFs in every possible class of situations, or be able to project the optimal DMF for each goal of each agent in each situation from the limited set of cases stored.

Using another track, instead of applying intelligence to selectively searching the large state space, we can apply intelligence to limit the state space size itself. All three sets of the DMF are targets to limit search: (1) the deciding agents, (2) the controlled agents, and (3) the goals. For instance, Martin's ADMF implementation limits the numbers of goals considered (Martin, 2001). Each agent begins considering only its own goals. Only if another agent suggests forming a DMF does an agent consider adding other goals. Thus, the goal state space is initially small and iteratively grows until a satisficing DMF is found. While this approach is computationally efficient and performs better than static DMFs, it has not been shown to discover optimal DMFs.

The best strategy for constraining the size of the DMF search space depends on the relative proportion of goals and agents. As shown above, the search space is $2^{(\mathbf{d} \log \mathbf{v} + \mathbf{c} \log \mathbf{u})\mathbf{m}}$ for **d** deciding agents with **v** gradations of voting strength, **c** controlled agents with **u** gradations of commitment, and **m** goals. Since the terms are multiplied in the exponent, any reduction in either will provide exponential gains in search. Obviously, limiting the gradations of voting strength and level of commitment limit the search space. The simple case of equal or no voting strength representation in the DMF has a linear relationship between **d** + **c** and **m**. Therefore, the most benefit is gained by reducing the number of agents under consideration if **d** + **c** < **m** and reducing the number of goals if **d** + **c** < **m**.

Not surprisingly, many methods of limiting the search space come naturally to people. For instance, by partitioning the space of agents to sets **C** and **D** independent of goals, one reduces both **d** and **c**. This partition, in effect, creates a "ruling class" and an "under-class" of agents, although the sets need not be disjoint. Fixed multiple level hierarchies reduce the search space for each agent even further. Furthermore, if the hierarchy is horizontally specialized, then the search space is reduced since only a limited number of agents will have the expertise to be responsible for certain classes of goals, and conversely, the number of goals to be considered for grouping into a DMF can sometimes be constrained to a subset of goals. Thus, in relatively static environments, fixed hierarchies can have performance advantages by reducing the effort agents require to find satisficing DMFs. A similar mechanism caches successful decision-making structures from the past, as in the case-based reasoning approach. Brooks and Durfee show that emergent fixed size *congregations* of agents have constant performance as the number of agents scales in an information sharing domain, with the best number of congregations shown empirically to be one-half the number of agents (Brooks and Durfee, 2002).

Another state-space limiting mechanism is identifying agents not as individuals, but by roles. For instance, when an agent needs an accounting task done, the agent need not consider every other agent in the system to accomplish the task, but only the accountant agents, again narrowing the search space (but requiring the typing agents by roles and the definitions of relationships between goals and role types). Alternatively, one can consider agents with the same role as interchangeable, or members of an equivalence class, and only evaluate the worth of the class once, instead of once per agent. (Rovatsos and Wolf, 2002) (Barbuceanu, 1997) (Tambe, 1997) Note that roles are orthogonal to organizations: an agent working within a task-specialized group may send a task to the accounting department, while a locally autonomous agent can look up accountant agents in a directory, or "yellow pages" service.

The size of the space of Global Decision-Making Frameworks is intractably large for an exhaustive search or enumeration. Brute-force methods of adapting Decision-Making Frameworks will not suffice for more than a few agents with a few goals. The space of possible organizations is extremely large with many factors that may possibly affect Decision-Making Framework performance. Learned and designer-implemented case- and rule- bases must use a principled approach using an understanding of the problem, situation, and multi-agent system structures to select Decision-Making Frameworks.

## Conclusion

This paper quantifies the problem of an agent searching for a decision-making organization to best fit a situation by quantifying the search space of possible organizations given the respective decision-making organizational representation. First, the space of all possible Multi-Agent System decision-making organizations can be shown to be infinite, even for finite numbers of agents and goals, due to the infinite allocations of voting and commitment strength among agents in the organization. Therefore, any search must constrain the space of organizations in some way. This paper explored the size of the search space containing all possible organizations given respective models for representing/describing decision-making organizations. As the representation constraining the decision-making organization description changes, the size of possible organization to be explored also changes.

A *Decision-Making Framework,* defined in prior work (Barber and Martin, 2001b), represents an organization purely by which agents decide the allocation of sub-goals for which goals to which set of agents. The Decision-Making Framework (DMF) space for any one goal set for a given group of **n** candidate deciding and enacting agents is $2^{\mathbf{n}}$, if all agents can be in both the deciding and the controlled sets. Modeling **v** gradations of agent voting strength in the DMF and **u** gradations of commitment to the DMF increases the number of possible organizations for a single goal set to $\mathbf{2^{d \log v + c \log u}}$.

A *Global Decision-Making Framework (GDMF)* is the partitioning of the set of intended goals into distinct Decision-Making Frameworks. A weak bound is calculated on the number of distinct sets of decision-making organizations in a GDMF as $\mathbf{O}\ (\mathbf{2^{(d \log v + c \log u)m+1}})$, for **d** deciding agents with **v** gradations of voting strength, **c** controlled agents with **u** gradations of commitment, and **m** goals. A tighter bound is possible based on the combinatorial mathematics concept of the *Bell number*, which is the number of ways a set can be partitioned, or divided into non-empty subsets. This bound is $\theta(B_{m^*}(2^{2n}))$, where **n** is the number of agents in the system and **m** is the number of goals. $B_m$ is exponential in the size of **m**, so the number of decision-making organizations in a GMDF is exponential in both the number of agents and the number of goals.

Several strategies are proposed for avoiding this combinatorial explosion. First, the system may use a heuristic rule base to select an organization quickly. A case base with a well-designed distance metric and a good strategy for choosing which cases to remember may also beneficially limit the organization set for consideration. In domains it is possible to create a compact normative theory of how agents should interact, which can guide the ADMF capability more reliably than fast and frugal heuristics. Finally, as in many problems in the NP domain, a greedy search in a limited domain may be quite effective. Martin used a greedy algorithm to add agents until a DMF with sufficient predicted performance was found for respective goal(s) (Martin, 2001). Others have shown the utility of arbitrarily limiting the number of organizations or using roles to limit the agents considered to accomplish a specific goal (Brooks and Durfee, 2002) (Rovatsos and Wolf, 2002) (Barbuceanu, 1997) (Tambe, 1997).

Like many problems in Artificial Intelligence, finding an optimal Decision-Making Framework is NP-Complete. This paper presents tight bounds on the space of organizations under several different DMF representations. The difficulty of finding the absolute best performing organization also leads to accepting approximation of the best or even merely sufficient decision-making organizations. Future work includes the development and characterization of algorithms acknowledging the nature of the space.

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