

Auctioning Contracts in a Task Allocation among Self-interested Agents

Shigeo Matsubara

NTT Communication Science Laboratories

NTT Corporation

2-4 Hikaridai, Seika-cho, Soraku-gun, Kyoto 619-0237 Japan

matsubara@cslab.kecl.ntt.co.jp

Abstract

We propose a new mechanism for a task allocation problem among self-interested agents. This paper focuses on the process after the task is allocated as well as the task allocation process itself. That is, in the former process, a contractor is not necessarily motivated to attain the task at a sufficient level if the outcome partially depends on factors outside the contractor's behavior. Although this problem may happen, e.g., in content delivery services in peer-to-peer networks, previous research efforts have not given attention to this problem. To keep the quality of a task achievement at a sufficient level, we have to find an efficient allocation of tasks and induce each contractor's effort. However, solving this problem is difficult because the contractee cannot ascertain each contractor's effort or the contractor's capabilities in handling the task. To solve this problem, we propose a new mechanism that auctions contracts. More specifically, the mechanism first finds an efficient allocation of the tasks and then calculates a contract based on the result of the auction. We theoretically analyze the mechanism and prove that the mechanism guarantees that each contractor reveals its true information in a single-task case. Moreover, we show that our method can reduce the contractee's operation costs by using computer simulation.

Introduction

Many multiagent researchers have discussed task allocation problems among self-interested agents and have tried to give these problems a theoretical framework based on economics and game theory, e.g., auctioning tasks (Sandholm 1996; Fujishima, McAdams, & Shoham 1999; Wellman *et al.* 2001; Parkes, Kalagnanam, & Eso 2001; Yokoo, Sakurai, & Matsubara 2001). However, these research efforts have not given sufficient attention to the process after the task is assigned, namely, whether each agent is motivated to carry out its task, although some research projects have focused on exchange processes (Sandholm & Lesser 1995).

As an example domain, consider content-delivery service in peer-to-peer networks (Parameswaran, Susarla, & Whinston 2001), which is a promising way for distributing contents. In these services, we assume that a content creator

agent (i.e., contractee) delegates its distribution task to content distributors (i.e., contractors).

In this situation, a contractee faces the problem of minimizing its total operation costs. Here, we assume that different contractors incur different costs for storing contents and delivering them to others. This leads to the problem of the contractee not being able to learn the true value of each contractor's cost, namely, the problem of asymmetric information. One solution to attain an efficient allocation of tasks is to use an auction.

However, in content-delivery services, it is not sufficient to only allocate the task. To keep the quality of services at an appropriate level, the contractee has to induce each contractor to behave appropriately after an allocation has been made. For example, if the load of service provision becomes large because of a concentration of service requests, the contractee has to induce the contractor to assign additional CPU resources to the task, although no contractor is motivated to provide additional resources without compensation.

One solution to this problem is to give a reward according to the result of service provision, namely, to give a higher reward if service provision succeeds. The problem that needs to be solved is to minimize the payments to contractors while still guaranteeing each contractor's voluntary participation, and this problem has been discussed in contract theory (Salanié 1997). However, this simple method cannot find an efficient task allocation.

From the above discussion, it is necessary to develop a way to keep the quality of services at an appropriate level while keeping the operation costs low even when a contractee cannot directly observe the behavior of contractors or their information (the initial cost incurred for providing services). This is a quite difficult problem because we have to solve two problems simultaneously: (1) how to allocate the tasks among contractors, and (2) how to devise a contract that can induce each contractor's appropriate behavior.

To solve this problem, we have developed an approximation method in which contracts are auctioned. More specifically, we first calculate an allocation of tasks so that social surplus is maximized in terms of initial cost and then calculate a contract that can assure a sufficiently large profit for the contractee.

The contributions of this paper are (1) introducing a problem of whether contractors are motivated to carry out

awarded tasks, (2) proposing a mechanism that can auction a contract, (3) proving that this mechanism can induce contractors' true declarations in a single-task case, and (4) experimentally showing that this mechanism can reduce the contractee's operation costs.

Related researches are being done in the contract theory, where an agent selection problem in a principal-agent model has already been discussed (McAfee & McMillan 1986; Laffont & Tirole 1987; Sappington 1991). These researches assume that the principal has a single task to be allocated and if an agent is selected, he/she always completes the allocated task. However, especially in Internet environments, tasks may not be completed due to factors outside the agent's behavior. This paper deals with a situation that an agent may fail to complete the allocated task. This is different from previous researches.

The rest of this paper is organized as follows. First, we describe the formal model, and then we propose a new mechanism for a single-task allocation problem. Next, we discuss multiple-task cases. Finally, we give our concluding remarks.

Model

This section gives a formal model to enable rigorous discussion. In the trading place, there exist a contractee agent, multiple contractor agents, and consumer agents. The contractee has content to deliver. The contractor who is allocated a distribution task distributes contents to consumers. Consumers report the quality of services (e.g., the time required to receive the content) to the contractee. Then, the contractee pays a reward to the contractor.

In this framework, a contractor may have its own task as well as the allocated task, thus it faces a resource allocation problem. The amount of the cost for service provision for the allocated tasks k_i depends on contractor i 's capabilities such as the CPU speed and knowledge about handling contents. We call these capabilities the contractor's technology.

Assumption 1 Contractor i 's technology to provide services related to the allocated tasks k_i is characterized by $\alpha_i(k_i)$.

For simplicity, we designate the technology $\alpha_i(k_i)$ as α_i .

In addition, contractor i assigns its resources to service provision for the allocated tasks k_i . For example, assigning more CPU resources enables a quick response to service requests, although contractor i suffers a loss by suspending the other tasks.

Assumption 2 The amount of assigned resources by contractor i is called the contractor's effort, which is denoted by e_i . e_i is chosen from the interval $[0, \bar{e}]$. We assume that contractor i 's loss caused by its effort e_i is equal to e_i .

Another interpretation of effort e_i is that the contractor obtains additional resources (e.g., memory, knowledge) from others by paying an amount of money corresponding to e_i .

Assumption 3 The result of service provision takes one of two states: success or failure.

The result of service provision is affected by other factors such as network congestion. Therefore, we introduce the following probability function.

Assumption 4 The result of service provision by contractor i is determined probabilistically based on contractor i 's technology and effort. Let $p(e_i; \alpha_i)$ denote the probability for success in providing services when contractor i 's technology is α_i and its effort is equal to e_i .

We call this probability function the contractor i 's performance profile. This probability can be viewed as quality of services.

Here, we introduce an order relation in terms of the value of technology α_i .

Assumption 5 If $p(e; \alpha_i) > p(e; \alpha_j)$ in $[0, \bar{e}]$, we say that contractor i has a smaller value of technology than contractor j .

The smaller the contractor's technology becomes, the more efficient its service provision is. That is, a contractor who has a smaller technology value can provide the same level of quality of services at lower cost compared to other contractors who have larger technology values. Note that given a task specification, the value of technology α_i is uniquely defined for each contractor i , while the value of effort e_i depends on contractor i 's decision.

Assumption 6 The probability for success in providing services by contractor i , $p(e_i; \alpha_i)$, is an increasing concave function of contractor i 's effort e_i and a decreasing concave function of contractor i 's technology α_i . Additionally, $p(e; \alpha_i)$ and $p(e; \alpha_j)$ do not intersect with each other in $[0, \bar{e}]$, if $\alpha_i \neq \alpha_j$.

This means that if the probability of success in providing services is already high, it becomes difficult to increase the probability by investing additional resources.

Assumption 7 The contractee cannot observe contractors' technologies and efforts.

Assumption 8 The contractee pays a reward w_i^H to contractor i if the contractor's service provision succeeds and pays a reward w_i^L if its service provision fails. If contractor i is not allocated any tasks, $w_i^H = w_i^L = 0$.

Contractor i 's expected utility, $U_i(e_i)$, is defined as follows.

Definition 1

$$U_i(e_i) = p(e_i; \alpha_i)w_i^H + (1 - p(e_i; \alpha_i))w_i^L - e_i$$

The contractor is risk-neutral.

On the other hand, the contractee's operation cost is defined as follows.

Definition 2

$$\sum_n ((1 - p(e_i; \alpha_i))q + p(e_i; \alpha_i)w_i^H + (1 - p(e_i; \alpha_i))w_i^L)$$

where n represents the number of contractors and q represents the unit loss caused by the failure of service provision.

In this expression, the first term represents the decrease in the contractee's profit caused by the failure of service provision, and the second and third terms represent the payments to the contractor. The contractee is risk-neutral.

Mechanism

In this section, we propose a new mechanism that determines the allocation of the task and contract (the amounts of reward). This section deals with the case where a single task is allocated to a single contractor, and the next section discusses multiple-task cases.

In content-delivery services, if a task (the level of quality of services) is specified to deal with at least five service requests per minute, a contractor is not willing to additionally assign its resources and deal with six service requests per minute if there is no compensation. Here, if the marginal cost caused by making an effort of e_i is smaller than the contractee's marginal profit by increasing the probability of success in service provision, the contractee has a chance to increase its profit by paying compensation to the contractor and inducing the contractor's effort of e_i .

However, it is difficult to obtain a contract that minimizes the contractee's operation cost because there are two kind of unknown information: technology and effort. Therefore, as an approximation method, we develop a mechanism that first obtains an allocation that maximizes social surplus in terms of declared performance profile and then calculates contracts based on the allocation obtained in the first step.

Mechanism for determining an allocation/contract

The procedure of our mechanism is as follows.

1. The contractee announces a task.
2. Each contractor reports its technology, namely, its performance profile (which may or may not be true) to the contractee. Any reported values of other contractors remain undisclosed to the contractor.
3. The contractee finds the contractor i who reports the lowest technology value.
4. The contractee calculates a contract (w_i^H, w_i^L) and offers it to contractor i .
5. Contractor i decides whether to accept the contract (w_i^H, w_i^L) .
6. If contractor i rejects the contract (w_i^H, w_i^L) , the task is not allocated to any contractor.

The calculation method of the contract (w_i^H, w_i^L) in step 4 is described below.

Behavior of a contractor

Although discussions in this subsection and the next subsection are based on existing contract theory, the other results are from our original work.

In this subsection, we examine what kind of contract should be offered to a contractor to induce it to select an effort level of e . To induce contractor i to select an effort level of e , the following incentive compatibility constraint must hold.

$$\begin{aligned} & p(e; \alpha_i)w_i^H + (1 - p(e; \alpha_i))w_i^L - e \\ & \geq p(e_i; \alpha_i)w_i^H + (1 - p(e_i; \alpha_i))w_i^L \\ & \quad - e_i \quad (\text{where } e_i \neq e) \end{aligned}$$

This constraint means that the utility obtained by selecting an effort level of e must be larger than or equal to that obtained by selecting another effort level.

Concerning the incentive compatibility constraint, contract theory tells us that if both the monotone likelihood ratio condition (MLRC) and the convexity of distribution function condition (CDFC) hold, we can apply the first-order approach (Salanié 1997). MLRC means that the larger the probability of success in providing services is, the larger the likelihood that a higher effort level is selected. CDFC intuitively means that the contractor's marginal profit for an additional effort probabilistically decreases.

Because this paper assumes that the number of possible results of service provision is two and the probability $p(e_i; \alpha_i)$ is an increasing concave function of e_i , we can conclude that the two conditions of MLRC and CDFC hold. Therefore, we can apply the first-order approach. The first-order approach means that the first-order partial derivative of contractor i 's utility with respect to e_i is equal to 0. That is, the following expression must hold.

$$p'(\alpha_i, e_i)(w_i^H - w_i^L) = 1 \quad (1)$$

Next, we examine the participation constraint. The participation constraint means that no contractor suffers any loss by signing a contract. If the participation constraint does not hold, contractors are not willing to sign a contract. Here, we assume that if a contractor is not allocated any tasks, its utility is equal to 0. Therefore, the participation constraint is represented as follows.

$$p(e_i; \alpha_i)w_i^H + (1 - p(e_i; \alpha_i))w_i^L - e_i \geq 0 \quad (2)$$

From the contractee's viewpoint, it is sufficient to give the contractor the minimum amount of reward that the contractor is willing to enter in the contract. Therefore, we deal with the participation constraint as follows.

$$p(e_i; \alpha_i)w_i^H + (1 - p(e_i; \alpha_i))w_i^L - e_i = 0 \quad (3)$$

From conditions (1) and (3), the amounts of rewards are calculated as follows.

$$w_i^H = e_i + \frac{1 - p(e_i; \alpha_i)}{p'(e_i; \alpha_i)} \quad (4)$$

$$w_i^L = e_i - \frac{p(e_i; \alpha_i)}{p'(e_i; \alpha_i)} \quad (5)$$

The reward for failure may become negative, although the expected utility of the contractor never becomes negative. This means that, in some cases, we may have to introduce some exchange mechanisms for inducing the contractor to make a payment (Matsubara & Yokoo 2000). However, by allowing the reward for failure to take a negative value, the contractee's operation cost can be reduced compared to that in a simple piecework system that pays a reward for success and nothing for failure.

Note that the method of selling the store, namely, selling the entire output to contractors at a flat fee, cannot be applied because the contractee's expected profit may become negative (Rasmusen 2001).

Behavior of a contractee

In the previous subsection, we obtain a contract (w_i^H, w_i^L) that induces contractor i to select an effort level of e . Based on this result, we examine what effort level needs to be set in order to minimize the contractee's operation cost.

The objective function of the contractee is given as follows.

$$\min_{e_i} ((1 - p(e_i; \alpha_i))q + p(e_i; \alpha_i)w_i^H + (1 - p(e_i; \alpha_i))w_i^L)$$

By substituting the expressions (4) and (5) for the above expression, the best contract for the contractee is obtained by calculating e_i that satisfies the following expression.

$$p'(e_i; \alpha_i) = 1/q$$

Here, let e_i^* denote the e_i that satisfies this expression. e_i^* is used in the next subsection.

Properties of the mechanism

In the discussion in the previous two subsections, we assume that the declared performance profile is used for calculating the amounts of rewards. However, contractor i may report a false performance profile. Actually, contractor i can obtain additional profit by overstating the value of α_i . If contractors declare false values and there is no equilibrium, we cannot predict what allocation is realized and how much cost the contractee owes. Therefore, to induce a contractor's truth declaration, we use the second lowest declared value, α_j ($j \neq i$), instead of the lowest declared value of α_i . We designate the declared value of α_i as $\tilde{\alpha}_i$ and the second lowest declared value of α_j ($j \neq i$) as $\tilde{\alpha}^{(2)}$.

We insert the following step into the procedure of the mechanism proposed above between steps 3 and 4.

3.5 Set the value of α_i to $\tilde{\alpha}^{(2)}$.

By using the second lowest declared value, we give up the notion of minimizing the contractee's operation cost. However, we believe that calculating a contract based on a socially efficient allocation in terms of technology is an appropriate method of approximation. The relation between a socially efficient allocation and revenue maximization is discussed in (Monderer & Tennenholtz 2001).

In this case, from the expressions (1), (4), (5), contractor i selects an effort level of e_i that satisfies the following expression.

$$p'(e_i; \alpha_i) \frac{1}{p'(e_i^*; \tilde{\alpha}^{(2)})} = 1 \quad (6)$$

Therefore, it is no longer guaranteed that contractor i selects e_i^* . However, the participation constraint holds.

Proposition 1 *Even if we calculate a contract by using $\tilde{\alpha}^{(2)}$ as the value of α_i , the participation constraint of contractor i still holds if contractor i does not overstate its value of α_i .*

Proof Let e_i^{**} denote the value of e_i that satisfies expression (6). Here, $U_i(e_i^{**}) \geq U_i(e_i^*)$ holds. If contractor i selects an

effort level of e_i^* , expected utility, $U_i(e_i^*)$, is calculated as follows.

$$\begin{aligned} & p(e_i^*; \alpha_i)w_i^H + (1 - p(e_i^*; \alpha_i))w_i^L - e_i^* \\ &= p(e_i^*; \alpha_i)(e_i^* + \frac{1 - p(e_i^*; \tilde{\alpha}^{(2)})}{p'(e_i^*; \tilde{\alpha}^{(2)})}) \\ & \quad + (1 - p(e_i^*; \alpha_i))(e_i^* - \frac{p(e_i^*; \tilde{\alpha}^{(2)})}{p'(e_i^*; \tilde{\alpha}^{(2)})}) \\ & \quad - e_i^* \\ &= \frac{1}{p'(e_i^*; \tilde{\alpha}^{(2)})}(p(e_i^*; \alpha_i) - p(e_i^*; \tilde{\alpha}^{(2)})) \end{aligned}$$

Because $\alpha_i \leq \tilde{\alpha}^{(2)}$ holds, $p(e_i^*; \alpha_i) > p(e_i^*; \tilde{\alpha}^{(2)})$ holds. Therefore, we obtain that $U_i(e_i^*) > 0$, and thus the participation constraint of contractor i is satisfied. \square

Proposition 2 *In the proposed mechanism, it is best for contractor i to report its true value of α_i .*

Proof First, we examine the case where contractor i wins the auction if it declares the true value. Contractor i cannot manipulate $\tilde{\alpha}^{(2)}$ because $\tilde{\alpha}^{(2)}$ is a value reported by another contractor. Therefore, even if contractor i overstates the value of α_i , as long as it is the winner of the auction, $\tilde{\alpha}^{(2)}$ does not change. Therefore, expected utility of contractor i does not change. If contractor i understates the value of α_i , it cannot obtain any additional utility because $\tilde{\alpha}^{(2)}$ does not change.

Next, we examine the case where contractor i loses the auction if it declares the true value. If the contractor understates the value of α_i and becomes the winner of the auction, $\alpha_i > \tilde{\alpha}^{(2)}$ holds.

Let e_i^{**} denote the value of e_i that satisfies expression (6). Contractor i 's expected utility U_i takes the maximum value at an effort level of e_i^{**} . $U_i(e_i^{**})$ is calculated as follows.

$$\begin{aligned} & p(e_i^{**}; \alpha_i)w_i^H + (1 - p(e_i^{**}; \alpha_i))w_i^L - e_i^{**} \\ &= e_i^* - e_i^{**} \\ & \quad + \frac{1}{p'(e_i^*; \tilde{\alpha}^{(2)})}(p(e_i^{**}; \alpha_i) - p(e_i^*; \tilde{\alpha}^{(2)})) \end{aligned}$$

First, we examine the case where $e_i^{**} < e_i^*$. From the assumption on the probability $p(e_i; \alpha_i)$, the following inequalities hold.

$$p'(e_i^*; \tilde{\alpha}^{(2)}) < \frac{p(e_i^*; \tilde{\alpha}^{(2)}) - p(e_i^{**}; \tilde{\alpha}^{(2)})}{e_i^* - e_i^{**}} < p'(e_i^{**}; \tilde{\alpha}^{(2)})$$

By transforming this expression, the following inequality is obtained.

$$\begin{aligned} e_i^* - e_i^{**} &< \frac{p(e_i^*; \tilde{\alpha}^{(2)}) - p(e_i^{**}; \tilde{\alpha}^{(2)})}{p'(e_i^*; \tilde{\alpha}^{(2)})} \\ &< \frac{p(e_i^*; \tilde{\alpha}^{(2)}) - p(e_i^{**}; \alpha_i)}{p'(e_i^*; \tilde{\alpha}^{(2)})} \end{aligned}$$

By substituting this inequality for the expression of the expected utility, we can obtain the following inequality.

$$U_i(e_i^{**}) < e_i^* - e_i^{**} + e_i^{**} - e_i^* = 0$$

Therefore, the expected utility of $U_i(e_i^{**})$ becomes negative.

Second, we examine the case where $e_i^{**} > e_i^*$. From the assumption on the probability $p(e_i; \alpha_i)$, the following inequalities hold.

$$p'(e_i^{**}; \tilde{\alpha}^{(2)}) < \frac{p(e_i^{**}; \tilde{\alpha}^{(2)}) - p(e_i^*; \tilde{\alpha}^{(2)})}{e_i^{**} - e_i^*} < p'(e_i^*; \tilde{\alpha}^{(2)})$$

By transforming this expression, the following inequality is obtained.

$$\begin{aligned} e_i^{**} - e_i^* &> \frac{p(e_i^{**}; \tilde{\alpha}^{(2)}) - p(e_i^*; \tilde{\alpha}^{(2)})}{p'(e_i^*; \tilde{\alpha}^{(2)})} \\ &> \frac{p(e_i^{**}; \alpha_i) - p(e_i^*; \tilde{\alpha}^{(2)})}{p'(e_i^*; \tilde{\alpha}^{(2)})} \end{aligned}$$

By substituting this inequality for the expression of the expected utility, we can obtain the following inequality.

$$U_i(e_i^{**}) < e_i^* - e_i^{**} + e_i^{**} - e_i^* = 0$$

Therefore, the expected utility of $U_i(e_i^{**})$ becomes negative.

Lastly, we examine the case where $e_i^{**} = e_i^*$. In this case, the expected utility of $U_i(e_i^{**})$ becomes negative because $p(e_i^{**}; \alpha_i) < p(e_i^*; \tilde{\alpha}^{(2)})$.

That is, contractor i cannot obtain positive utility by understating its value of α_i . \square

This is a desirable property of the mechanism because contractors do not have to spy on other contractors' technologies, which makes the system stable.

Evaluations

We evaluated to what extent the proposed mechanism can reduce the contractee's operation cost. We compared our mechanism with two other mechanisms: ideal and fixed-quality cases. In the mechanism named ideal, under the assumption that a contractee knows the true value of each contractor's technology, the contractee calculates the contract. On the other hand, in a mechanism named fixed-quality, first a contractee determines the level of quality of services, second a contractee determines a task allocation by using an auction but does not change the level of quality of services based on the auction result. Instead, the contractee pays an amount of money equal to the second lowest bid if the contractor's service provision succeeds and pays nothing if the contractor's service provision fails.

We assume that each contractor's performance profile takes the following form.

$$p(e_i; \alpha_i) = \sqrt{ae_i}$$

We examine two cases: easy task and hard task. In an easy task, the value of a of each contractor is drawn from a uniform distribution over $[0.0, 1.0]$, while in a hard task, the value of a of each contractor is drawn from a uniform distribution over $[0, 0, 0.1]$. The unit loss caused by failure in providing services, q , is set to 1.0 and 2.0. In addition, we set $p(e_i; \alpha_i)$ to 0.6 and 0.8 in the fixed-quality mechanism.

Figures 1 and 2 show the results of the contractee's operation cost in an easy task and a hard task, respectively. The x-axis represents the number of bidders (contractors), while the y-axis represents the contractee's operation cost.

These figures elucidate the following.

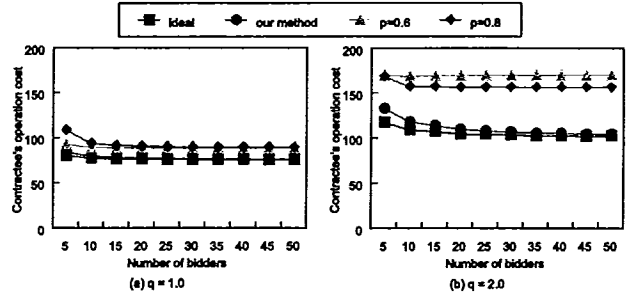


Figure 1: Experimental results (easy task)

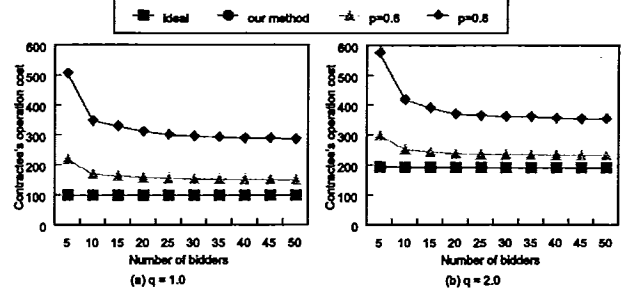


Figure 2: Experimental results (hard task)

- The proposed mechanism can reduce the contractee's operation cost more effectively than the fixed-quality mechanism. As the unit loss of q becomes large, the amount of reduction also becomes large.
- As the number of bidders increases, the performance of the proposed mechanism becomes closer to that of the ideal mechanism.

Although this evaluation is a preliminary one, the obtained results show that our mechanism is a promising way for keeping the contractee's operation cost low. Thorough evaluations of our mechanism are one of our future works.

Multiple-task Cases

So far, we have restricted our discussions to the single-task case. However, sometimes a contractee may want to allocate the same content to multiple contractors in order to avoid a concentration of service requests to a contractor. In this paper, we assume that the marginal cost with respect to an additional unit increases for each contractor.

The procedure of our mechanism is as follows. Here, we assume that m units exist.

1. The contractee announces a task.
2. Each contractor reports its technology, namely, its performance profile (which may or may not be true) to the contractee. Any reported values of other contractors remain undisclosed to the contractor.
3. The contractee finds the m contractors whose value of α_i is from the lowest to the m -th lowest.

4. For each winner, if the winner i who has technology values of $\alpha_{i1}, \alpha_{i2}, \dots, \alpha_{im}$ wins m_i units, the contractee finds the m_i lowest rejected bids, $\tilde{\alpha}_1^{(2)}, \tilde{\alpha}_2^{(2)}, \dots, \tilde{\alpha}_{m_i}^{(2)}$. Set the value of α_{i1} to $\tilde{\alpha}_1^{(2)}$, α_{i2} to $\tilde{\alpha}_2^{(2)}$, \dots , and α_{im_i} to $\tilde{\alpha}_{m_i}^{(2)}$.
5. The contractee calculates contracts (w_i^H, w_i^L) and offers them to contractor i .
6. Contractor i decides whether to accept the contract (w_i^H, w_i^L) .
7. If contractor i rejects the contract (w_i^H, w_i^L) , the task is not allocated to any contractor.

The calculation method of the contract (w_i^H, w_i^L) is the same in the previous section.

In this case, the participation constraint holds.

Proposition 3 *Even if we calculate a contract by using $\tilde{\alpha}_i^{(2)}$ as the value of α_{ii} , the participation constraint of contractor i still holds if contractor i does not overstate its value of α_{ii} .*

Because this proposition can be proved in a similar way to the proof of proposition 1, we omit the proof.

Proposition 4 *In the proposed mechanism, it is best for contractor i to report its true values of α_i , if the marginal cost with respect to an additional unit increases for each contractor.*

To prove this proposition, we use the following three lemmas.

Lemma 5 *In the proposed mechanism, contractor i cannot obtain an additional unit of utility by overstating its value of α_{ii} if the allocation does not change.*

Because this proposition can be proved in a similar way to the proof of proposition 2, we omit the proof.

Lemma 6 *In the proposed mechanism, contractor i cannot obtain an additional unit of utility by understating its value of α_{ii} .*

Because this proposition can be proved in a similar way to the proof of proposition 2, we omit the proof.

Lastly, we examine the possibility of a demand reduction lie (Ausubel & Cramton 1998).

Lemma 7 *In the proposed mechanism, contractor i cannot obtain an additional unit of utility by reducing demand.*

Proof Suppose that contractor 1 has technology values of α_{11} and α_{12} , contractor 2 has a technology value of α_{21} , contractor 3 has a technology value of α_{31} and $\alpha_{11} < \alpha_{12} < \alpha_{21} < \alpha_{31}$ hold. In addition, suppose that two units exist. If contractor 1 reports the true values of technology, contractor 1 wins two units. The contracts are calculated based on α_{21} and α_{31} . On the other hand, if contractor 1 reports only α_{11} , contractor 1 wins one unit. The contract in this case is calculated based on α_{31} . In the former case, contractor 1 can choose which contracts, contract based on α_{21} or contract based on α_{31} , to be applied to the first unit. The remaining contract is applied to the second unit. Thus, contractor 1's utility is greater than or equal to the sum of utility from the contract based on α_{31} for the first unit and utility

from the contract based on α_{21} for the second unit. It is obvious that an amount of this utility is larger than that in the demand reduction case. Other cases can be discussed in the similar way. \square

From the above three lemmas, we can conclude that proposition 4 holds.

Concluding remarks

This paper developed a new mechanism to determine the allocation of tasks and calculate a contract to solve the incentive problem in content delivery services in peer-to-peer networks. This problem is difficult to solve because a contractee cannot observe the contractors' efforts or their technologies for handling the task.

To solve the problem, we proposed a mechanism that auctions contracts by auctioning a pair of rewards for success and failure. More specifically, the mechanism first finds a socially efficient allocation of the tasks and then calculates a contract based on the result of the auction. By analyzing the mechanism through game theory, we showed that the mechanism guarantees that each contractor reveals its true information in a single-task case and a single-item multiple-unit case where the marginal cost with respect to an additional unit increases. Moreover, experimental results showed that our mechanism can reduce the contractee's operation cost compared to a naive mechanism.

Although Laffont and Tirole addressed the problem of a contractee not knowing the contractors' cost parameters or efforts, they assumed that the relation between the cost parameters and efforts is restricted to a special form. This can reduce the problem to one of an unknown parameter.

One of our future works includes developing a mechanism for single-item, multiple-unit case where a marginal cost with respect to an additional utility decreases and for multiple-item cases.

Acknowledgments

The author wishes to thank Yoshifumi Ooyama, and Kiyoshi Kogure for their support during this work at NTT Communication Science Laboratories, and Makoto Yokoo and Yuko Sakurai for their helpful discussions.

References

- Ausubel, L. M., and Cramton, P. 1998. Demand reduction and inefficiency in multi-unit auctions. Working Paper.
- Fujishima, Y.; McAdams, D.; and Shoham, Y. 1999. Speeding up ascending-bid auctions. In *Proceedings of the Sixteenth International Joint Conference on Artificial Intelligence (IJCAI-99)*, 554–559.
- Laffont, J.-J., and Tirole, J. 1987. Auctioning incentive contracts. *Journal of Political Economy* 95(5):921–937.
- Matsubara, S., and Yokoo, M. 2000. Defection-free exchange mechanisms for information good. In *Proceedings of the Fourth International Conference on Multi-Agent Systems (ICMAS-2000)*, 183–190.

- McAfee, R. P., and McMillan, J. 1986. Bidding for contracts: a principal-agent analysis. *Rand Journal of Economics* 17(3):326–338.
- Monderer, D., and Tennenholtz, M. 2001. Asymptotically optimal multi-object auctions. Working Paper.
- Parameswaran, M.; Susarla, A.; and Whinston, A. B. 2001. P2P networking: An information-sharing alternative. *IEEE Computer* 34(7):31–38.
- Parkes, D. C.; Kalagnanam, J.; and Eso, M. 2001. Achieving budget-balance with Vickrey-based payment schemes in exchanges. In *Proceedings of the Seventeenth International Joint Conference on Artificial Intelligence (IJCAI-01)*, 1161–1168.
- Rasmusen, E. 2001. *Games and Information: an introduction to game theory*. Blackwell Publishers, 3rd edition.
- Salanié, B. 1997. *The Economics of Contracts*. MIT Press.
- Sandholm, T. W., and Lesser, V. R. 1995. Equilibrium analysis of the possibilities of unenforced exchange in multi-agent systems. In *Proceedings of the Fourteenth International Joint Conference on Artificial Intelligence (IJCAI-95)*, 694–701.
- Sandholm, T. W. 1996. Limitation of the Vickrey auction in computational multiagent systems. In *Proceedings of the Second International Conference on Multi-Agent Systems (ICMAS-96)*, 299–306.
- Sappington, D. E. M. 1991. Incentives in principal-agent relationships. *Journal of Economic Perspectives* 5(2):45–66.
- Wellman, M. P.; Walsh, W. E.; Wurman, P. R.; and MacKie-Mason, J. K. 2001. Auction protocols for decentralized scheduling. *Games and Economic Behavior* 35(1):271–303.
- Yokoo, M.; Sakurai, Y.; and Matsubara, S. 2001. Robust combinatorial auction protocol against false-name bids. *Artificial Intelligence Journal* 130(2):167–181.