Does Learning by Market Participants Make Financial Markets Complicated?

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Abstract
In this study we rethought efficient market hypothesis from a viewpoint of complexity of market participants’ prediction methods and market price’s dynamics, and examined the hypothesis using simulation results of our artificial market model. As a result, we found the two difference from the hypothesis. (a) Complexity of markets was not fixed, but changed with complexity of agents. (b) When agents increased the complexity of their prediction methods, structure of dynamic patterns of market price didn’t disappear, but it can’t be described by equation of any dimensions.

Introduction
Are you surprised if the performance of financial specialists’ forecasts is the same as that of randomly generated forecasts?

In the field of economics, the theory of financial markets called the efficient market hypothesis was proposed in the 70s, and it has caused many arguments till today. By this hypothesis, the movement of the price of financial markets is a random walk, and cannot be predicted. Therefore, the performance of all the forecasts is the same. Theories of financial engineering which developed greatly today are based on this hypothesis, and they assume financial prices as the stochastic process.

Although many statistical verification of the hypothesis was performed using actual data, since the hypothesis included a market participant’s expectation formation, it has not been verified directly. In recent years, however, the artificial market approach which builds a virtual market model and performs a simulation into a computer appeared, and researches in this approach try to verify the hypothesis directly(Chen & Yeh 1996; Chen, Yeh, & Liao 1999; 2000).

This study rethink the efficient market hypothesis from the new viewpoint of the relation between the complexity of market participants’ prediction formulas and the complexity of the movement of a market price. And this study examines the hypothesis from the simulation result using the artificial market model.
Then, what is the motivation to which each market participant complicates his prediction formula? Joshi et al. (Joshi, Parket, & Bedau 2000) think that it is because the situation similar to the prisoner’s dilemma game has occurred. In their artificial market model, taking in the technique of the moving average of a technical analysis to a prediction method, and raising the dimension of a prediction formula from 0 to 1 corresponds to the default strategy of the prisoner’s dilemma game. On the other hand, not using a technical analysis for prediction corresponds to the cooperation strategy. From the simulation result, the two following conditions for becoming a prisoner’s dilemma situation were seen.

Condition 1 If one raises his prediction dimension, his prediction becomes more accurate and the profit of his dealings result increases. Thus, the motivation of the default strategy exists.

Condition 2 However, when everybody raised the dimension, the movement of the market price became more complicated, and the prediction accuracy has fallen rather than the time of everybody’s not using the technical analysis.

Thus, since everybody raised the dimension of his prediction formula in pursuit of profits, the prediction accuracy becomes worse than before.

In the following sections, by the artificial market simulation, we analyze the complexity of a market and the prisoner’s dilemma situation when a prediction dimension becomes larger.

Artificial market model

The artificial market is a virtual financial market with 50 virtual dealers (agents) in a computer. One financial capital and one non-risk capital exist in this artificial market. Each agent expects the movement of the financial price, and he changes the position of the financial and non-risk capital so that the utility of his expected profit may become the maximum. In the artificial market, one term consists of four step of expectation, an order, price determination, and learning, and time progresses discretely by repeating these four steps.

Expectation

Each agent expects the change value of the financial price of this term using the weighted sum of the change value of past financial price. That is, in this study, since fundamentals information does not exist in a market, the agents expects the change value of the financial price only by the technical analysis.

The expectation formula of each agent is auto the regressive integral moving average model ARIMA(n, 1, 0), where n means the number of the terms of the price changes used for expectation. The larger n is, the larger the dimension of an expectation formula is. Thus in this study, n is regarded as the complexity of each agent’s expectation.

The expectation formula is as follows, when \( P_t \) is the financial price of this term which is not yet determined and \( y_t \) is the expectation the change of financial price \( (P_t - P_{t-1}) \).

\[
\hat{y}_t = \sum_{i=1}^{n} b_i y_{t-i} + e_t \\
= x'_t b_t + e_t
\]

Here, \( e_t \) is the normal distribution whose average is 0 and standard deviation is 0.1, \( b_t \) is a vector with the coefficient of the prediction formula\(^1\), \( (b_1, \cdots, b_n)' \), and \( x_t \) is a vector of the explanation variables of the prediction formula, i.e., the past price changes\(^2\), \( (y_{t-1}, \cdots, y_{t-n})' \).

Order

It is assumed that each agent has the utility function of expected profit with risk avoidance. Then the optimum quantity of the position of the financial capital with the maximum utility, \( q_t^* \), is proportional to the expected change value \( y_t \) of the formula (1).

\[
q_t^* = a y_t,
\]

where \( a \) is a coefficient. Each agent’s amount of orders \( o_t \) is the difference between the optimum position \( q_t^* \) and the current position \( q_{t-1} \).

\[
o_t = q_t^* - q_{t-1}
\]

If the market price \( P_t \) is lower (higher) than his expected price \( (P_{t-1} + y_t) \), each agent order to buy (sell). The amount of order is \( o_t \).

\[
\begin{align*}
\text{If } o_t &> 0, \\
&\begin{cases} 
\text{Buy } o_t & (P_t \leq P_{t-1} + y_t) \\
\text{No action} & (P_t > P_{t-1} + y_t)
\end{cases} \\
\text{If } o_t &< 0, \\
&\begin{cases} 
\text{No action} & (P_t < P_{t-1} + y_t) \\
\text{Sell } o_t & (P_t \geq P_{t-1} + y_t)
\end{cases}
\end{align*}
\]

Price determination

All the orders of 50 agents in the market are accumulated, and the market price of this term is determined as the value where the demand and supply are balanced. Dealings are transacted between the buyer who gave the price higher than a market price, and the seller of a lower price.

Learning

Each agent updates the coefficients \( b_t \) of the prediction formula (1) using the successive least-squares method with the information on the change \( y_t \) of the newly determined market price\(^3\). The least-squares method is as follows (Harley 1981).

\[
b_{t+1} = b_t + (X'X_t)^{-1} x_t (y_t - X'_t b_t),
\]

\(^1\)The initial value of the coefficients \( b_0 \) is given with the uniform random numbers from -1 to 1.

\(^2\)At the start, the initial values of price \( x_0 \) are generated by the normal distribution whose average is 0 and standard deviation is 1.

\(^3\)When \( n = 0 \), the prediction value is a random number and learning is not performed.
where $X_t$ is a learning matrix which starts by $X_0 = 100 \times I$ ($I$ is a unit matrix), and is updated by the following formula.

$$
(X_t'X_t)^{-1} = (X_{t-1}'X_{t-1})^{-1} - (X_{t-1}'X_{t-1})^{-1}x_tX_t'(X_{t-1}'X_{t-1})^{-1}
$$

$$
f_t = 1 + x_t'(X_{t-1}'X_{t-1})^{-1}x_t
$$

**Simulation result**

In the next section, we examine the complexity of the market and the prisoner’s dilemma situation when the prediction dimension became large using the artificial market model.

**Merit of complicating a prediction formula**

We investigated the merit of complicating the prediction formula. The dimensions of 25 agents’ prediction formulas was set to $n$, and the dimension of the prediction formula of the other 25 agents was $n+1$. Each simulation had 4000 terms which consisted of the four steps in section . The averages of forecast errors were calculated both about the agent group with $n$ dimensions and about the group of $n+1$ dimensions. The forecast errors were the difference between each agent’s prediction value and a market price. The initial value of random numbers was changed and 100 simulations was carried out. Figure 1 shows the difference between the forecasts errors of the group with $n+1$ dimensions and those of the group of $n$ dimensions.

As a result, when the number of dimensions in the prediction formula was small, the forecast error became large, as the number of dimensions increased. That is, the conditions 2 of the prisoner’s dilemma situation in the section were hold. However, it has converged to the fixed value when the number of dimensions was larger than three.

**Development of the complexity of a market**

In order to examine the independence of the complexity of the movement of a market price from the complexity of each market participant’s prediction formula, we carried out the correlation dimension analysis. All 50 agents have the prediction formulas of the same $n$ dimension. We carried out the simulation with 4000 terms 100 times. After having accumulated the forecasts errors in 4000 terms and taking an average of 50 agents in 100 simulations. (Fig.2).

As a result, when the number of dimensions in the prediction formula was small, the forecast error became large, as the number of dimensions increased. That is, the conditions 1 of the prisoner’s dilemma situation in the section are hold. However, when the number of dimensions becomes large, the merit of complicating prediction formulas disappears.

**The demerit in the whole market**

We examined whether the prediction of prices becomes harder in the whole market as increase of the dimension of prediction formulas. In this simulation, 50 prediction formulas of all agents were the same $n$ dimension. We carried out the simulation with 4000 terms 100 times. After having accumulated the forecasts errors in 4000 terms and taking an average of 50 agents in 100 simulations. (Fig.2).

As a result, when the number of dimensions in the prediction formula was small, the forecast error became large, as the number of dimensions increased. That is, the conditions 2 of the prisoner’s dilemma situation in the section were hold. However, it has converged to the fixed value when the number of dimensions was larger than three.

**Development of the complexity of a market**

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As a result, when a prediction dimension was 0, the the correlation dimension curve was convex downward like the theoretical value of a random walk (fig. 3a). That is, there is no structure in the dynamics of the market price. However, when the prediction dimension increase a little, the correlation dimension curve was convex upward and saturated

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4 Since the calculation of averages were impossible when the market price had diverged, we carried out simulations until we could get 100 simulations whose paths did not diverge.

5 The path to diverge was not seen when all agents’ prediction formula was the same dimension.

6 The procedure of the correlation dimension analysis was described in (Nakajima 1999; 2000).
(fig. 3b). Thus, the structure that could be described by an equation of a finite dimension appeared in the dynamics of the market price. Furthermore, when the prediction dimension was raised, the correlation dimension curve became a straight line (fig. 3c). Thus, the correlation dimension curve was neither convex downward like a random walk nor saturated. That is, there was a structure in the dynamics of the market price, but it could not be described by an equation of any finite dimension.

According to Nakajima (Nakajima 1999; 2000), as a result of analyzing Tokyo Stock Exchange Stock Price Index data, the logarithm of a correlation dimension went up linearly like this simulation result in fig. 3c. That is, when each agent’s prediction dimension increases, like the price data in the real-world, the dynamics of the price in the artificial market can be described roughly by an equation of some dimensions. And the more precise description is also attained by increasing the number of dimension. However, the movement of price data cannot be described completely by an equation of any finite dimensions. That is, the number of the variables related to the movement cannot be specified completely.

**New efficient market hypothesis**

The simulation results are summarized as follows.

- When each market participant’s prediction dimension is 0, the movement of a market price resembles a random walk. If the prediction dimension increases, the structure which can be described by an equation of a finite dimension appears in the movement of price.

- Therefore, if each agent increases his prediction dimension, since the prediction dimension approaches to the dimension of the price determination formula and his prediction becomes more accurate. Thus, the merit of complicating prediction formulas exists. However, if everybody increases their prediction dimension, prediction accuracy becomes smaller than before. That is, it will become the prisoner’s dilemma situation.

- If everybody continues to increase the prediction dimension in the prisoner’s dilemma situation, the movement of a market price come to have the structure that can not be described completely by an equation of any finite dimensions.

The structure of the movement of a market price changed as market participants changed their prediction formulas. That is, the complexity of market participants and the complexity of a market are not independent unlike the efficient market hypothesis. The simulation results also suggests that the structure of the dynamics of price data did not disappear when market participants continue to complicate their prediction formulas. In the final state, however each market participant increases his prediction dimension, he can not predict the market price completely.

In such the state where there is no “correct answer” of learning, it is thought that a procedure of learning by each market participant becomes the key factor to the movement of a market price in addition to a result of learning. As Kichiji(Kichiji 2000) said, the efficiency of learning by a market participant, the difference in the cognitive framework, the interaction between market participants, and the method of informational choice, etc. become important.

Another key point is the mechanism of market price determination. In this study we assumed that the market price were determined discretely as an equilibrium price. Alternatively we can assume that the market price is determined continuously as a transaction price of each dealing. The mechanism of market price determination is the mechanism how to accumulate the individual complexity on the complexity of a market. Therefore, it has large influence on the relation between the complexity of market participants’ prediction formulas and the complexity of the movement of a market price. It is interesting to examine whether the same simulation can be acquired when the mechanism of market price determination changes.

**Conclusion**

This study examined an efficient market hypothesis using artificial market approach. As a result, the following two points different from an efficient market hypothesis were found.

- While the prediction dimension of agents is small, the structure which can be described to the movement of a market price exists, and the motivation of increasing the prediction dimension exists.

- Even if the market participant increases the prediction dimension, the structure of the movement of a market price does not disappear. Finally, however each market participant increases his prediction dimension, he can not predict the market price completely.

As future works, we want to investigate the influence of (a) the procedure of learning by a market participant and (b) the mechanism of the price determination on the relation between the complexity of market participants’ prediction formulas and the complexity of the movement of a market price.

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**References**


Figure 3: Correlation dimensions: X-axis is the logarithm of embedding dimensions. A solid line is an average of the correlation dimension of 100 paths. A dotted line is the theoretical value of a random walk.