Non-Rational Agents Explain GARCH Model: 
Agent Simulation for Behavioral Finance

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Abstract
GARCH (Generalized Autoregressive Conditional Heteroscedasticity) is a macro level model to estimate the volatility of financial markets. Although the model is very fundamental in the financial and economic domain, however, there have been no clear explanation about the model from the micro-level financial behaviors. This paper develops agent-based simulation models, which consists of simple agents with rational and/or non-rational decision making functionalities for investment. Using the simulation model with both rational and non-rational agents, the paper has shown that the behaviors of the non-rational agents with the characteristics of prospect theory coincide with the estimation by GARCH model.

Key Words: Multiagent-based simulation for economic issues, Society dynamics, Self-organizing systems and emergent organization, Financial Engineering

INTRODUCTION

Understanding the mechanisms of financial markets is critical because of the rapid deployment of e-commerce and the rigidity of economic systems. So far, economic analysts have used macroscopic mathematical models to understand the mechanisms. The classical models in finance are analytical, thus, make assumptions regarding the market and the behavior of individuals operating in the market. The relations of such analytical models and the mental models of the individuals or agents have not become clear, yet (Arthur 1997), (Levy 2000). We do not have had rigid foundations of the validity of the macro-level analytical models. In this paper, we address agent-based approach and develop simulation models to evaluate financial market behaviors.

The rest of the paper is organized as follows: In Section 2, define the problems we would like to uncover. In Section 3, we describe our agent-based simulation models, then in Section 4, using the models, intensive experiments are carried out. Section 5 is devoted to the discussion of the experiments and related work. In Section 6, some concluding remarks will be given.

OBJECTIVES OF THE RESEARCH

The following three models are often used to estimate the volatility of stock market data: Ordinary Least Squire (OLS) Model, Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model (Bollerslev 1986), and Asymmetric GARCH Model (Glosten 1993). If the agents would follow the usual assumptions of the analytical models: they are rational about market information and maximize their expected utilities, they have homogeneous expectations regarding future distribution of returns, they all have same holding period, and so on. OLS model would coincide with the real market phenomena.

OLS Model:
\[ \epsilon_t \sim N(0, \sigma^2), \text{ where } \sigma^2 \text{ is constant,} \]

GARCH Model:
\[ \sigma_t^2 = \alpha + \sum_{i=1}^{p} \alpha_i \cdot \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \cdot \sigma_{t-i}^2, \]

Asymmetric GARCH Model:
\[ \sigma_t^2 = \alpha + \sum_{i=1}^{p} (\alpha_i \cdot \epsilon_{t-i}^2 + \gamma_i \cdot S_{t-i} \cdot \epsilon_{t-i}^2) + \sum_{i=1}^{q} \sigma_{t-i}^2, \]

\[ S_{t-i} = \begin{cases} 1 & \text{if } \epsilon_{t-i} < 0, \\ 0 & \text{if } \epsilon_{t-i} \geq 0, \end{cases} \]

where, \( t, y, \epsilon, \) and \( \sigma^2 \) are time periods, estimation values, errors, and the variance of Normal distribution, respectively. The coefficients of \( a, b, \) and \( c \) are respectively parameters to be determined by real data. The analytic models so far are only able to explain how OLS model works.

To apply agent-based models to financial economic domain, in this research, we define the following two problems. These problems have been frequently reported in the finance literature using real world data, however, they have not been yet uncovered (Arthur 1997), (Levy 2000).

- Examining the Roles of non-rational Agents in the Market:
Friedman states in (Friedman 1953) that by the law of natural selection, non-rational investors should not survive and only rational investors would remain in the market. But, the
statements seem not to hold. We also examine this via agent-based simulation.

- Agent Characteristics to Coincide with (A-)GARCH Model:
As shown above, (Asymmetric-)GARCH Model shows the best sit to the real data. Real market behaviors often show the price change similar to the ones GARCH Model generates. Using agent-based simulation, we would like to determine the agent characteristics to explain GARCH model. Our simulation models are at the similar complexity to the ones used in microscopic simulation (Levy 2000) and Sugarscape (Epstein 1996). The agents have a small set of decision rules to determine their attitudes about the market conditions. They are represented by simple equations. We will design the agents, which are neither game-theoretic ones nor simple ones with the KISS principle in (Axelrod 1997), because we will validate the agents' behaviors compared with real phenomena. We also will design the agents with complex functionalities as reported in (Takadama 1999), (Terano 1998). In OCS (Organizational Learning-oriented Classifier System) (Takadama 1999) the agents learn rules represented by classifiers using Genetic Algorithms and change good ones with each other. In TRURL (Terano 1998) the agents make decisions using multi attribute decision functions, communicate each other, and move around to gather together if they have similar knowledge. Compared with them, the agents in this paper are so simple that we are able to analyze the characteristics of them.

**AGENT-BASED MODELING FOR THE FINANCIAL MARKETS**

In this section, we describe the agent-based models, which are extension of traditional finance models. First, we describe the components of the market, then determine the trading rules of the investor agents.

**Components of the Market**
The market we define on the computer consists of 1,000 investor agents. Each agent trade the assets as either individual stocks or riskless assets. At each simulation step, the benefits or losses will occur based on the Brownian motion, and each agent trades its asset based on its benefits/losses and past pricing information. One of the unique points of the agents we design is that the non-rational agents have the characteristics with Prospect Theory in socio-psychology. Prospect Theory is reported in e.g., Kahneman et al. (Kahneman 1979). The theory states the decision making model of human beings have the characteristics that (1) the decision is made based on the change from some reference points; (2) the decision attitude is different when the benefit or loss will occur; (3) the loss is considered to be larger than the benefit; and (4) the decision is made based on subjective probability of the events. In our agents model, the agents with Prospect Theory think the loss is twice as much as the benefit (Kagel 1995), (Shleifer 2000). Table 3 is a summary of the market components. Figure 1 also shows the outline of the agent architecture.

**Decision Making of the Investor Agents**
All the investor agents determine their asset allocation based on the sum of (i) long term expected return ratio (SAA) and (ii) short term expected return ratio. The model is known as Black and Litterman Model in (Arthur 1997). The decision making strategies of the agents in our simulation model have very small difference. The SAA part is common to the all the agents, however, the decision on the short term expected return ratio is different. The rational agents determine the short term expected return ratio is calculated by Forecast_short statement in Figure 3. On the other hand, the ones of the irrational agents are determined by the same line of the following Figures 4, 5, and 6.

In the simulation cycle, the market price is determined by the value with which the demand and the supply are coinciding. The general decision-making algorithm is shown in Figure 2. In the following subsubsections, we explain how the investor agents determine their short term prediction values.

**Prediction of Rational Investor Agents**
The prediction price of rational agents is determined by the dividend discount model. The benefits of the market are open to the agents in our model, the prediction price is computed based on the open market price. Figure 3 illustrates the decision making algorithm.

**Prediction of Investor Agents with Prospect Theory**
The agents with Prospect Theory think the loss is twice as much as the benefit. More detail, they measure the loss twice more than the real one when the most recent price is lower
%Input Investor's Information
Equilibrium_Return: Average return ratio implied by the asset allocation (common to all the investors)
Average return ratio: 2\sigma^2 stockweight + r_f,
stockweight: Ratio of the amount of total stocks with the total assets in the market
\sigma_1: Standard deviation of long term expected return ratio (common to all the agents, set to 1.0 x 10^-3)
Forecast_short(): Short term expected return (different in each type of agents; described below)
\sigma_2(): Standard deviation of short term expected return ratio (different in each type of agents; described below)
\lambda: Ratio of risk avoidance (common to all the agents, set to 1.25)
\delta: Required return ratio of the stock benefit (common to all the agents, set to 10%/200days)
\tau_f: Risk free rate (common to all the agents, set to 5%/200days)
Fund: Total assets of the agents(initial value: 2000(common))
%Input Lower and Upper Bounds of Investment Ratio
Lower_Bound: Lower Bound of Investment Ratio
Upper_Bound: Upper Bound of Investment Ratio
%Calculate The Agents Return
Return = (1/\sigma_1 * Equilibrium_Return + 1/\sigma_2())/(1/\sigma_1 + 1/\sigma_2())
%Solve the optimization problem to calculate %the optimal stock weight:w
%Max(1 + Return)w + (1 + r_f)(1 - w) - \lambda \sigma^2 w^2
% s.t.w \leq Lower_Bound
% w \geq Upper_Bound
w = (Return - r_f)/(2\lambda \sigma^2)
w = Upper_Bound, if w \leq Lower_Bound
w = Lower_Bound, if w \geq Lower_Bound

Figure 2: Decision Making Algorithm in Common

Forecast_short(Value_t){
Value_{t+1} = Profit_t/\delta + Profit_t
Forecast_short = Value_{t+1}/Value_t - 1.0
\% \sigma_2(){\sigma_2 = 0.2/200^{0.5}}
\% Profit_t: Profit at period t (known)
\% \delta: Required return ratio of the stock (0.2/200^{0.5})

Figure 3: Decision Making Algorithm of Rational Agents

EXPERIMENTS
We implement the agent models in Matlab. The experiments using the agent models in the previous section are designed

than the prediction price. The prediction price is determined by the same way of the rational agents. Figure 4 shows the decision making algorithm.

Forecast_short(Value_t){
Value_{t+1} = Profit_t/\delta + Profit_t
If (Value_{t-1} < ReferencePointValue and Value_{t+1} < ReferencePointValue) Then
Value_{t+1} = ReferencePointValue - 2.25(ReferencePointValue - Value_{t+1})
Forecast_short=(Value_{t+1}/Value_t - 1)(1 + \epsilon), where \epsilon \sim N(0,0.01)
\%ReferencePointValue: Past Trading Price(2 or 5 days ago)

Figure 4: Decision Making Algorithm of Agents with Prospect Theory

Prediction of Investor Agents with Over Confidence Characteristics The agents with over confidence characteristics think the risk of the stock market smaller. For example, the over confidence agents invest more to stocks than the rational agents. The prediction price is computed based on dividend discount model. Figure 5 shows the decision making algorithm.

Forecast_short(Value_t){
Value_{t+1} = Profit_t/\delta + Profit_t
Forecast_short = Value_{t+1}/Value_t - 1.0
\% \sigma_2(){\sigma_2 = k x 0.2/200^{0.5}}
\% Profit_t: Profit at period t (known)
\% \delta: Required return ratio of the stock (0.2/200^{0.5})

Figure 5: Decision Making Algorithm of Agents with Over Confidence Characteristics

Prediction of Trend Predictor Agents The trend predictor agents determine the prediction price by extrapolating the averages of the most recent ten days stock price data. Figure 6 shows the decision making algorithm.

Forecast_short(Value_t){
Trend The recent 10 days average of stock price return
Value_{t+1} = Value_{t-1} \times Trend^2 + Profit_t \times Trend
Forecast_short = Value_{t+1}/Value_t - 1.0
\% \sigma_2(){\sigma_2 = 0.2/200^{0.5}}

Figure 6: Decision Making Algorithm of Trend Predictor Agents

EXPERIMENTS
We implement the agent models in Matlab. The experiments using the agent models in the previous section are designed
as follows: (1) How do the over confidence agents affect the market behaviors? (2) How do the agents with Prospect theory affect the market behaviors? These series of experiments (1) and (2) are to uncover the first research objective. (3) Experiments to measure the volatility change in the market. They address the second research objective.

**Effects of Over Confidence Agents**

We set the ratio of rational agents and trend predictor agents to 500:500, and the trend predictor agents have over confidence characteristics. From the experiments shown in Figures 7 and 8, the results are also contradictory with theoretical models. This is because the over confidence agents tend to make much more trade than the others.

![Figure 7: Changes of Trading Prices with Over Confidence Agents (500:500 no Constraints)](image1)

![Figure 8: Changes of Cumulative Excess Returns with Over Confidence Agents (500:500)](image2)

**Effects of Agents with Prospect Theory**

We set the ratio of rational agents and agents with Prospect Theory to 500:500, and both kinds of agents makes decisions based on the expected utility maximization. From the experiments shown in Figures 9 and 10, the results are also contradictory with theoretical models. Interesting phenomena are that when the theoretical prices decrease, the trading prices also tend to randomly decrease. The cases also suggest that if the agents would not have rational decision making policies, the trading prices would become different from the theoretical ones and the rational agents would have less benefits.

![Figure 9: Changes of Trading Prices with Prospect Theory Agents (500:500 no Constraints)](image3)

![Figure 10: Changes of Cumulative Excess Returns with Prospect Theory Agents (500:500 no Constraints)](image4)

**Analyzing Volatility of the Market**

Before estimating the simulation models, We examine the three models with real data. In each model, we will estimate the value $y_t = c + \xi_t$.

Table 1 and 2 are summaries to apply the above three models to real market data about MSCI (Morgan Stanley Capital International) stock index and Dow Jones individual stock market data, respectively. The date are taken during recent 5679 periods.

<table>
<thead>
<tr>
<th>Table 2: Applying the Models to MSCI Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>OLS</td>
</tr>
<tr>
<td>GARCH</td>
</tr>
<tr>
<td>ASYMETRY</td>
</tr>
</tbody>
</table>
On the parameters of GARCH and Asymmetric GARCH models, we use \( p=1 \) and \( p=1 \), that is, GARCH(1,1) and AGARCH(1,1), which are most frequently used in the real data analysis. The values of each item represents how the real coincide with the model measured by AIC (Akaike Information Criteria). The shaded items are the best fit one in each row. From the AIC values, it is clear that Asymmetric GARCH model is the best for the estimation. These real data analysis suggests that we must develop novel theory to be able to well explain the real phenomena in the economic and financial domain.

### Table 3: Applying the Models to Dow Jones Individual Stock Data

<table>
<thead>
<tr>
<th>Simulation Number</th>
<th>GARCH</th>
<th>ASYMMETR GARCH</th>
<th>ASYMMETRIC</th>
<th>OLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10474</td>
<td>10591</td>
<td>10779</td>
<td>10591</td>
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<tr>
<td>2</td>
<td>10474</td>
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<td>10</td>
<td>10474</td>
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</tr>
</tbody>
</table>

To understand the characteristics of volatility of the market and the decision making procedures of the agents, we apply the three models OLS, GARCH, and Asymmetric GARCH to the agent simulator. To evaluate the accuracy of the models, we employ AIC already shown in Tables 1 and 2.

We set various kinds of agent sets as follows: (1) Rational agents only (2) The number of noise predictor agents and trend predictors is set to 500:500. The noise predictor agents predict random values during the simulation. (3) The number of trend predictor agents and rational agents is set to 500:500. (4) The number of agents with Prospect Theory and rational agents is set to 500:500. (5) The number of the simulation experiments are 10. The results are summarized in Table 2. Each item in the table shows the value of AIC. The shaded items are best ones in each simulation run. The results have suggested that in Cases (1), and (2), as implied from the traditional analytic models, OLS model shows the best, however, in Case (3) and (4), GARCH and Asymmetric GARCH models are respectively show the best fit. Compared with the real data in Table 1 and experimental results in Table 2, we draw the conclusion that the proposed agent-base simulation model explains the gap between the macro level analytic models and micro level agent-based simulation.

### RERALTED WORK AND DISCUSSION

Conventional finance theory has developed several prediction models for asset allocation price based on the assumptions that the investors behave rationally, that is, they make decisions in order to maximize their expected utilities. Ingersoll (Ingersoll 1987) have investigated the forms of the utility functions. Hawson et al. (Hawson 1993) have reported Bayesian theoretic decision making is rational if new information will sickeningly arrive when the time passes. Baik and Litterman model (Black 1992) also suggest that the effectiveness of Bayesian theoretic decision making. They consider that the macro level theoretical prices can be derived from the fundamentals of financial markets. However, as Shiller (Shiller 2000) has suggested, real markets are developing their own theories to explain the current prices. On the contrary to such conventional analytic models, in this paper, we employ agent-based simulation models, which are able to ground both macro level phenomena and micro level mental models. Recent agent-based (Axelrod 1997), (Epstein 1996) or microscopic simulation studies (Levy 2000) have frequently suggested the effectiveness of the approaches. However, the research in the literature tends to show too artificial results: The results are so good that the desire of a model builder is already built in the models, and they have very weak relationship to the real world phenomena (Terano 1998). In the proposed models in the paper, we have tried to address the issues. The experimental results using the proposed models have drawn the implications: (1) The "efficient market hypothesis" does not hold when there are some kinds non-rational agents in the market; (2) The roles of such non-rational agents are essential to explain the real world phenomena, in the Market; and (3) Using macro level (A-)GARCH model, micro level Prospect Theory, and agent-based simulation we are able to bridge the theoretical models and real phenomena. These statements are answers to the issues addressed in Section 2.

### CONCLUDING REMARKS

This paper have proposed agent-based simulation models for economic and the finance domain in order to bridge the gap between macro- and micro-level concepts. The models consist of simple agents with rational and/or non-rational decision making functionalities for investment to virtual markets. Although the models are so simple, but the experimental results using both rational and non-rational agents have shown that (1) the efficient market hypothesis does not hold even if there are rational decision making agents in the model, (2) the non-rational decision making agents remains, even when there are some rational ones, and (3) the behaviors of the agents with the characteristics of prospect theory coincide with the estimation by GARCH model. Our future work includes (1) to develop automated tuning meth-
ods for the simulators, (2) to examine both macro and micro level theories to bridge real world phenomena and theoretical models, and (3) to develop much more complex systems to simulate various levels of social and economic behaviors.

References


