The Influence of Investor Sentiment on the Formation of Golden-cross and Dead-cross

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Abstract
The so-called "golden-cross" and "dead-cross" are said to be useful signals to forecast market trends. In this paper, we focus on the Japanese stock market where gold-crosses and dead-crosses are empirically considered as useful investment signals. First, we examined the usefulness of these signals by using historical Japanese stock price data. The results confirmed that these crosses were useful as confirmatory signals for forecasting market trends. The results also showed that the minimum length of period (days) of useful moving averages is shorter in the case of golden-cross than in that of dead-cross. Second, we tried to identify the underlying reasons for the usefulness of the crosses. Because a model, which assumed all investors were rational financial experts, failed to explain the usefulness of the crosses, we were able to assume that the crosses reflected investors' irrationality or behavioral bias: conservativeness and representativeness about trends (Tversky and Kahneman 1974). We then developed a model that incorporated this bias. Based on simulations using this model, we identified the mechanism with which these crosses closely relate to investors' irrationality or behavioral bias. The analysis also revealed investors' tendency that they were convinced by a bull trend more easily and quickly than by a bear trend. This finding is in line with what is generally observed as investors' bull-bias in the Japanese stock market.

1. Introduction
1.1 Definition of golden-cross and dead-cross
Ever since Joseph Granville (1960) introduced, the moving average line has been used as a tool for judging conversion of a trend. The following two patterns, using the moving average lines, are considered as "buy" or "sell" signals: When a shorter (e.g., 50-day) moving average line crosses over a longer one (e.g., 200-day), from below, while the both lines are rising, it is a major buy signal called golden-cross, indicating that the market is in a bull trend; The reverse is called as dead-cross, a sell signal.

Although Fama (1970) tried to deny the usefulness of these crosses by using a random walk model, the golden-cross and dead-cross are still supported by many experienced investors.

1.2 Purpose of the study
The so-called golden-cross and dead-cross are said to be useful signals of market trend forecast in two aspects. One is that these crosses can signal (or spot) a trend's change. And the other is that they can confirm the formation of a new sustainable trend. There have been researches that tried to show that the crosses were able to catch trend changes in several markets, such as US stock, US treasury bonds, Japanese Yen forex, etc. However, there has been no research on the Japanese stock market, particularly about the usefulness as a confirmatory signal of a new sustainable trend.

First, we identified, by applying historical price data, the usefulness of those cross in the Japanese stock market with regard to the above mentioned two aspects. Second, we tried to explain, by investors' behavioral biases, the reason for the usefulness of the crosses, an approach similar to several previous studies explaining abnormal phenomena in financial market by the behavioral biases (Andrei S. 2000). In this paper, we first verify whether a random walk model, which assumed all investor being rational financial experts, can explain the usefulness of the crosses. Then we develop a model incorporating investor's behavioral biases (hereafter we will call this model as an "investor sentiment model"). Similarly, we verify whether an investor sentiment model can explain the usefulness of the crosses. If a random walk model fail to explain the usefulness of the crosses but investor sentiment model can explain it, we will show that the investors' behavioral bias is the reason that made the crosses effective as a buy or sell signal.

1.3 Previous studies about Golden-cross and Dead-cross
Several people tried to analyze the usefulness of golden- and dead-crosses. Among them, Stephan Tayler (1990) tried to analyze it in the currency futures market, by quantifying the profitability of an investment strategy, i.e., buying a fixed amount when a golden-cross appeared and selling off the same fixed amount when a dead-cross appeared. Nauzer Balsara et al (1996) also applied similar investment strategy to such markets as Comex gold, treasury bonds, soybeans and Japanese Yen forex. They examined the usefulness of the crosses and of the optimal sets of dual moving lines. In Nauzer's paper, the short-term moving averages range from 3 to 15 days and the long-term one range from 9 to 45 days.
Both papers concluded that, although the dual moving line system was an effective technical trading rule to some extent, there was found no universally effective set of lines that didn't depend on specific periodic markets.

In this paper, we will examine the usefulness of these signals not only as an indicator of a trend change but also as an indicator of the continuity of a new trend that has started. In addition, we focus on behavioral biases of investors. We identify the underlying reasons for the usefulness of the golden-cross and dead-cross signals by using a model that incorporates behavioral biases.

1.4 Construction of this paper
First, in Section 2, we quantify the usefulness of these signals by calculating various crosses' performance by using Japanese historical data. In Section 3, we verify whether or not the phenomenon found in Section 2 can be simulated by the random walk model. In Section 4, we introduce a model that incorporates a behavioral bias. Finally, in Section 5, we try to explain the underlying reasons for the usefulness of the signals.

2. Analysis Using Historical Data
The purpose of the analysis is to verify whether golden- and dead-cross are useful as a buy or sell signal and what time-spans should be applied.

2.1 Data
We used the historical daily closing prices of the following stocks and indices from August 27, 1991 to December 27, 2001 on the Tokyo Stock Exchange.

2.2 Procedure of the Analysis
The following definitions are used for this analysis.
(1) Monitor the crossing between a n-day short-term moving average line and a 2n-day long-term moving average line.

(2) When a short-term moving average line crosses over a long-term one, from below, while the both lines are rising, this is defined as a golden-cross.

(3) When a short-term moving average line crosses over a long-term one, from above, while both lines are falling, this is defined as a dead-cross.

(4) While both short-term and long-term moving average lines are rising but do not cross, it is defined as a "quasi-golden-cross".

(5) While both short-term and long-term moving average lines are falling but do not cross, it is defined as a "quasi-dead-cross".

(6) We measure the usefulness of a signal for a trend change, by quantifying the profitability of an investment strategy, i.e., buying a fixed amount at the time of a golden-cross appears and selling off the same fixed amount at the time of a dead-cross appears. These are compared with the average profitability during the investigated span.

(7) To measure the usefulness of the crosses as signals for forecasting a new trend's continuity that has just been formed, the return on investment is calculated as (2.1).

\[ rp = \left( \frac{\text{ave}_{n} - 1}{P_0} \right) \text{/} m \]

\[ \text{ave}_{n} \text{ is the } m\text{-day average price after the event (golden-cross, dead-cross, quasi-golden-cross or quasi-dead-cross)} \]

(8) We set a significant level to be 5% and do two side t-tests and check whether there is a significant difference between the performance of a golden-cross and quasi-golden-cross or between that of a dead-cross and quasi-dead-cross.

2.3 Result
The difference in performance between golden-cross and quasi-golden-cross as signals for continuity of a new trend is shown in Fig.1. In Fig.1 (also Fig.2, Fig.3, Fig.4, Fig.5, Fig.6 and Fig.7), the horizontal axis represents \( n \), i.e. the period of each short-term moving average line, and the vertical axis represents the corresponding average price change per day. We changed \( n \) from 10 to 100 and fixed \( m \) to 90.\(^2\) The difference between dead-cross and quasi-dead-cross is shown in Fig.2.

As shown in Fig.1, a golden-cross is not significantly useful for forecasting a new trend's continuity when relatively shorter moving average lines are applied, while it becomes significantly useful when longer ones (\( n>43 \) days) are applied. Likewise, as is shown in Fig.2, a dead-cross is not significantly useful for forecasting a new trend's continuity when relatively shorter moving average lines are applied, while it becomes significantly useful when longer ones (\( n>66 \) days) are applied.

Moreover, the minimum period of moving average when a golden-cross becomes useful is shorter than the case of

\(^1\) Experienced investors often use approximately one-to-two ratio in picking up a short-term and long-term moving averages, e.g., a pair of 13 and 25 weeks lines or a pair of 90 and 200 days ones. Accordingly, we decided to use a pair of \( n \)-days and \( 2n \)-days lines in the analysis.

\(^2\) We only show the result where \( m \) was fixed 90 due to space limitation. However, the similar results were also observed when we fixed \( m \) to other values such as 30, 60 and 120.
dead-cross.

The usefulness as a trend change "spotter", when changing n from 10 to 100 is shown in Fig. 3. In Fig. 3, the horizontal axis represents n and the vertical axis represents profitability of the investment strategy, described in Section 2.2, par day. Fig. 3 shows that golden-crosses and dead-crosses are useful as an indication of a trend change to some extent. However, there seems no universal rule about the set of the lines for showing signals of a trend change.

We can conclude that those crosses are effective as signals for identifying the continuity of a new trend when relatively longer-term moving average lines are applied.

3. Random Walk Model
We tried to verify if a random walk model, which assumed all investor being rational under a weak form of an efficient market hypothesis (EMH), explain the usefulness of the crosses as signals that showed continuity of a new trend. If a random walk model can not explain the usefulness of the crosses.

3.1 The Model Used.
We assume a world where investors are rational and the price are not effected by the past price data.

Here, we assumed that a market satisfied the following conditions. (These Conditions (1)-(4) will also apply to the investor sentiment model in Section 4)

1. In the market, only one asset X is traded at regular intervals. (In this paper, the number of transaction is fixed as 30,000)
2. When we denote X's price at time t (hereinafter "t" or "today") as Pt, the price of the immediate future (hereinafter "t+1" or "Tomorrow") P_{t+1} satisfies (3.1).
   \[ P_{t+1} = P_t + y_t (y_t = \pm \Delta, \Delta: const) \]
   (3.1)
   In short, the price moves only at a constant notch.
3. There is no news which affects the price of X. Every knows the theoretical price of X (which is denoted as P_z).
4. Investors decide the Tomorrow's price of X (or the probability that the price will rise Tomorrow) based on the above mentioned theoretical price and on the series of prices in the past.

We assume that investors predict future prices based on a random walk model. In short, investors pay no attention to previous prices. However, obviously no tendency would appear in a pure random walk model in which investors even pay no attention to the theoretical price. Because investors expect the probabilities that a price would go up or down to be equal, the expectation of 90-day average price after any event should be zero.

Even if prices go up or down, investors predict a price by referring to the theoretical price based on the assumption that they would not deviate significantly. Therefore, the higher the current spot price is, the lower the investors' estimated probability that the future price will go up may become. Also, the lower the current spot price is, the lower the investors' estimated probability that the future price will go down may become.

We will determine the tomorrow's price by (3.2).

\[ f(p_t) = \exp(-w \cdot (p_t - p_z)^2) \]
\[ Pr(p_{t+1} > p_t) = \begin{cases} f(p_t) & (p_t - p_z \geq 0) \\ 1 - f(p_t) & (p_t - p_z < 0) \end{cases} \]

3.2 Result.
We analyzed the usefulness of golden- and dead-cross as signals by using the price data that were obtained through the simulation of the random walk model. We calculated average price change per day in order for measuring the
performance of the crosses as the trend confirming signals. No significant difference was found between a golden-cross and a quasi-golden-cross or between a dead-cross and a quasi-dead-cross. This result indicates that the random walk model cannot explain the reason why a golden-cross or a dead-cross is a useful as the trend confirming signals, whatever value was set to \( w \). The result is shown in Fig.4 and Fig.5 (\( w \) was fixed to be 0.01).

![Random walk model simulation for Golden-cross](image)

![Random walk model simulation for Dead-cross](image)

4. Investor Sentiment Model
The results of Section 3 indicates that we cannot explain why a golden-cross or a dead-cross can be a useful signal as shown in Section 2 if all investors are rational in a weak form efficiency of the EMH.

In this section, we present a new model to capture the result of Section 2 by incorporating the ideas of the following psychological study. This model will be called an investor sentiment model.

Now, we assume that a market also satisfies the conditions (1)-(4) in Section 3.1. However, we assume that investors predict the future prices not based on random walk model but based on upward and downward biased estimation.

4.1 Behavioral Biases
As a nature of the human being in the case of problem solution, when a certain and unexpected phenomenon happens, people do not believe it easily or they may not notice the importance of that event. However, if a certain phenomenon occurs continuously, they start to think that the phenomenon will continue for a while without any reasoning. The former (not to believe certain phenomenon easily) is called as man's "conservativeness" (Edward 1968) and the latter (after a serious of phenomena, to start to believe the continuity of the phenomenon blindly) is called as "representativeness" (Tversky and Kahneman 1974). Moreover, it is known that conservativeness suddenly (not gradually) disappears after certain phenomenon occurs continuously, while representativeness appears instead.

The conservativeness and representativeness affect price trend in a market. When the price fluctuation of a stock comes into a new trend without significant news, people do not believe in the change immediately. A phenomenon contrary to the previous trend would not immediately lead to the conversion of sentiment. However if a new phenomenon continues, a sudden sentiment conversion would occur at a certain point of time.

Andrei S. (2000) focused on these phenomena and constructed an investor sentiment model to explain anomaly phenomena such as the return reversal effects (De Bondt and Thaler 1985). By taking his model into account, we constructed a model to explain why the golden-cross and dead-cross can be useful signals.

4.2 Upward and Downward Biased Estimation
Our model, like Andrei's model, focuses on conservativeness and the representativeness. It is assumed that investors use the previous prices of a stock for predicting the future prices. Here, we assume that investors predict the future prices based on dual-state Marcov model: Upward trend or downward trend. If an investor thinks that the market is in an upward trend, he or she thinks that the price tend to rise. If an investor thinks that the market is in a downward trend, he or she thinks that the price will fall. In other words, if an investor thinks that the market is in a downward trend, he or she thinks that the price will fall.

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If an investor thinks the market is in an upward trend (\( S_t=1 \)),

\[
\text{Pr}(y_t > 0 | S_t = 1, p_t = 0) = \pi
\]

: Estimated probability that the price will rise

(4.1)

\[
\text{Pr}(y_t < 0 | S_t = 1, p_t = 0) = 1 - \pi
\]

: Estimated probability that the price will fall

(4.2)

\[0 < \pi < \frac{1}{2}\]

(4.3)

If an investor think the market is in an upward trend (\( S_t=1 \)),

\[
\text{Pr}(y_t > 0 | S_t = 1, p_t = 0) = 1 - \pi
\]

(4.4)

\[
\text{Pr}(y_t < 0 | S_t = 1, p_t = 0) = \pi
\]

(4.5)

The transition probability is fixed as

\[
\begin{pmatrix}
\text{Pr}(S_t = 1 | S_{t-1} = 1) & \text{Pr}(S_t = 1 | S_{t-1} = -1) \\
\text{Pr}(S_t = -1 | S_{t-1} = 1) & \text{Pr}(S_t = -1 | S_{t-1} = -1)
\end{pmatrix} = \begin{pmatrix}
1 - \lambda_1 & \lambda_1 \\
\lambda_2 & 1 - \lambda_2
\end{pmatrix}
\]

(4.6)

Now we set \( q_t \) to be the probability that an investor thinks the state to be in a downward trend at time \( t \).
Therefore, the following probabilities are given.

\[
\text{Pr}(y_1 > 0 | \rho_1 = 0) = (1 - q_1)(1 - \pi) + q_1\pi = g(t) \tag{4.7}
\]
- Investor's estimated probability that the price will rise

\[
\text{Pr}(y_1 < 0 | \rho_1 = 0) = 1 - g(t) = (1 - q_1)\pi + q_1(1 - \pi) \tag{4.8}
\]
- Investor's estimated probability that the price will fall

\[q_{t+1} = \frac{((1 - \lambda_1)q_t + \lambda_2(1 - q_t))\pi}{((1 - \lambda_1)q_t + \lambda_2(1 - q_t))(1 - \pi)} \quad \text{(if } y_{t-1} > 0) \tag{4.9}
\]

\[q_{t+1} = \frac{((1 - \lambda_1)q_t + \lambda_2(1 - q_t))(1 - \pi)}{((1 - \lambda_1)q_t + \lambda_2(1 - q_t)(1 - \pi) + (\lambda_1q_t + (1 - \lambda_2)(1 - q_t))(1 - \pi))\pi \quad \text{(if } y_{t-1} < 0) \tag{4.10}
\]

4.3 Determination based on the Biases and the Theoretical Price
Investors predict the future prices based on the estimation explained in Section 4.2 and the theoretical price explained in Section 3.1. In this model, investors' prediction follows fundamentally a behavioral bias explained in Section 4.2, but the model fulfills the property that a higher current price leads to a lower probability, estimated by the investors, of further price increase. Therefore we determine the tomorrow's price as (4.11).

\[
\text{Pr}(y_1 > 0) = \begin{cases} f(p_1) \cdot g(t) & (p_1 - p_x \geq 0) \\ 1 - f(p_1) \cdot (1 - g(t)) & (p_1 - p_x < 0) \\ \exp(-w \cdot (p_1 - p_x)^2) \cdot (q_1 + (1 - q_1)(1 - \pi)) & (p_1 - p_x \geq 0) \\ 1 - \exp(-w \cdot (p_1 - p_x)^2) \cdot (q_1(1 - \pi) - (1 - q_1)\pi) & (p_1 - p_x < 0) \end{cases} \tag{4.11}
\]

4.4 Optimization
This model involves many parameters (\(\pi, w, \lambda_1, \lambda_2\)). We optimized the parameters\(^3\) so that the simulation results closely replicate that of the actual data (Fig.1, Fig.2) in terms of the period (days) of moving average lines when the crosses became useful for forecasting a new trend's continuity.

4.5 Result of the Golden-cross and Dead-cross Performance
We measure the usefulness of golden- and dead-cross by using the price data obtained from the simulation of investor sentiment model, in the same manner as it was done in Section 2. By adjusting parameters (to \(\pi = 0.39, w = 0.01, \lambda_1 = 0.001, \lambda_2 = 0.008\), we succeed in reproducing the results as shown in Fig.6 and Fig.7\(^4\).

Fig.6 shows that a golden-cross in a relatively shorter

\[^3\] \(\pi\) ranges from 0 to 0.5, adjusted in increment of 0.01. \(w\) ranges from 0 to 1, adjusted in increment of 0.001. \(\lambda_1\) ranges from 0 to 0.5, adjusted in increment of 0.001. \(\lambda_2\) ranges from 0 to 0.5, adjusted in increment of 0.001.

\[^4\] In case of the optimal solution, the degree of conformity is significantly higher in predominance compared to the other combination.

4.6 Result of Analysis on Investor Sentiment
We analyzed how the investor sentiment changes immediately before a cross emerged by calculating the average \(q_t\) before and after the cross. Fig.8 is the result when the span of the shorter moving average is fixed to 80 days (\(n = 80\)). In Fig.8 the horizontal axis represents days before formation of golden-cross and the vertical axis represents \(q_t\). Fig.8 shows a sudden and rapid change of investor sentiment from downward bias to upward bias.
5. Discussion

5.1 Reasons for the Usefulness of the Crosses
From this model, we identified the mechanism with which those crosses closely relate to investors’ irrationality or behavioral bias. In addition, from Fig.8, we can say that, in most of the case, golden-cross and dead-cross are considered to be relevant to the change of investors’ sentiment from downward bias to upward bias or vice-versa. Consequently, we conclude that the crosses can be signals indicating consensus that a new trend has been formed.

If the period is too short, the crosses may not become effective because the so-called “cheat” or “noise” are picked up. However, if the period is long enough, the crosses can forecast a sustainable trend. Therefore, after a golden-cross formation, an upward tendency continues for a while under the influence of bull bias. Similarly, after a dead-cross formation, a downward tendency continues for a while under the influence of bear bias.

However, there seem no obvious rules about a “right” period of the moving averages in which the crosses are particularly effective for identifying a trend change. If you choose a relatively shorter period, the cross may pick up deceiving moves while, if you choose relatively longer period, the cross may pick up true trends too late.

5.2 What We Can Say from the Parameter?
From the model, the peculiarity of the Japanese stock market that has been empirically known was also identified.
After optimization, each parameter was calculated as \( \pi = 0.39, \ w = 0.01, \ \lambda_1 = 0.001, \ \lambda_2 = 0.008. \) \( \lambda_1 \) represents the probability that the investors think the market trend from upward to downward. \( \lambda_2 \) represents the probability that the investors think the market trend from downward to upward.

Therefore, \( \lambda_1 < \lambda_2 \) represents that the investors are less responsive to downward movements than upward movements. This conclusion corresponds to what is often observed in the Japanese stock market as one of its peculiarities.

The Japanese investor is said to be "bull-biased", i.e., to believe a bull trend more easily and quickly than a bear trend. A fact supporting the existence of this bull-bias is that, in a stock margin transaction, long positions always exceed short positions (Nikkei News Article 2001). In other words, more people prefer to "go long" than to "go short". The simulation result \( (\lambda_1 < \lambda_2) \) supports the existence of this bull-bias.

6. Conclusion
By using the historical Japanese price data, it was confirmed that golden- and dead-cross are effective as signal for continuity of a new trend if appropriate days (relatively longer days) of moving averages are used, a point that was not thoroughly discussed by the previous works. Moreover, a golden-cross becomes an effective signal sooner (in shorter days) than a dead-cross in terms of days of moving averages.

From the simulation, we succeeded to identify the mechanism with which those crosses closely relate to investors' irrationality or behavioral bias, which relationship has not been well discussed. The analysis also revealed investors' tendency that they are convinced by a bull trend more easily and quickly than by a bear trend. This finding is in line with what is empirically observed as investors' bull-bias in the Japanese stock market.

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