The Nine Comments on the RCC Theory

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Abstract
This paper comments on nine aspects of the Region Connection Calculus (the RCC theory) with conclusions as follows: (1) regions in the RCC theory shall not be points; (2) the identical relation is a deviation from its classic usage; (3) the operator is un-determinate; (4) it is unclear about the notion of region in the theory; (5) atomic regions have zero thickness, instead of non-zero thickness as is assumed in the theory; (6) there is room for simple distance relations in the RCC theory; (7) there is no room for the congruence relation in the theory; (8) properties of the connection relation are not sufficiently characterized; (9) the connection relation formalized in the theory is not primitive for spatial representations in general.

Motivation
The Region Connection Calculus (the RCC theory) has been accepted as one dominant theory in the QSR (Qualitative Spatial Representation) community. However, we found some mysteries in the RCC theory during our research work. In this paper, we list nine mysteries and provide our understanding and comment. The paper is structured as follows: We first very briefly introduce the RCC theory, as we assume that readers are quite familiar with it; then we list nine questions and present our comments and answers, if any.

An outline of the RCC theory
The RCC theory (Randell et al. 1992) has two axioms to govern the connection relation 'C' between regions: (1) for any region \( x \), \( x \) connects with itself; (2) for any regions \( x \) and \( y \), if \( x \) connects with \( y \), then \( y \) also connects with \( x \). The two axioms are formalized as follows.

\[
\forall x [C(x, x)] \\
\forall xy [C(x, y) \rightarrow C(y, x)]
\]

Several spatial relations are further formalized using the connection relation. For example, that region \( x \) disconnects from region \( y \), \( DC(x, y) \), is defined as \( \neg C(x, y) \). That \( x \) is a part of \( y \), \( P(x, y) \), is defined as \( \forall z [C(z, x) \rightarrow C(z, y)] \). That region \( x \) is a proper part of region \( y \), \( PP(x, y) \), is defined as \( PP(x, y) \equiv P(x, y) \wedge \neg P(y, x) \).

Can regions be points?
In this section, we show that in the RCC theory a region cannot be a point. That two regions overlap (\( O \)) is defined in the theory as that there exists a region which is part of both of the two regions, i.e., \( O(x, y) \equiv \exists z [P(z, x) \wedge P(z, y)] \).

That two regions are externally connected (\( EC \)) is defined as that they are connected and do not overlap, i.e. \( EC(x, y) \equiv C(x, y) \wedge \neg O(x, y) \), which means that their closures share a point and there is no region which is shared by both of them. If the theory allows a region to be a point, that there is no region which is shared by both of them will further lead to that there is no points which is shared by both of them. This is unfortunately equivalent to that the two regions are disconnected, and this makes the definition of \( EC \) self-contradictory. Therefore, a region in the RCC theory cannot be a point. Following the same lines of arguments, we can come to the conclusion that regions in the RCC theory cannot be lines, curves, etc.
What does identical mean?

In the RCC theory, two regions $x$ and $y$ being identical (EQ or $=$) is defined as that each is part of the other. This is equivalent to that for any region $z$, $z$ connects with $x$, if and only if $z$ connects with $y$, i.e., $\forall z[\text{C}(z, x) \leftrightarrow \text{C}(z, y)]$. As $\text{C}(x, y)$ is interpreted in the theory as closures of $x$ and $y$ share a common point, two regions being identical means their closures are perfectly coincided. That is, if region $x^2 + y^2 \leq 4$ and region $x^2 + y^2 < 4$ are two regions in the RCC theory, they will be identical; if region $x^2 + y^2 \leq 4, (x, y) \neq (0, 0)$ and region $x^2 + y^2 \leq 4$ are two regions in the RCC theory, they will be identical. This means that the identical relation in the RCC theory is a deviation from its classic theory, which can be stated as follows: $x = y$, if, and only if, $x$ has every property which $y$ has, and $y$ has every property which $x$ has, (Tarski 1946, pp. 55). Therefore, in the classic sense, neither region $x^2 + y^2 \leq 4$ and region $x^2 + y^2 < 4$, nor region $x^2 + y^2 \leq 4, (x, y) \neq (0, 0)$ and region $x^2 + y^2 \leq 4$ are identical. They only have the same closure. This questions the unique denotation of the $\iota$ operator in the RCC theory.

Which region does the $\iota$ operator denote?

In the RCC theory the sum $(x, y)$ function is defined as the region $w$ such that for any region $z$ connecting with $w$, if and only if $z$ connects with $x$ or $y$.

$$\text{sum}(x, y) \overset{\text{def}}{=} \{w | \forall z[\text{C}(z, w) \equiv [\text{C}(z, x) \lor \text{C}(z, y)]]\}$$

Let $x$ and $y$ be two non-closed regions in the RCC theory, then the question is raised: which region can be the sum of $x$ and $y$, the joint region of $x$ and $y$, the joint region of $\text{cl}(x)$ and $y$, the joint region of $x$ and $\text{cl}(y)$, the joint region of $\text{cl}(x)$ and $\text{cl}(y)$, or others? — there are indeed many regions which are identical with the joint region of $x$ and $y$ in the RCC theory. Classically, $\iota y[\text{so-and-so}]$ means 'the so-and-so' and 'the so-and-so' will be false if there are no so-and-so’s or several so-and-so’s (Russell 1956, pp.92). Therefore, the $\iota$ operator in the RCC theory is a deviation from the classic usage. Our question is: Can $\iota y[\text{so-and-so}]$ mean that any region $y$ so-and-so and that all these regions are identical in the RCC theory? The advantage of this explanation is that the uniqueness of the $\iota$ descriptor is guaranteed inside the RCC theory. Let us examine this explanation by considering one property of the complement function defined in the RCC theory. The $\text{compl}(x)$ function is defined in (Randell et al. 1992, pp.168) as the region $y$ such that for any region $z$ two conditions hold: (1) $z$ connects with $y$ if and only if $z$ is not non-tangential proper part of $x$; (2) $z$ overlaps $y$ if and only if $z$ is not part of $x$.

$$\text{compl}(x) \overset{\text{def}}{=} \{y | \forall z[\text{C}(z, y) \leftrightarrow \neg \text{NTPP}(z, x)] \land [\text{O}(z, y) \leftrightarrow \neg \text{P}(z, x)]]\}$$

It holds that $\forall x[\text{compl}([\text{compl}(x)] = x]$. Let regions $x$, $y$, $w$, and $v$ be regions as illustrated in Figure 1. Region $x$ consists of two parts: an extended part and a curve; region $y$ is the extended part of $x$; the complement of region $x$ is identical with the complement of region $y$ in the RCC theory. And the complement of the complement of region $x$ is region $y$, not identical with region $x$. The theorem $\forall x[\text{compl}([\text{compl}(x)] = x]$ will not be held. We would conclude that $\iota y[\text{so-and-so}]$ cannot be explained as the $y$ so-and-so in the RCC theory, and that if it is explained as any $y$ so-and-so with the condition that all these $ys$ are identical inside the RCC theory, there must be some restrictions on the notion of regions in the RCC theory, otherwise some theorems in the theory will not be held.

What kinds of regions are regions?

The RCC theory stipulates what kinds of regions are regions as follows: for every region $x$ there is region $y$ such that $y$ is non-tangential proper part of $x$, (Randell et al. 1992, pp.169).

$$\forall x \exists y[\text{NTPP}(y, x)]$$

From this stipulation we know immediately that neither points nor curves should be called as regions in the theory, as we know that they do not have non-tangential part. The question is: Does this stipulation exclude other kinds of regions? Is region $x$ in Figure 1 a region in the theory? As region $x$ has a non-tangential proper part, it should be a valid region in the theory. In general, if there is a region $r$ which has non-tangential proper part, then for any region $p$ including points and curves, the sum of $r$ and $p$ has non-tangential proper part. Suppose the RCC theory were a country whose
citizens are *regions*, the above stipulation serves as a judge who decides whether a person is a citizen of the country or not. This judge would be rather ‘forgiving’ — for any person who is not a citizen of the country, he only needs to find a citizen of the country and say to the judge, “we are together”. Then, the judge will say, “you both as a whole is a citizen of the country and say to the judger, “we are to-

gion that in some work the notion of atomic regions has been introduced inconsistency, as mentioned in (Randell et al. 1992, pp. 173-174). What we would mention here is that for any regions, z and w, if z connects with x, w connects with x, then z connects with w.

∀z, w[C(z, x) ↔ C(w, x)]

**Proof**

This theorem shows that for any regions w and z, if they connect with an atomic region, they will connect with each other. Formula (5) ∀z[C(z, x) ↔ C(w, z)] is equivalent with ∀w[C(z, x) ↔ C(w, z)], which means that for any region z, if an atomic region connects with z, then it is part of z. All these imply that an atomic region has zero thickness, instead of non-zero thickness. Including regions with zero thickness into the theory would cause inconsistency. And including atomic regions into the theory indeed introduced inconsistency, as mentioned in (Randell et al. 1992, pp. 173-174). What we would mention here is that in some work the notion of point is characterized just the same as the notion of the atomic region. For example in (Smith 1996, pp. 289), a point x, written as Pt(x) is defined as ∀y[Pt(x) → y = x], which means for any y, if y is part of x, then y is identical to x. From this point, we will be easier to understand why introducing atomic regions into the theory would cause inconsistency — atomic regions are in fact points, and regions in the theory cannot be points.
Is there room for distance in the RCC theory?

In this section, we discuss whether there is room for distance relations in the RCC theory. The answer seems positive, as this is at least evidenced in (Cohn and Bennett 1997, 7): An alternative interpretation of C might be given informally by saying the distance between the two regions is zero. That is, the RCC theory can also be viewed as a calculus whose primitive relation is the distance between two regions is zero. Further, let us think about the distance relation as follows: ‘distance-less-than-or-equal-to-one-foot(\(x, y\))’, read as “region \(x\) is less than or equal to one foot away from region \(y\)”. This distance relation is reflexive and symmetrical: \(x\) is less than or equal to one foot away from itself (reflexive); if \(x\) is less than or equal to one foot away from \(y\), then \(y\) is less than or equal to one foot away from \(x\) (symmetrical). This is also a valid interpretation for the \(C\) relation. Generally, let \(p\) be a non-negative real number, ‘distance-less-than-or-equal-to-\(p\)-meter(\(x, y\))’ is a reflexive, and symmetrical relation and therefore can be an interpretation of \(C\). When \(p\) is zero, the theory is a connection calculus; when \(p\) is greater than zero, the theory would be a simple distance calculus. The distance between \(x\) and \(y\) being more than \(p\) meter can be represented by \(\neg C(x, y)\). We conclude that although the RCC theory is named as a calculus with the connection relation, it can also be used as a calculus for simple distance relations.

Can the congruence relation between regions be delineated in the RCC theory?

The congruence relation (the geometric equivalence) is one of the main topics in geometry, (Flegg 1974). Two objects being congruent means that one can be perfectly placed on the other, as illustrated in Figure 3. In the RCC theory the congruence relation between regions should be interpreted as the closure of one region can be perfectly placed on the closure of the other. Our question is: can the congruence relation between regions be represented in the RCC theory? This is equivalent to ask: is there such a formula \(G(x, y)\) that \(G(x, y)\) holds if and only if regions \(x\) and \(y\) are congruent, where \(G(x, y)\) is constructed by the relation \(C\) and notions in the first order logic. If \(G(x, y)\) is such a formula, then \(\neg G(x, y)\) is such that when \(y\) is congruent with \(x\), it does not hold, and that when \(y\) is not congruent with \(x\), it holds. This is also a valid formula to determine whether two regions are congruent, but in a non-conventional way – just like, in some countries people give positive attitude by shaking the head, and give negative attitude by nodding the head. Therefore, the property of \(G(x, y)\) can be stated as follows:

Are properties of the connection relation sufficiently characterized by the axioms?

The RCC theory uses the connection relation \(C\) as the primitive relation. Two axioms characterize properties of the connection relation as reflexive and symmetrical: \(\forall x[C(x, x)]\) and \(\forall xy[C(x, y) \rightarrow C(y, x)]\). We claim that the two properties do not sufficiently characterize properties of the connection relation. If it were, the two axioms should be able to deduce any property of the connection relation. Let us think about one property as follows: if two regions are connected, then for any region \(p\), we can find a place for it, so that \(p\) can connect with the two regions. This property even holds, when \(p\) is a point — if \(p\) is located at any of the shared part of the closures of the two connected regions, \(p\) will connect with both of them. This property can also be stated as follows: for any regions \(x\), \(y\), and \(z\), if \(x\) connects with \(y\), then there is region \(z'\) such that \(z'\) is congruent with
z, and that \( z' \) connects with \( x \) and \( y \), as illustrated in Figure 4. As the congruence relation can not be delineated in the RCC theory, this property of the connection relation cannot be represented either. So, we conclude that the RCC theory does not delineate all primitive properties of the connection relation. However, this will not be a problem, if those delineated properties of the connection relation can construct all spatial properties that the theory wishes to represent.

**Is C primitive for spatial relations?**
The relation C is primitive in the RCC theory, as all the spatial relations appear in the theory are defined through it. However, the fact that the axioms in the RCC theory can neither characterize nor deduce one property of the connection relation leads to the question: are there spatial relations which can be interpreted through the connection relation but cannot be represented in the RCC theory? If yes, we will have to conclude that C is not primitive for spatial relations, even the connection relation is. In the above section, we have shown that the congruence relation cannot be developed through C. In the following part, we will show that the C relation in the RCC theory cannot describe more than two distance relations in one interpretation. By doing so we argue that the C relation in the RCC theory is not a primitive relation for spatial relations, in the sense that it cannot develop sufficient distance relations.

The definition of distance in general has been discussed in mathematical philosophy and general topology, (Russell 1903), (Kelley 1955). However, such mathematical or philosophical perspectives to view the notion of distance relation do not meet the motivation for grounding the RCC theory which took the naive point of view: From the standpoint of our naive understanding of the world, this topological structure is arguably too rich for our purposes, . . . . Given the choice between two possible theories used for formal representing space, the ease by which a person can understand and use the theory must be taken into account. The basic part of the original theory . . . required the user to be familiar with general topology . . . . We thought this restriction could be eased . . . . (Randell et al. 1992, pp. 166). We therefore will take a naive point of view to analyze one understanding of distance relations, and show why such understanding is beyond the descriptive power of the C relation. Ancient Egyptians used the notion of Royal Egyptian Cubit to describe distances. The length of a Cubit is defined as the length of the forearm from the bent elbow to the tip of the extended middle finger plus the width of the palm of the hand of the Pharaoh or King ruling at that time. The distance between two objects being no more than one Cubit can be understood as there is a cubit which can connect with the two objects. If C is interpreted as one kind of distance relation, e.g. \( C(x, y) \) is interpreted as the distance between \( x \) and \( y \) is no more than one Cubit, there will be two basic distance relations distinguished: the distance between two regions is less than or equal to one Cubit, C, and the distance between two regions is more than one Cubit, \( \neg C \). The question is raised: Can distance relations, such as less than two/three/four Cubits be described in the RCC theory? To answer this question, we shall first make clear the meaning of the distance being less than two Cubits. If the distance between \( x \) and \( y \) being less than or equal to one Cubit means that there is a region which is congruent with Cubit such that this region connects with \( x \) and \( y \), the distance between \( x \) and \( y \) being less than or equal to two Cubits shall mean that there are two regions which are congruent with Cubit and such that when they connect with one another, they shall connect with \( x \) and \( y \). However, as we have shown that there is no room for the congruence relation in the RCC theory, there is also no room for relations which are based on the congruence relation, such as distance relations.

**References**