(Some) Default and Non-Monotonic Aspects of Qualitative Spatial Reasoning A Preliminary Report

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Abstract

We present some scenarios where default and/or non-monotonic reasoning patterns are either necessary or useful for the modelling of dynamic spatial environments. The identified instances bear a direct relationship to the fundamental epistemological issues relevant to the frame and ramification problems; these are cases where a typical use of non-monotonicity is necessary at a meta-theoretical or domain-independent level. Furthermore, non-monotonic reasoning is also necessary whilst modelling the appearance and disappearance of spatial objects – these phenomena, considered inherent to dynamic spatial systems, essentially involve the representation of an incompletely known domain of discourse. The final case, from the viewpoint of this paper, involves the use of non-monotonic reasoning for modelling causal explanation tasks in an abductive manner. Indeed, the non-monotonic patterns we illustrate are only indicative of the ones encountered and accounted for in the context of our key task of developing a situation calculus based domain-independent qualitative spatial theory that is usable in diverse dynamic spatial domains. The identification of other similar patterns and the general utility of non-monotonic reasoning from a specific spatial reasoning viewpoint is an important research agenda, and this paper calls for a further investigation of the same within the mainstream qualitative spatial reasoning domain.

Introduction

The integration of specialised spatial representation and reasoning techniques within general common-sense and/or logic-based reasoning frameworks is an important next-step for their applicability in realistic domains. This integration is non-trivial and requires unification along ontological, representational and computational fronts, i.e., a paradigm such as 'Reasoning about Space, Actions and Change' needs to be pursued and promoted. Indeed, this is also closely related to the general problem pertaining to the sub-division of endeavours in AI and the development of a unifying semantics for logic-based common-sense reasoning and other specialised reasoning domains such as qualitative spatial reasoning. The point is adequately summed up by Shanahan [1995]:

> '*If we are to develop a formal theory of commonsense, we need a precisely defined language for talking about shape, spatial location and change. The theory will include axioms, expressed in that language, that capture domain-independent truths about shape, location and change, and will also incorporate a formal account of any non-deductive forms of commonsense inference that arise in reasoning about the spatial properties of objects and how they vary over time*'

Past work has pursued the broad agenda of integrating the specialisation of 'qualitative spatial reasoning'[1] within general logic-based frameworks in AI. Specifically, as a means to narrow down this problem of integration, the emphasis has been on operationalising a 'dynamic spatial systems' approach for the modelling of changing spatial environments [Bhatt and Loke, 2008, Bhatt, 2008]. Here, a dynamic spatial system is regarded as a specialisation of the dynamic systems [Sandewall, 1994] concept for the case where spatial configurations undergo change as a result of interaction within the environment, i.e., as a result of explicitly identified events and actions within the system being modelled. In this context, we have investigated the construction of a 'domain-independent qualitative spatial theory', which may be utilised in diverse application scenarios that involve the modelling of dynamically varying spatial information. Grounded in this previous work, we identify and illustrate some instances where default and/or non-monotonic reasoning patterns are either necessary or useful for representing and reasoning about dynamic spatial environments. The identified instances are a direct product of our attempt to operationalise the aforementioned notion of a qualitative spatial theory within the framework of the situation calculus, which is a general formalism for modelling dynamic domains [McCarthy and Hayes, 1969]. Specifically, and in so far as the scope of this paper is concerned, the identified requirements for non-monotonic reasoning fall within the following conceptual categories:

- Epistemological: These instances bear a direct relationship to the fundamental epistemological issues relevant to the frame and ramification problems, i.e., these are cases

[1] A comprehensive review of qualitative spatial reasoning can be found in [Cohn and Hazarika, 2001]

where appeal to non-monotonicity is necessary at a meta-theoretical level

- Phenomenal: Non-monotonicity is also necessary whilst modelling phenomena involving appearance and disappearance of spatial objects – this essentially involves representing an incompletely known domain of discourse. We hypothesize that modelling of other phenomenal aspects that are considered intrinsic to dynamic spatial systems will lead to the identification of other scenarios whereas non-monotonic reasoning is necessary and/or useful
- Reasoning tasks: The final case, given the scope of this paper, involves the use of non-monotonic reasoning for modelling causal explanation tasks in an abductive manner. Here, the application of non-monotonic reasoning is intrinsic to an abductive formalisation of explanation [Shanahan, 1993, 1997]

Indeed, the non-monotonic patterns we identify and illustrate are only indicative of the ones encountered and accounted for in the context of a limited task involving the development of the situation calculus based domain-independent qualitative spatial theory [Bhatt and Loke, 2008, Bhatt, 2008]. The identification of other patterns and the general utility of non-monotonic reasoning from a specific spatial reasoning viewpoint is an open and important research agenda, and this paper calls for a further investigation of the same within the mainstream qualitative spatial reasoning domain.

Dynamic Spatial Environments and Need for Non-Monotonicity

Application domains such as intelligent systems, spatial information systems, temporal and event-based GIS and cognitive robotics involve the modelling of dynamically varying spatial information in one way or another. These application domains require modes of reasoning that are richer than those provided by conventional constraint-based reasoning techniques that are prevalent in the qualitative spatial reasoning domain [Renz and Nebel, 2007]. Of course, the utility of the conventional apparatus is not disputed, rather, the point here is that a level-of-abstraction higher than that provided by such techniques is essential. For instance, at lower-levels, constraint-based reasoning techniques remain applicable, but at higher (application) levels, other forms of reasoning, e.g., in the form of spatial planning and explanation, in the context of general logic-based reasoning frameworks are both necessary as well as useful for applicability in realistic domains such as the ones aforementioned.

Consider, for instance, a logic-driven intelligent system that, amongst other things, has a spatial component involving representing and reasoning with qualitative spatial knowledge. One application requirement here is to model reasoning tasks that involve non-monotonic knowledge updates in the form of spatial information assimilation (e.g., by way of explanation) and other forms of useful reasoning tasks involving spatial property projection and spatial planning. Indeed, if the spatial component is to be based on existing (qualitative) theories of space, it is essential that such theories (precisely, qualitative spatial calculi pertaining to differing aspects of space such as topology [Randell et al., 1992], orientation [Moratz et al., 2000]) be embedded with the general logic-based framework under consideration. Most importantly, it is essential that the following high-level axiomatic semantics of the calculi being embedded be preserved:

1. jointly exhaustive and pair-wise disjoint property of the relations
2. other basic properties of the relations including symmetry, asymmetry and transitivity
3. compositional inference and consistency maintenance
4. a primitive notion of change based on the principle of conceptual neighbourhoods [Freksa, 1991]
5. relative entailments between calculi when more than one spatial calculus is being modelled in a non-integrated manner (i.e., there exist separate composition tables)

Additionally, it is also desired that phenomenal aspects (e.g., *appearance* of new objects) considered inherent in typical dynamic spatial systems be accounted for within the spatial theory that is required to be embedded within the general logic-based reasoning framework. The main objective of such an embedding is that all modes of reasoning that are supported by the containing logical framework, which in our case is the situation calculus, can then be directly exploited for modelling useful reasoning tasks in the context of the spatial component. However, the endeavour is fraught with several complexities – if a true integration of the specialisation of qualitative spatial reasoning within a general logic-based reasoning framework is to be achieved, key requirements from a specific 'dynamic spatial systems' viewpoint need to be accounted for:

1. Qualitative physics: seamless integration of a domain-independent 'qualitative physics' that is based on existing qualitative theories of space
2. Causal and teleological aspects: support for modelling and reasoning with dynamic teleological and causal accounts of a system or process in addition to the representation of the underlying (qualitative) physics (i.e., an integration of the 'how' and the 'why' aspects of spatial change)
3. Epistemological issues: investigation of the implications of some of the fundamental epistemological problems (frame, ramification and qualification), which have otherwise assumed a primary significance within the symbolic artificial intelligence domain, for the special case of dynamic spatial systems
4. Non-monotonic reasoning: incorporation of non-monotonic or default forms of inference, which is necessitated by the requirement to model human-like common-sense reasoning patterns and the need to account for the fundamental epistemological issues in (3), and
5. Concurrency in the spatial domain: an account of concurrency and continuity for the specialised spatial domain in the context of existing temporal reasoning approaches

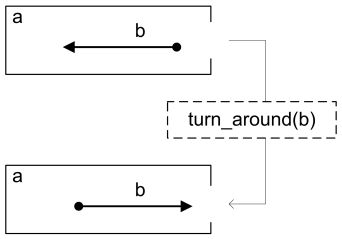

Figure 1: Spatial Property Persistence

Indeed, there exist no clear benchmarks for the fulfilment of each of the above-stated requirements. In so far as the scope of this paper is concerned, we focus on the aspects that are related to requirements (1), (3) and (4) from above.

Default and Non-monotonic Aspects of Spatial Reasoning

Several forms of non-monotonic inference are useful, and in some cases even necessary, when reasoning about changing spatial relationships within a dynamic spatial environment. The objective in this paper is to intuitively illustrate the cases using examples; axiomatisation and treatment using the situation calculus formalism have been covered in detail elsewhere [Bhatt and Loke, 2008, Bhatt, 2008]. Although not strictly necessary, basic familiarity with the representational aspects of situation calculus will certainly aid the discussion to follow:

Spatial Property Persistence ('the frame problem')

Spatial property persistence, i.e., the intuition that the spatial relationship between two objects typically remains the same, is one default reasoning pattern rooted in the frame problem that is identifiable within the spatial context. For instance, assuming that dynamic topological and orientational information constitutes the state descriptions corresponding to the unique '*situations*'[2], the problem is that of formalising the intuition that the topological relationship between two objects or the orientation of an object relative to another '*typically*' remains the same, unless if there is '*cause*', whatever be the nature of such cause, to believe to the contrary. Consider Fig. 1, which qualitative depicts the relationship of an *agent*, modelled as a directed line-segment ('b') to a containing object ('a') that is interpreted as a *room*. Given that the spatial relationship of the agent with that of the room is that of containment, i.e., $inside(b,\ a)$, the problem of spatial property persistence is that of formalising the intuition that this containment relationship persists in the situation resulting from the occurrence of an action such as $turn\_around$. At least one other instance, addressing this line of investigation, can be found in the work of Shanahan [1995]. Within a real-valued co-ordinate system, Shanahan investigates the

[2]A situation can be interpreted as a unique node within a branching-tree structured situation space. Corresponding to each such node or 'situation' is a state description denoting which dynamic properties hold in that situation.

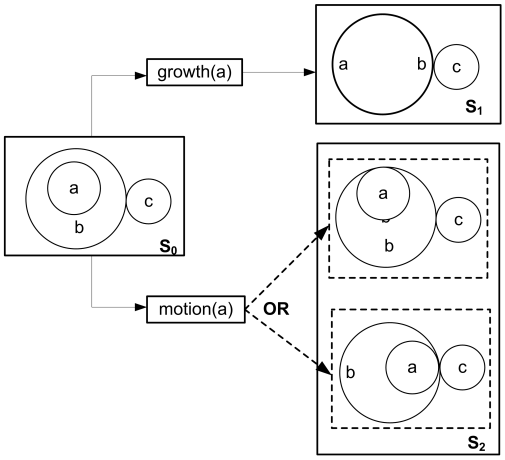

Figure 2: Compositional Constraints and Ramifications

default reasoning pattern required to model a different kind of property persistence, also connected to the frame problem, required to model the common-sense law that '*space is typically empty*'.[3] For instance, an agent would need to make such a default assumption before moving itself or moving other objects to a certain region of space or when other domain specific occurrences have happened.

In the context of the situation calculus formalism, a generic frame assumption incorporating the principal of inertia whilst deriving the successor state axioms [Reiter, 1991] is sufficient to handle this and other similar scenarios that involve spatial property persistence.[4]

Global Consistency of Spatial Information ('the ramification problem')

Spatial situation descriptions denoting configurations of domain objects, i.e., by way qualitative spatial relationships relevant to one or more spatial dimensions that hold between the objects of the domain, must be globally consistent in adherence to the compositional constraints of the underlying qualitative space, i.e., all composition theorems as applicable for a particular spatial domain being modelled (e.g., topology, orientation) should be satisfiable for the spatial situation descriptions. The notion of compositional consistency also includes those scenarios when more than one aspect of space is being modelled in a non-integrated way, i.e., relative dependencies between mutually dependent spatial dimensions that are modelled explicitly too should be satisfiable. Ensuring these two aspects of global consistency of spatial information is non-trivial because the compositional constraints contain indirect effects in them thereby necessi-

[3]Depending on the purpose for which such a default assumption is applied, one could argue that this also has connections to the qualification problem. But this is a different issue altogether since the common-sense law remains the same.

[4]Successor state axioms constitute the causal laws of the domain – these determine what changes as a result of an occurrence within the (spatial) system being modelled [Reiter, 1991].

tating a solution to the ramification problem [Finger, 1987], i.e., the problem of indirect effects. In the context of the situation calculus, Lin [1995] illustrates the need to distinguish ordinary state constraints from indirect effect yielding ones, the latter being also referred to as *ramification constraints*. This is because when ramification constraints are present, it is possible to infer new effect axioms (or simply effects) from explicitly formulated (direct) effect axioms together with the ramification constraints. Simply speaking, ramification constraints lead to what can be referred to as '*unexplained changes*', which is clearly undesirable within a theory of change. In the spatial representation task, i.e., the embedding of qualitative spatial calculi within the spatial theory [Bhatt, 2008], indirect effect yielding constraints are a recurring problem – as mentioned already, modelling composition theorems and axioms of interaction (using ordinary state constraints) leads to unexplained changes since the resulting constraints contain indirect effects in them. For instance, this is evident whilst performing compositional inference with the (*three*) mutual spatial relationships involving the trivial case of *three* objects o_1, o_2 and o_3 – when o_1 and o_2 undergo a transition to a different qualitative state, this also has an indirect effect on the relationship between o_1 and o_3 since the relationship between the latter two is constrained by the compositional constraints of the relational space. As an example, consider the illustration in Fig. 2 – the scenario depicted herein consists of the topological relationships between three objects 'a', 'b' and 'c'. In the initial situation 'S_0', the spatial extension of 'a' is a *non-tangential part* of that of 'b'. Further, assume that there is a change in the relationship between 'a' and 'b', as depicted in Fig. 2, as a result of a direct effect of an event such as *growth* or an action involving the *motion* of 'a'. Indeed, as is clear from Fig. 2, for the spatial situation description in the resulting situation (either 'S_1' or 'S_2'), the compositional dependencies between 'a', 'b' and 'c' must be adhered to, i.e., the change of relationship between 'a' and 'c' must be derivable as an indirect effect. In a trivial scenario, such as the present one, consisting of few objects, it could be correctly argued that the indirect effects can be completely formulated as direct effects. However, for a more involved scene description n objects and complete *n-clique* descriptions consisting of $n(n-1)/2$ spatial relationships for every spatial domain (e.g., topology, orientation, size) being modelled is impractical and error prone. The situation is only complicated given that fact that some of the spatial domains being modelled could be inter-dependent.

Ramification, Causality and Non-Monotonic Reasoning Whilst the details are not relevant here, it suffices to point out that a solution to the problem of ramifications for this particular case (of ensuring global compositional consistency of spatial scene descriptions) is obtainable from the general works of Lin and Reiter [1994], Lin [1995]. The solution basically involves appeal to causality (i.e., modelling all ramification yielding constraints in the form of causal rules) and applying non-monotonic reasoning (using circumscription) to minimise the effects of occurrences whilst deriving the successor state axioms or the causal laws

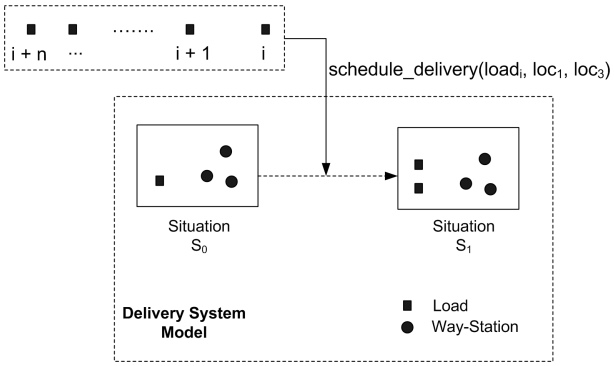

Figure 3: Appearance Events - Delivery Example

of the domain. Note that this manner of deriving the successor state axioms is an extension to the original approach proposed by Reiter [1991], where only a solution to the frame problem is included under a general 'completeness assumption' stipulating that there are no indirect effects within the domain theory. The application of this approach to ensuring global consistency of spatial scene descriptions has been illustrated in detail in [Bhatt and Loke, 2008].

Phenomenal Aspect - Appearance and Disappearance of Objects

Appearance of new objects and disappearance of existing ones, either abruptly or explicitly formulated in the domain theory, is characteristic of non-trivial dynamic spatial systems. In robotic applications, it is necessary to introduce new objects into the model, since it is unlikely that a complete description of the robot's environment is either specifiable or even available. Similarly, it is also typical for a mobile robot operating in a dynamic environment, with limited perceptual or sensory capability, to lose track of certain objects because of issues such as noisy sensors or a limited field-of-vision. As an example, consider a '*delivery scenario*' in which a vehicle/robot is assigned the task of delivering '*object*(*s*)' from one '*way-station*' to another (see Fig. 3). In the initial situation description, the domain consists of a finite number of '*way-stations*' and deliverable '*objects*'. However, the scheduling of new objects for delivery in future situations will involve introducing new 'objects' into the domain theory. For example, an external event[5] such as '$schedule\_delivery(new\_load,\ loc_1,\ loc_3)$' introduces a new object, namely '*new_load*', into the domain.

Appearance and disappearance events involving the modification of the domain of discourse are not unique to applications in robotics. Even within the projected next-generation of event-based and temporal geographic information systems, appearance and disappearance events are regarded to be an important typological element for the modelling of dynamic geospatial processes [Claramunt and Thériault, 1995, Worboys, 2005]. For instance, Claramunt and Thériault [1995] identify the basic processes used to define a set

[5]External events are those occurrences that do not have an associated occurrence criteria and may therefore occur abruptly.

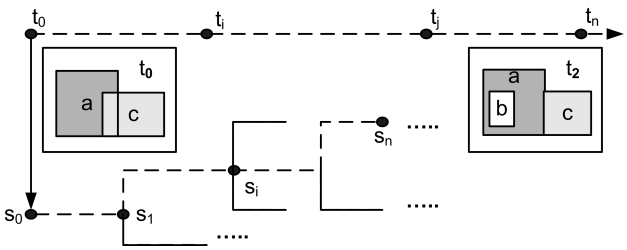

Figure 4: Appearance Events - Propagating Existential Facts

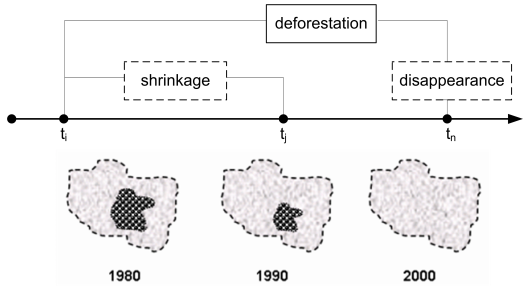

Figure 5: GIS Domain - Deforestation

of low-order spatio-temporal events which, among other things, include appearance and disappearance events as fundamental. Similarly, toward event-based models of dynamic geographic phenomena, Worboys [2005] suggests the use of appearance and disappearance events at least in so far as single object behaviours are concerned. We regard that such phenomena, being intrinsic to a typical dynamic spatial system, merit systematic treatment.

Maintaining and Propagating Existential Facts From a representational viewpoint, introducing new objects in the domain poses a problem since there is no general way to deal with an incompletely known domain of discourse. For instance, let $< s_0, s_1, s_2, \ldots, s_n >$ denote a situation-based linear history or one branch within the branching-tree structure of the overall situation space (see Fig. 4). From a dynamic spatial system perspective, each state corresponding to every situation with this history is primarily a set denoting the spatial configuration of objects in that situation. Further assume that an object 'b', that is unknown or not a part of the dynamic 'spatial configuration set' in the initial situation 's_0', comes into existence (by an appearance event) in a later situation, say 's_2'. At this point, it is necessary to incorporate the non-existence of 'b' in the situations preceding 's_2' by (non-monotonically) propagating its non-existence backwards into the situation-based history. In fact, appearance of previously unknown objects is the only reason '*existential facts*' about objects need to be included as propositional fluents / dynamic properties at a domain-independent level. The case of disappearing objects is trivial and simply involves negating and object's existential status upon the occurrence of disappearance events. Indeed, an object that is known but has disappeared may not participate in spatial relationships with other objects, until such a time when it reappears.[6]

Reasoning Requirement – Explanation as Abduction

Explanation, in general, is regarded as a converse operation to temporal projection essentially involving reasoning from effects to causes, i.e., reasoning about the past [Shanahan, 1989]. Precisely, given a set of time-stamped observations or snap-shots (e.g., observation of a mobile-robot or time-stamped GIS data), the objective is to explain which events and/or actions may have caused the resulting state-of-affairs. In the context of the situation calculus formalism, Shanahan [1993, 1997] proposes a non-monotonic approach that utilises circumscription as a basis of minimisation (of effects) and explanation derivation (in terms of potential occurrences). We have specialised this approach toward the formulation of an abductive occurrence-driven causal explanation task, where a set of time-ordered observations (e.g., pertaining to spatial configurations) may be explained in terms of the spatial actions and events that may have caused the observed state-of-affairs [Bhatt, 2008].

The non-monotonicity required in modelling explanation tasks is characteristic to modelling explanation problems abductively in general, rather than being peculiar to spatial reasoning tasks. However, one aspect of this non-monotonicity is characteristic to a spatial reasoning task – in deriving minimal models or explanations of observations consisting of changing spatial configurations, it is possible that the derived explanations may be *inadequate*, i.e., may not include domain-specific occurrences that have caused the observed changes. For instance, consider a geographic information system domain / scenario as depicted in Fig. 5. At a domain-independent level (i.e., at the level of a general spatial theory), the scene may be described using topological and qualitative size relationships. Consequently, the only changes that are identifiable at the level of the spatial theory are *shrinkage* and eventual *disappearance* – this is because a domain-independent spatial theory may only include a generic typology of spatial change at the most. However, at a domain-specific level, these changes could characterise a specific event (or process) such as, for instance, *deforestation*. The hypotheses or explanations that are generated during a explanation process should necessarily consist of the domain-level occurrences in addition to the underlying (associated) spatial changes (as per the generic typology) that are identifiable. That is to say, that the derived explanations be '*adequate*' and more or less take a form such as: '*Between time-points t_i and t_i, the process of deforestation is abducible as one potential hypothesis*'. To achieve this adequacy, a model-filtration heuristic that disregards those models (i.e., explanations) that do not include any domain-specific (spatial) occurrences (actions or events) leads to explanations that are adequate, if such explanation exists per se – this is because minimal models that only consist of a domain-independent explanation (e.g., in the form

[6]Presently, upon reappearance, it is presumed that an object's identity is maintained.

of *shrinkage*, *disappearance* and a temporal-order between these two) would be excluded by such a filtration heuristic.

Other potential solution to achieve adequacy is to include high-level or domain-specific predicates that relate the domain-independent occurrences (as per the typology) to arbitrary high-level processes that have a domain-dependent interpretation. Notwithstanding the fact that we regard both potential solutions to the problem of achieving adequacy to be rather rudimentary or ad-hoc solutions, it must be pointed out that the model-filtration approach is more general and does not presuppose any information of the domain-independent typology on the part of a domain modeller.

Outlook

We have identified and illustrated some instances where default and/or non-monotonic reasoning patterns are either necessary or useful for the modelling of dynamic spatial environments. Although the details have not been included here, each of the identified instances has been grounded using the situation calculus formalism, using a broader context of modelling dynamic spatial systems [Bhatt and Loke, 2008]. Property persistence, i.e., the spatial relationship between two objects '*typically*' remains the same, is one default reasoning pattern connected to the frame problem that was encountered here. Similarly, the ramification problem arises whilst modelling constraints containing indirect effects (e.g., compositional constraints, axioms of interaction between inter-dependent spatial domains). It is also evident that several forms of non-monotonic inference are useful (e.g., in deriving the successor state axioms and also in explanation tasks), and in some cases even necessary (e.g., compositional constraints), when reasoning about changing spatial relationships between objects. Non-monotonic reasoning is also required to model explanation tasks in an abductive manner. Clearly, in addition to aspects already accounted for, a closer look at application-level use-cases that may benefit from default or non-monotonic reasoning patterns is an important next step and deserves utmost priority. The identification of 'common-sense' laws or reasoning patterns encountered in qualitative reasoning about space – patterns that are applicable in general, or for specific spatial reasoning tasks, is one of the most interesting and important research agendas, at least from the viewpoint of the future direction of this work.

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