# Propositional Attitudes in Non-Compositional Logic

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**Abstract**

Several authors analyzed propositional attitudes (*wish*, *fear*, *regret*, *glad*) by integrating their epistemic and deontic components. This paper extends previous work done by the author and presents a logical calculus inspired by Possibility Theory, a non-compositional version of fuzzy logic.

## Linguistic properties of *glad, regret, wish, fear*

**Fact 1:** The factual predicates *glad* and *regret* presuppose the agent's *knowledge* of the propositional content (Kiparsky & Kiparsky, 1971). Situations that cannot be known cannot be object of these attitudes.

(1) a. I'm so glad that he will return in episode seven.
b. #Joan is glad that she will win the Texas lottery scratchcard jackpot (Telegraph, 13 Aug 2011).

(2) a. You regret that he will move to Singapore.
b. #He regrets that there is an earthquake tomorrow.

**Fact 2:** The predicates *wish* and *fear* presuppose the agent's *lack of knowledge* of the propositional content (Gerner 2010a). Events that are known to happen cannot be object of wishes and fears.

(3) a. I wish that you'd grow up.
b. #I wish that I will get married tomorrow.

(4) a. Syria's neighbors fear that fighting could spread.
b. #John fears that he has an elder brother.

**Fact 3:** No agent who is right in his mind can wish or be glad about events that he values as bad. In the same vein, no event that is rated positively in the value system of a human agent can be regretted or feared by him.

(5) a. #The president was glad that his words had been misinterpreted.
b. #Bill regrets that a solution to his problems exists.



(6) a. #The king wished that the royal wedding would be spoiled by riots.
b. #I fear that someone, somewhere, may be happy.

**Constraints:** These facts about attitudinal predicates can be captured by Searle-style preparatory and propositional conditions (Searle, 1969, 1979; Gerner, 2010a,b,c).

| | Preparatory | Propositional |
|---|---|---|
| *glad* | the agent *knows* φ based on background information | φ is *not bad* in the value system of a human agent |
| *regret* | the agent *knows* φ based on background information | φ is *not good* in the value system of a human agent |
| *wish* | the speaker does not *know* φ based on background information | φ is *not bad* in the value system of a human agent |
| *fear* | the speaker does not *know* φ based on background information | φ is *not good* in the value system of a human agent |

## The logic of propositional attitudes

Propositional attitudes like *glad*, *regret*, *wish*, *fear* have epistemic and deontic components (Heim, 1992; Gerner, 2010a,b,c). The computation of these components can be modeled by necessity/possibility measures in the sense of Possibility Theory (Dubois & Prade, 1988; 2001).

Let SENT be the set of Boolean propositions and let the following epistemic and deontic modalities be given: NESS, POSS, OBLI, PERM: SENT $\rightarrow$ {0, 1} such that NESS (epistemic) and OBLI (deontic) are two necessity measures, and POSS (epistemic) and PERM (deontic) two possibility measures. We can capture the agent's epistemic state and deontic state of $\varphi \in$ SENT in the following way (Dubois & Prade, 2001: 40):

(7) The agent's epistemic state of φ
(NESS(φ), NESS(¬φ)) =
  a. (1,0) iff the agent *knows* φ;
  b. (0,1) iff the agent *knows* ¬φ;
  c. (0,0) iff the agent neither knows φ nor ¬φ;
  d. (1,1) iff the agent *knows* both φ and ¬φ

(8) The agent's deontic state of φ
(OBLI(φ), OBLI(¬φ)) =
  a. (1,0) iff φ is *good/binding*;
  b. (0,1) iff φ is *bad/forbidden*;
  c. (0,0) iff φ is neither *good* nor *bad* (permissible);
  d. (1,1) iff φ is both *good* and *bad*.

We exclude the case of contradictory beliefs and contradictory values in (5d) and (6d). The preparatory and propositional conditions on the predicates *glad*, *regret*, *wish*, *fear* can be formalized as follows.

| | Preparatory | Propositional |
|---|---|---|
| *glad* | NESS(φ) = 1 | OBLI(¬φ) = 0 |
| *regret* | NESS(φ) = 1 | OBLI(φ) = 0 |
| *wish* | NESS(φ) = 0 | OBLI(¬φ) = 0 |
| *fear* | NESS(φ) = 0 | OBLI(φ) = 0 |

These conditions can be used to define four Boolean measures GLAD, REGR, WISH, FEAR: SENT → {0, 1} that model the attitudinal predicates *glad*, *regret*, *wish*, *fear*.

(9) Definition of GLAD, REGR, WISH, FEAR
SENT → {0, 1}

$$\varphi \rightarrow \mathrm{GLAD}(\varphi) = \begin{cases} 1 \text{ if } \mathrm{NESS}(\varphi) = 1 \textit{ and } \mathrm{OBLI}(\neg\varphi) = 0 \\ 0 \text{ if otherwise} \end{cases}$$

$$\varphi \rightarrow \mathrm{REGR}(\varphi) = \begin{cases} 1 \text{ if } \mathrm{NESS}(\varphi) = 1 \textit{ and } \mathrm{OBLI}(\varphi) = 0 \\ 0 \text{ if otherwise} \end{cases}$$

$$\varphi \rightarrow \mathrm{WISH}(\varphi) = \begin{cases} 1 \text{ if } \mathrm{NESS}(\varphi) = 0 \textit{ and } \mathrm{OBLI}(\neg\varphi) = 0 \\ 0 \text{ if otherwise} \end{cases}$$

$$\varphi \rightarrow \mathrm{FEAR}(\varphi) = \begin{cases} 1 \text{ if } \mathrm{NESS}(\varphi) = 0 \textit{ and } \mathrm{OBLI}(\varphi) = 0 \\ 0 \text{ if otherwise} \end{cases}$$

As NESS(¬φ) = 1 – POSS(φ) and OBLI(¬φ) = 1 – PERM(φ), we can also represent the four measures as follows.

SENT → {0, 1}
  φ → GLAD(φ) = NESS(φ) × PERM(φ)
  φ → REGR(φ) = NESS(φ) × PERM(¬φ)
  φ → WISH(φ) = POSS(¬φ) × PERM(φ)
  φ → FEAR(φ) = POSS(¬φ) × PERM(¬φ)

This definition is part of a more general system in which we can replace '×' in the above definition by '∗' which in bivalent logics is interpreted as ordinary multiplication and in multi-valued logics as *continuous t-norm* (Hajék 1998: 28; Gottwald 2008). We can define the following **16** logical attitudes of which at least **four** are lexicalized in human languages: WISH, FEAR, GLAD, REGR..

(10) Definition of 16 propositional attitudes

| Predicate | | Formula | Gloss |
|---|---|---|---|
| | | POSS(φ) ∗ PERM(φ) | think possible & not-bad |
| | | POSS(φ) ∗ PERM(¬φ) | think possible & not-good |
| WISH(φ) | = | POSS(¬φ) ∗ PERM(φ) | not-know & not-bad |
| FEAR(φ) | = | POSS(¬φ) ∗ PERM(¬φ) | not-know & not-good |
| | | POSS(φ) ∗ OBLI(φ) | think possible & good |
| | | POSS(φ) ∗ OBLI(¬φ) | think possible & bad |
| | | POSS(¬φ) ∗ OBLI(φ) | not-know & good |
| | | POSS(¬φ) ∗ OBLI(¬φ) | not-know & bad |
| GLAD(φ) | | NESS(φ) ∗ PERM(φ) | know & not-bad |
| REGR(φ) | | NESS(φ) ∗ PERM(¬φ) | know & not-good |
| | | NESS(¬φ) ∗ PERM(φ) | think impossible & not-bad |
| | | NESS(¬φ) ∗ PERM(¬φ) | think impossible & not-good |
| | | NESS(φ) ∗ OBLI(φ) | know & good |
| | | NESS(φ) ∗ OBLI(¬φ) | know & bad |
| | | NESS(¬φ) ∗ OBLI(φ) | think impossible & good |
| | | NESS(¬φ) ∗ OBLI(¬φ) | think impossible & bad |

## Non-compositional logic

### Introduction

Dubois & Prade (2001) distinguish four types of logics by compositionality properties of the underlying confidence measure.

(11) A confidence measure *g*: SENT → V is defined by
  a. V = {0,1} or [0,1];
  b. $g(\underline{\mathbf{0}}) = 0$ and $g(\underline{\mathbf{1}}) = 1$;
  c. $g(\varphi \wedge \psi) \leq g(\psi)$ and $g(\varphi) \leq g(\varphi \vee \psi)$.

The following types of logics are based on the degree of compositionality of the confidence measure *g* (Dubois & Prade 2001: 55).

(12) *g* is fully compositional iff
  a. $g(\neg\varphi) = 1 - g(\varphi)$;
  b. $g(\varphi \vee \psi) = \max(g(\varphi), g(\psi))$;
  c. $g(\varphi \wedge \psi) = \min(g(\varphi), g(\psi))$.

As examples, we can mention the classical Boolean Logic (V = {0,1}, $g = \|\ \|$) or diverse fuzzy logics (Hájek 1998; Gottwald 2008).

(13) $g$ is compositional for negation only iff
$g(\neg\varphi) = 1 - g(\varphi)$.

Logics with this property are logics based on probability measures.

(14) $g$ is compositional for disjunction only iff
$g(\varphi \vee \psi) = \max(g(\varphi), g(\psi))$.

Possibility Logic defined by possibility measures is compositional for disjunction only (Zadeh, 1978; Dubois & Prade, 1988, 2001).

(15) $g$ is compositional for conjunction only iff
$g(\varphi \wedge \psi) = \min(g(\varphi), g(\psi))$.

Necessity measures are the dual measures of possibility measures; they are compositional for conjunction only and also lead to Possibility Logic (Dubois & Prade, 1988, 2001).

## The logic of confidence measures

Let $g$, $h$: SENT $\rightarrow$ V be two confidence measures and let $*$: $V^2 \rightarrow V$ be a *t-norm* which is a function that is (i) commutative and associative; (ii) non-decreasing for both arguments ($x_1 \leq x_2 \Rightarrow x_1 * y \leq x_2 * y$ and $y_1 \leq y_2 \Rightarrow x * y_1 \leq x * y_2$); (iii) absorbing ($1 * x = x$ and $0 * x = 0$).

**Negation**
We define the measure $g^{\neg}$: SENT $\rightarrow$ V by $g^{\neg}(\varphi) = g(\neg\varphi)$. The measure $g^{\neg}$ is compositional for $\neg$ iff $g$ is compositional for $\neg$.

(16) Lemma:
$g^{\neg}(\neg\varphi) = 1 - g^{\neg}(\varphi)$ iff $g(\neg\varphi) = 1 - g(\varphi)$.

The measure $g^{\neg}$ is non-compositional for $\wedge$ and $\vee$ if $g$ is only compositional for $\wedge$.

(17) Lemma:
If $g$ is only compositional for $\wedge$, then $g^{\neg}$ is non-compositional for $\wedge$ and $\vee$.

Proof:

a. Let us show that $g^{\neg}$ is non-compositional for $\wedge$:
As $g$ is non-compositional for $\vee$, there are $\varphi$, $\psi \in$ SENT such that $g(\varphi \vee \psi) > \max(g(\varphi), g(\psi))$. Let us pose $\xi = \neg\varphi$ and $\chi = \neg\psi$. We have $g^{\neg}(\xi \wedge \chi) = g(\varphi \vee \psi) > \max(g(\varphi), g(\psi)) \geq \min(g(\varphi), g(\psi)) = \min(g^{\neg}(\xi), g^{\neg}(\chi))$.

b. Let us show that $g^{\neg}$ is non-compositional for $\vee$:
As $g$ is a confidence measure, there are $\varphi$, $\psi \in$ SENT with $\min(g(\varphi), g(\psi)) < \max(g(\varphi), g(\psi))$. Let us pose $\xi = \neg\varphi$ and $\chi = \neg\psi$. It follows that $g^{\neg}(\xi \vee \chi) = g(\varphi \wedge \psi) = \min(g(\varphi), g(\psi)) < \max(g(\varphi), g(\psi)) = \max(g^{\neg}(\xi), g^{\neg}(\chi))$.

The same situation holds for $g^{\neg}$ if $g$ is compositional for $\vee$ only. The proof is omitted.

(18) Lemma:
If $g$ is compositional for $\vee$ only, then $g^{\neg}$ is non-compositional for $\wedge$ and $\vee$.

**Conjunction**
The confidence measure $g * h$ is compositional for $\wedge$ if and only if $g$ and $h$ are.

(19) Lemma:
$g{*}h(\varphi \wedge \psi) = \min(g{*}h(\varphi), g{*}h(\psi))$ iff
$g(\varphi \wedge \psi) = \min(g(\varphi), g(\psi))$ and
$h(\varphi \wedge \psi) = \min(h(\varphi), h(\psi))$.

Proof:
As $*$ is decreasing in both arguments, we have:
if $h(\varphi) \leq h(\psi)$, then $g(\varphi){*}h(\varphi) \leq g(\varphi){*}h(\psi)$;
if $h(\psi) \leq h(\varphi)$, then $g(\psi){*}h(\psi) \leq g(\psi){*}h(\varphi)$.
It follows that $\min(g(\varphi){*}h(\varphi), g(\psi){*}h(\psi)) =$
$\min(g(\varphi){*}h(\varphi), g(\varphi){*}h(\psi), g(\psi){*}h(\varphi), g(\psi){*}h(\psi))$.
Now it is obvious that $g(\varphi \wedge \psi){*}h(\varphi \wedge \psi) =$
$\min(g(\varphi){*}h(\varphi), g(\varphi){*}h(\psi), g(\psi){*}h(\varphi), g(\psi){*}h(\psi))$
iff $g(\varphi \wedge \psi) = \min(g(\varphi), g(\psi))$ and
$h(\varphi \wedge \psi) = \min(h(\varphi), h(\psi))$.

**Disjunction**
The confidence measure $g * h$ is non-compositional for $\vee$ independently of whether $g$ and $h$ are compositional.

(20) Lemma:
There are $\xi, \chi \in$ SENT such that
$g{*}h(\xi \vee \chi) \neq \max(g{*}h(\xi), g{*}h(\chi))$.

Proof:
As $g$ is a confidence measure, there is $\xi \in$ SENT with $g(\xi) < 1$ and $g(\neg\xi) < 1$. Let us pose $\chi = \neg\xi$. From the property of t-norm it follows that $g(\xi){*}h(\xi) < 1$ and $g(\chi){*}h(\chi) < 1$. Furthermore, we have $g(\xi \vee \chi){*}h(\xi \vee \chi) = 1$, as $g$ and $h$ are confidence measures. With these choices we have $\max(g(\xi){*}h(\xi), g(\chi){*}h(\chi)) < 1 = g(\xi \vee \chi){*}h(\xi \vee \chi)$.

### Propositional attitudes are non-compositional

As NESS/OBLI are necessity and POSS/PERM possibility measures, the lemmas in the preceding sections ensure the following compositionality properties.

| $g$ | compositional for | $h$ | compositional for |
|---|---|---|---|
| POSS(φ) | $\vee$ | PERM(φ) | $\vee$ |
| POSS(¬φ) | --- | PERM(¬φ) | --- |
| NESS(φ) | $\wedge$ | OBLI(φ) | $\wedge$ |
| NESS(¬φ) | --- | OBLI(¬φ) | --- |

The 16 propositional attitudes defined in the last section are fully non-compositional with one exception. The measure NESS(φ) ∗ OBLI(φ) is compositional for $\wedge$ only.

| Attitudes | | Definition | compositional for |
|---|---|---|---|
| | | POSS(φ) ∗ PERM(φ) | --- |
| | | POSS(φ) ∗ PERM(¬φ) | --- |
| WISH(φ) | = | POSS(¬φ) ∗ PERM(φ) | --- |
| FEAR(φ) | = | POSS(¬φ) ∗ PERM(¬φ) | --- |
| | | POSS(φ) ∗ OBLI(φ) | --- |
| | | POSS(φ) ∗ OBLI(¬φ) | --- |
| | | POSS(¬φ) ∗ OBLI(φ) | --- |
| | | POSS(¬φ) ∗ OBLI(¬φ) | --- |
| GLAD(φ) | | NESS(φ) ∗ PERM(φ) | --- |
| REGR(φ) | | NESS(φ) ∗ PERM(¬φ) | --- |
| | | NESS(¬φ) ∗ PERM(φ) | --- |
| | | NESS(¬φ) ∗ PERM(¬φ) | --- |
| | | NESS(φ) ∗ OBLI(φ) | $\wedge$ |
| | | NESS(φ) ∗ OBLI(¬φ) | --- |
| | | NESS(¬φ) ∗ OBLI(φ) | --- |
| | | NESS(¬φ) ∗ OBLI(¬φ) | --- |

## Illustrations

The predicates *glad*, *regret*, *wish* and *fear* encode propositional predicates that are non-compositional for ¬, $\wedge$, $\vee$. We illustrate below that English sentences mirror the logical properties. ("Counterfactual attitudes" are marked by '%'.)

### Negation (¬)

(21) WISH(φ) = 1 and WISH(¬φ) = 0
  a. The farmer wishes that it rains.
  b. The farmer wishes that it does *not* rain.

(22) WISH(φ) = 0 and WISH(¬φ) = 0
  a. #John wishes that New Year's Eve will fall on the 1st of January.
  b. %John wishes that New Year's Eve would*n't* fall on the 1st of January.

(23) REGR(φ) = 1 and REGR(¬φ) = 0
  a. John regrets joining the army.
  b. John regrets not joining the army.

(24) REGR(φ) = 0 and REGR(¬φ) = 0
  a. #Bill regrets that a meteorite will smash his house.
  b. #Bill regrets that a meteorite will not smash his house.

### Conjunction ($\wedge$)

(25) WISH(φ $\wedge$ ψ) = 0 and WISH(φ) $\wedge$ WISH(ψ) = 0 or 1
  a. Nancy wishes to marry Fred and Jim.
  b. Nancy wishes to marry Fred and she wishes to marry Jim too.

(26) REGR(φ $\wedge$ ψ) = 1 and REGR(φ) $\wedge$ REGR(ψ) = 0 or 1
  a. Nancy regrets having invited a four-star general and a peace activist.
  b. Nancy regrets having invited a four-star general and she regrets having invited a peace activist.

### Disjunction ($\vee$)

(27) WISH(φ $\vee$ ψ) = 1 and WISH(φ) $\vee$ WISH(ψ) = 1
  a. Mary wishes Fred or Bill to come.
  b. Mary wishes Fred to come or she wishes Bill to come.

(28) WISH(φ $\vee$ ¬φ) = 0 and WISH(φ) $\vee$ WISH(¬φ) = 1
  a. #Mary wishes that he is alive or dead.
  b. Mary wishes that he is alive or she wishes that he is dead.

(29) REGR(φ $\vee$ ψ) = 1 and REGR(φ) $\vee$ REGR(ψ) = 1
  a. Hilda regrets that Bill or Peter left New York.
  b. Hilda regrets that Bill left New York or she regrets that Peter did so.

(30) REGR(φ $\vee$ ¬φ) = 0 and REGR(φ) $\vee$ REGR(¬φ) = 1
  a. #Mary regrets having a boyfriend or not having a boyfriend.
  b. Mary regrets having a boyfriend or she regrets not having a boyfriend.

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