

# The Logic of Typical and Atypical Instances (LTA)

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## Abstract

The difference between typical instances and atypical instances in a natural categorization process has been introduced by E. Rosch and studied by cognitive psychology and AI. A lot of the knowledge representation systems are expressed in using fuzzy concepts but a degree of membership raises some problem for natural categorizations (especially to classification problems in anthropology, ethnology, archeology, linguistics but also in ontologies), but atypical instances of a concept cannot be apprehended adequately by different degrees from a prototype. Other formal approaches, as paraconsistent logics or non monotonic logics, conceptualize often atypical objects as exceptions. It had yet been developed an alternative way with the logics of determination of the objects (LDO). In this paper, we present the logics of typical and atypical (LTA) in order to give directly a logical approach of typicality / atypicality associated to a concept by a more common way than in LDO, in using only classes and not determination operators. It is introduced a distinction between predicative property and concept defined with its intension and its essence, a part of intension. A typical instance of a concept inherits all properties of intension; a typical instance inherits only properties of essence but it is a full member of the category associated to a concept and not a member with a weak degree of membership. In natural categorization, there are often instances (the exceptions) which do not inherit some properties of the essence; they cannot be considered as atypical instance and belong to the boundary of the category.

## Introduction

Following Rosch's investigations in cognitive psychology Rosch (Rosch, Mervis, 1975; Rosch, 1977) and the studies of natural categorizations (for instance : Le Ny, 1989), in Logics of Determination of Objects (LDO) (Descles, 2002; Descles, Pascu, 2006, 2011; Pascu 2006), it has been developed a formal approach of these problems for taking in account, by means rules, the distinction between typical and atypical instances of a concept. Indeed, *A beaver builds dams* does not at all means that *All beavers build dams* – classically analyzed in Predicate Calculus by an universal quantification ' $(\forall x) (\text{to-be-beaver} \Rightarrow \text{to-build}(x))$ ' – since, by this sentence, it is said that only typical beavers build dams; indeed there are beavers

(for instance the new born beavers does not build dams). How to conceptualize this notion of "typical instance" of a concept from a logical viewpoint? The distinction between typical and atypical instances of a given concept is very important to organize and to structure an ontology because all instances have not exactly the same inheritance properties. Furthermore, there are instances that cannot be conceptualized neither as typical instances, neither typical instances: they are instances located only at the boundary of the category defined by a concept; these instances are exceptions inside the category

In an applicative formalism (Church's  $\lambda$ -calculus or Curry's combinatory logic (Curry, Feys 1958), functional programming), an expression is built by applications of operators onto operands for building results. With the Church's *functional types*, different functional types of operators can be considered. In following G. Frege (1967/1893), a concept (or a predicative property or predicate) is always unsaturated (or uncompleted) whereas an object is always saturated (or completed). Thus, from an applicative viewpoint, a predicative property is thought as an operator and an object as being never an operator. A *predicative operation* is an application of a concept-operator to an-object-operand for building either a new operator or an operand of another concept or also a proposition (an absolute operand). In LDO, an unique canonical object ' $\tau f$ ' is associated to each predicative property ' $f$ '; the object ' $\tau f$ ' is the best representation, as an object, of the concept ' $f$ '; it is a *fully indeterminate typical object* of ' $f$ '. For example, the fully indeterminate typical object ' $\tau(\text{"to-be-a-man"})$ ' is associated to the predicative property "to-be-a-man"; it is expressed by the linguistic expression *a man*; this saturated object is different from the unsaturated property "to-be-a-man". *Determination operators* are also introduced in LDO; they are expressed by linguistic expressions as (...) *living New-York*, (...) *having U.S.-citizenship*) and are associated to properties (as "to-live-New-York" or "to-have-U.S.-citizenship"). Each determination operator generates more determinate objects; for example, the objects expressed by the nominal phrases as *a man living in New-York*, or *a man having U.S.-citizenship*, are more determinate than the typical

indeterminate object ‘a-man’ = ‘ $\tau$ (“to-be-man”)’. The indeterminate objects ‘a-man-having-two-hands’, ‘a-man having-two-legs’, ‘a-man-having-two-eyes’ are *typical representative objects* falling under the concept “to-be-a-man”, whereas the object ‘a-one-legged-man’ is an *atypical representative object* thought equally falling under “to-be-a-man”. All the *fully determinate objects* associated to a predicative property ‘f’ (or the fully determinate objects that fall under ‘f’) constitute the *extension* of ‘f’, noted ‘Ext(f)’. The set of more or less indeterminate objects, for instance the objects that are generated from typical object ‘ $\tau f$ ’ by means of determination operators associated to other properties, constitute the *expansion* of the concept ‘f’, noted ‘Exp(f)’. Of course:  $\text{Exp}(f) \supseteq \text{Ext}(f)$ . By this way, one can describe atypical objects among the more or less indeterminate objects. To account for this distinction, we must introduce *intension of a concept* and the *essence*, a part of intension, and articulate the intension to expansion and extension. We define later these two notions. It is important to not that “intension” used in this paper is not taken in Carnap’s sense but in following the traditional approach opened by the so called “La logique de Port Royal” (Arnauld, Nicole, 1662) in the XVII<sup>th</sup> century. The whole problem of typicality/atypicality led us no longer considered the duality between extension and intension, according to the well-known law: “intension (or “comprehension”) increases if and only if expansion (“étendue” in Port Royal’s terms) reduces”. We will explain later this lack of duality. With the formalism of LDO, there are taken into account not only predicative properties (or predicates), objects (nominal terms and propositions) and quantifiers (operators acting onto concepts) but also *determination operators* (directly expressed in natural languages by adjectives, relative constructions, genitive constructions...) and the possibility to generate more and less indeterminate objects from the *fully indeterminate typical object* ‘ $\tau(f)$ ’ associated to a predicative property ‘f’. Thus, LDO becomes an extension of the “classical” Language of First Order Predicates (LFOP).

Whereas the LDO uses determination operators, this paper describes a more directly logical approach of typicality/atypicality, independently of determinations and fully indeterminate typical objects, allowing us to take into account objects which being no longer atypical, are nevertheless on the external outer edges of the category (for instance at the boundary). Otherwise, ontologies of domains are structured networks of concepts and of classes of objects. Generally, in the building of ontologies, the problem of typical/atypical is not considered: atypical objects are viewed only as exceptions without doing a deep “logical” analysis of different kinds of objects (especially by introducing in the formalism the notions of intension and essence of a concept). An atypical object must be

considered as “fully belonging” to the category and not as an object located on the edges of the category. We start with an example, where is analyzed a concept and a resulting categorization when with typical, atypical objects and exceptions.

## Logical analysis of “to be inhabitant of X”

### Example

The concept “to be inhabitant of X” is clearly related to the other properties “to live in X”, “to have an address in X” and “to have duties”, and also “to pay taxes” and “to have the power to vote”. These properties are constituents of the intension of the analyzed concept. All inhabitants of the city ‘X’ are not substitutable for one another. Some of them, the citizens, have the citizenship, others, the foreigners residing in the city have not. When they are older than 18 years, the citizens must pay taxes and they are also the power to vote. The foreigners pay also taxes but, generally, they have not the power to vote, though in some countries some foreigner residents have also this power with the condition: they are residents during 5 years or more. One must consider also inhabitants without fix residence (homeless) or being irregular residents (for instance tramps without an identification card). The model associated to the concept “to be inhabitant of X” is presented by the network of the figure 1. In this network, the comprehension relations between properties of intension are expressed by arrows. The relation ‘ $u \rightarrow v$ ’ (‘u’ “comprehends” ‘v’) means that the property ‘u’ includes (or comprehends) the property ‘v’, that is:  $(\forall x) [u(x) \Rightarrow v(x)]$ . To understand the *concept* “to be inhabitant of X”, it is to take into account not only the constitutive predicative property “to be inhabitant of X” but also its intension and its essence defined by the class of properties “to live in X”, “to have an address in X”, “to have duties”. The essence is the heart of the concept but it not the concept as a whole. It is the pair intension / essence which gives the formal tools to understand and to manipulate a concept. This network raises also other predicative properties: “to have citizenship”, “to be a foreigner”, “to be a resident”, “to be a non-resident”. A family of classes of objects structures the class of instances of the concept “to be inhabitant of X”. However, these classes are not defined in duality with the predicative properties. For example, if we consider the comprehension relation between “to be citizen under 18 old” and “to be inhabitant with U.S. citizenship”, we see that objects falling under the first concept does not inherit the properties “to have the power to vote” and “to pay taxes”. Similarly, the foreign residents are foreign inhabitants who pay taxes without

having always the power to vote since only some foreign residents have obtained this power.

## Definitions

A *predicative property* ‘f’ (relative to individual objects) is an operator acting onto the fully determinate objects of a domain ‘D’ giving as results one of truth values  $\{T, \perp\}$ : an object ‘x’ “is falling under f” (or “is not falling under ‘f’”) when ‘f(x) = T’ (or ‘f(x) =  $\perp$ ’). This definition is exactly identical to Frege’s concept. However, following (Descles, Pasqu, 2011), we consider that a *concept*, noted ‘ $\wedge f$ ’, is more complex than its underlying predicative property ‘f’. Indeed, a concept is defined with its associated intension, whereas a predicative property is alone and not associated with an intension. Thus, a concept is characterized on one hand, by a structured “intension” compound by other concepts more general associated to it and organized in a network, structured by the closure ‘ $\rightarrow^*$ ’ of a *comprehension relation* ‘ $\rightarrow$ ’, and, on the other hand, the typical objects inherit all the properties of its intension but atypical objects inherit only a part of intension, called the *essence* of the concept. The definitions of the extension, the intension and the essence of a predicative property ‘f’ are given as follows:

$$\begin{aligned} \text{Ext}(f) &= \{x; f(x) = T\} \\ \text{Int}(f) &= \{g; f \rightarrow^* g; (\forall x) (f(x) = T \Rightarrow g(x) = T)\} \\ \text{Ess}(f) &= \subseteq \text{Int}(f) \end{aligned}$$

For each concept ‘ $\wedge f$ ’, there is a property ‘f’ associated with: 1°) an *intension*, a class of predicative properties comprehended by ‘f’, either directly or by transitivity, extracted from the network associated to the concept ‘ $\wedge f$ ’ and 2°) an *essence*, a specific part of the intension, a class of properties necessarily comprehended by the concept ‘ $\wedge f$ ’. The essence contains non-contradictory properties. The typical instances must inherit all non-contradictory properties of the intension, while atypical instances inherit only a part of the properties of intension and, necessarily all the properties of essence.

We denote by ‘ $\text{Ext}_*(f)$ ’ (or ‘ $\text{Ext}(\text{Int}(f))$ ’), the extension of *typical instances* which inherit all predicative properties of the intension  $\text{Int}(f)$  of the concept ‘ $\wedge f$ ’; and by ‘ $\text{Ext}(\text{Ess}(f))$ ’ (noted also by ‘ $\text{Ext}(\wedge f)$ ’) the extension of all instances falling under predicative properties of the essence, hence:

$$\begin{aligned} \text{Ext}_*(f) &= \text{Ext}(\text{Int}(f)) = \{x \in \text{Ext}(f); \forall g \in \text{Int}(f) [g(x) = T]\} \\ \text{Ext}(\text{Ess}(f)) &= \{x \in \text{Ext}(f); \forall h \in \text{Ess}(f) [h(x) = T]\} \end{aligned}$$

with the following inclusions:

$$\begin{aligned} \text{Int}(f) &\supseteq \text{Ess}(f) \supseteq \{f\} \\ \text{Ext}_*(f) &= \text{Ext}(\text{Int}(f)) \subseteq \text{Ext}(\text{Ess}(f)) \subseteq \text{Ext}(f) \end{aligned}$$

The extension of *atypical instances* is defined as follows:

$$\text{Ext}_*(f) = \{x \in \text{Ext}(\text{Ess}(f)); \exists g \in \text{Int}(f) [g(x) = \perp]\}$$

The extension of the instances in the boundary of the category is:

$$\text{Ext}_*(f) = \{x \in \text{Ext}(f); \exists h \in \text{Ess}(f) [h(x) = \perp]\}$$

An instance ‘x’ of a concept ‘ $\wedge f$ ’ is an *atypical instance* of this concept when there is a property ‘g’ of the intension of ‘f’, such that ‘g’ is in the essence of ‘f’ and ‘g’ is not inherited by this instance ‘x’. An instance ‘z’ of the concept ‘ $\wedge f$ ’ is out of the category when it falls under the predicative property ‘f’, but there is a property ‘h’ of the essence of ‘f’ such that ‘h’ is not inherited by ‘z’. This instance, a marginal instance of the concept ‘ $\wedge f$ ’, is an exception in the categorization defined by ‘ $\wedge f$ ’; it must be distinguished from atypical instances.

For example, ‘a one-legged man’ is an atypical instance of the concept “to be man” when we put the property “to have two legs” in its intension. Now, ‘a dead man’ should not be considered as an atypical instance among men since it does not inherit the property “to-be-alive”, considered as belonging to the essence on the concept “to-be-a-man” but it still remains an instance of the concept “to-be-a-man”. The definition of  $\text{Ext}_*(f)$  us allows to apprehend this kind of instances which can not be assimilated with atypical instances in any case.

Let us introduce two new distinctions between instances. The first one is modeled by a subclass of all instances of ‘ $\wedge f$ ’ inheriting at least a property ‘g’ of the intension of ‘f’. This class, noted by ‘ $\text{Ext-Int}(f)$ ’, is defined as follows:

$$\text{Ext-Int}(f) = \{x \in \text{Ext}(f); \exists g \in \text{Int}(f) [g \neq f \ \& \ g(x) = T]\}$$

**Remark :** The intensional-extension  $\text{Ext-Int}(f)$  of ‘f’ is different from the extension of the intension,  $\text{Ext}(\text{Ess}(f))$ .

The second distinction is the class  $\text{Ext}_*(f)$ :

$$\text{Ext}_*(f) = \{x \in \text{Ext}(f); \forall g \in \text{Int}(f) [g \neq f \ \& \ g(x) = \perp]\}$$

The class  $\text{Ext}_*(f)$  contains all the instances which inherit none of the properties associated to ‘f’, (neither the properties ‘g’ from its intension, nor the properties ‘h’ from its essence), except ‘f’ itself. These instances no longer fall under the concept ‘ $\wedge f$ ’, but they still remain related to the property ‘f’. However, such an instance is an instance of the property ‘f’ and no an instance of the

concept ‘ $\wedge f$ ’. It is located to the extreme limit of the category (the exterior-boundary of the category considered as an abstract place (Desclés, 2012):

$$\text{Ext}_t(f) = \text{Ext}(f) - \text{Ext-Int}(f)$$

The following relations are deduced:

$$\begin{aligned} \text{Ext}(f) &\supseteq \text{Ext-Int}(f) \supseteq \text{Ext(Ess}(f)) \supseteq \text{Ext(Int}(f)) \\ \text{Ext}(f) &\supseteq \text{Ext}_t(f) \supseteq \text{Ext}_t(f) \end{aligned}$$

An instance ‘ $x$ ’ of a concept ‘ $\wedge f$ ’ (‘ $f$ ’ is the main property characterizing the concept itself) has an intension formed by all the properties characterizing ‘ $f$ ’, ‘ $f$ ’ itself, and other more specific properties characterizing specifically this instance ‘ $x$ ’. The intension of each instance ‘ $x$ ’, noted ‘ $\text{Int}(x)$ ’, is computed in starting from all these characteristic properties of ‘ $x$ ’ and in taking also the properties of the network but by avoiding contradictions in the inheritance by ‘ $x$ ’, that is the properties which are not in contradiction with an inferior property: an instance ‘ $x$ ’ cannot have, in the same time, in its intension ‘ $\text{Int}(x)$ ’, a property ‘ $g$ ’ and its negation ‘ $N_1(g)$ ’ (where ‘ $N_1$ ’ is a predicative negation defined, in the framework of Combinatory Logics (Curry, Feys, 1958), from the negation ‘ $N_0$ ’ acting only onto propositions) according to the logical principle of non contradiction. The algorithmic process for building the intension ‘ $\text{Int}(x)$ ’ of an instance ‘ $x$ ’ is directed by the following computing principle: taking the characteristic properties of the instance and the characteristic properties of an upper property without extracting from the network a property which should be in contradiction with these inferior characteristic properties. The network of properties, associated to the concept ‘ $\wedge$ to be an inhabitant of  $X$ ’ is presented in the figure 1.

### The analysis of the example

In the figure 1, we present the network of the concept “to be inhabitant of  $X$ ”; its essence is compound by 3 properties (“to live  $X$ ”, “to have an address in  $X$ ”, “to have duties”); at the essence, are adjunct two other properties of the intension: “to pay taxes”, “to have the power to vote”. All typical inhabitants inherit five properties. The typical inhabitants are the citizens of the city  $X$  who are older than 18 years. The intension of the more specific property “to be citizen inhabitant” contains the intension of the concept “ $\wedge$ to be inhabitant of  $X$ ”. The corresponding extensional class is the class B. The instances of the class B’ are the atypical citizen (among all citizen) who are old below 18 years: they have not the power to vote and do not pay taxes. The intension of these instances is computed 1°) by removing these two properties from the intension of typical citizens and 2°) by adding the property “to be old below

18”. Thus, there is no contradiction among properties characterizing all elements of the class B’.

The class of foreigners living in  $X$  is a subclass of the more general class of inhabitants of  $X$ . The instances of this class have a specific property “do not have the power to vote”; this property does them atypical instances in the class of inhabitants. In this last class, there is the subclass C of atypical foreign inhabitants who are older than 18 years and resident during 5 years or more, having obtained the power to vote, the intension of C is computed in starting from the intension of foreign residents by removing the property “not have the power to vote” (but not the property “to pay taxes”) and by adding the complementary property “to have the power to vote” and also the restrictive properties “to have more than 5 years of residence” and “to be older than 18 years”.

The class F is the class of irregular inhabitants. They are neither typical, nor atypical because they do not inherit some essential properties of the concept “to be inhabitant of  $X$ ”. In general they have no address in the city  $X$ , they are homeless. Whereas they live  $X$ , they are on the border of the category defined by “ $\wedge$ to be inhabitant of  $X$ ”, they are not instances of this concept: they do not inherit the properties which contribute to structure the extensional classes of the category) but they fall under the property “to be inhabitant of  $X$ ”; they are not located outside the category because they are still related to the property and inherit, may be, some properties, for instance “to have duties” since they are living  $X$ : they are only marginal elements and exceptions in the categorization but not at all atypical instances.

The structure of the category of all instances falling under the concept “ $\wedge$ to be inhabitant of  $X$ ” is presented in the figure 2 (the properties are represented by ellipses and the instances by rectangles). The classes A, B, B’, C, C’, D, E, F are subclasses of Ext(“to-be-inhabitant-of- $X$ ”).

- A = Ext (“to be citizen old under 18 years”)
- B = Ext (“to be citizen older than 18 years”)
- B’ = Ext (“to be citizen older than 18 years”)
- C = Ext (“to be foreign resident & older than 18 years & to be resident during 5 years or more”)
- C’ = Ext (“to be a foreign resident old below 18 years”)
- D = Ext (“to be a foreign resident & to be a resident during 5 years or more”)
- E = Ext (“to be a foreign no resident inhabitant”)
- F = Ext (“to be an irregular inhabitant”)

The class A is a class of atypical instances among the class of inhabitants of  $X$ . The class B is a class of all typical instances among citizens; the class B’ is a class of atypical instances among the class of citizens. The class C is a class of atypical instances of foreign resident (they obtained the power to vote); the class C’ is a class of



typical instances among foreign residents. The class D is a class of typical instances among foreign residents (they pay taxes but they have not the power to vote). The class E is a class of atypical instances of foreign inhabitants of X (they do not pay taxes), the typical instances of foreign inhabitants being the class of foreign residents who pay taxes and have not the power to vote. The class F is a class whose all the objects fall under the concept “to be inhabitant of X” but these objects are exceptions and are in the boundary of the extension (when we give a quasi-topological structure to extensions (Desclés, Guentchéva, 2010); being without address, they have no home, no identification cards, they do not pay taxes, whereas they live in X.

## Conclusions

A *concept* (called “concept” in Frege's terminology and also in the conceptualization worked by classical logics) is a *predicative property* apprehended with an intension and an essence, yielding different sub-classes of objects. All properties are organized in a network with some pairs of properties (a property and a complementary property). It is obvious that the Logic of Typical and Atypical Instances (LTA) can be closely related to the Logic of Determination of Objects (LDO), but LTA is conceived without involving the notions as *determination*, *more or less indeterminate objects*, *fully indeterminate objects*. In LTA, a categorization process is based only on properties associated to concepts and works with algorithms that build the intensions and extensions of fully determinate objects and classes of these objects. It is well known that the Zadeh's fuzzy logics (Zadeh, 1965) introduced a degree of membership (a number between 0 and 1); with this approach, an atypical object is considered as a member with a degree less than 1, and, by consequence, this atypical object is not considered as a full object of the category but only a member with a inferior degree. Thus, one-legged men (or blind, or one-handed) would not be fully men but only men with not an upper degree from the general fact that a typical man has two legs (two eyes, two hands...). This interpretation of atypicality is not at all acceptable. On the contrary, in LDA, all atypical objects are full members of the category defined by the essence of a concept.

Approaches as non-monotonic logics or paraconsistent logics (Da Costa, 1986; Beziau, 2007) offer different kinds of reasoning to manage exceptions but do not consider directly the notion of atypicality. When one builds and organizes an ontology, it is necessary to take into account not only the classes of typical objects but also the classes of atypical objects and the exceptions. Generally, the algorithmic process for manipulating the typical objects are

not very complex and more regular than a process to atypical instances. The discussion about the meaning of a concept is often unclear (not only in philosophy and human sciences but also in natural sciences and in computer science) and may be strongly controversial. The categorization process must explain what is the intension (associated to all properties comprehended by a concept), what are the properties inside of essence, in order to clearly and identify all typical instances and to take also into account atypicality of some instances and do not mingle atypical instances and exceptions.

The distinction intension/essence (and typical/atypical instances) can change with the time and with specific, social or cultural users of a categorization. For instance, general and professional users have not exactly the same representations of a given domain of objects. An electric car is now an atypical car (since it is difficult to run beyond 150 miles without to refill batteries) but, in the future, an electric car will be a typical car and, perhaps, at this moment, a car running with fuel will become an atypical car. In a categorization, when typical and atypical objects are clearly identified, there are objects related to a property but these objects are not “good instances” of the associated concept. When an object falls under almost properties of the intension of a concept but not under one property of its essence, it cannot be considered as an atypical object but only as an exception. In this case, it is perhaps interesting to discuss about properties of the essence and perhaps to decide the change of the essence by including this exception as an atypical object or, by a contrary way, to decide that the exception becomes a no member of the category. For instance, after discussion among scientists, it has been decided that the “planet” Pluto, considered in a first time, as an atypical planet was not yet a planet but only a sidereal object which turns around the solar.

The logical notions relative to different kinds of objects taken into account by LTA and LDO are very close to investigations about a general theory of objects studied by A. Meinong (Meinong, 1899) and followers (to see for instance (Chislom, 1982), (Husserl et al. 2007), (Leclercq, 2012)). It will be interesting to compare these different approaches of non-classical logics to evaluate their relevance for AI and knowledge representations.

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