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Decision Tables Aggregation in Rough Sets Approximation

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Abstract

The Dominance-based Rough Set Approach (DRSA) is an extension of Rough Sets Theory to handle multicriteria classification problems by authorizing preference-ordered attributes. The DRSA assumes the existence of a single decision table while real-world decision problems imply generally several experts with different decision tables. The objective of this paper is to propose an algorithm for the aggregation of a set of decision tables, as a first step for approximating these tables. The algorithm is illustrated using real-world data.

Introduction

The Dominance-based Rough Set Approach (DRSA) (Greco, Matarazzo, and Słowiński 2001) is an extension of rough sets theory (Pawlak 1991) to handle multicriteria classification problems by authorizing preference-ordered attributes. The input data for DRSA are often structured in a decision table where rows correspond to decision objects and columns correspond to attributes. The attributes used in rough approximation in multicriteria classification problems are often divided into two disjoint subsets: a subset of *condition attributes* and a subset of *decision attributes*. The DRSA assumes the existence of a single decision table. However, multicriteria classification problems generally imply different experts having different and conflicting objectives and preferences, each with its decision table.

The approximation of several decision tables has been addressed by several authors (Bi and Chen 2007; Chakhar and Saad 2012; Chen, Kilgour, and Hipel 2012; Greco, Matarazzo, and Słowiński 2006). The first step of rough approximation of decision tables consists in the aggregation of these tables into a collective decision table with one collective decision attribute. The objective of this paper is to propose an algorithm for the aggregation of a set of decision tables as a first step to rough approximation of these tables. The algorithm is illustrated using real-world data.

The rest of paper is organized as follows. Section 2 presents the background and set decision tables aggregation problem. Section 3 presents the aggregation algorithm. Section 4 presents a numerical example. Section 5 concludes the paper.

Decision tables aggregation problem

In rough sets theory, information regarding the *decision objects* is often structured in a 4-tuple *information table* $\mathbf{S} = \langle U, Q, V, f \rangle$, where U is a non-empty finite set of objects and Q is a non-empty finite set of attributes such that $q : U \to V_q$ for every $q \in Q$. V_q is the domain of attribute q. $V = \bigcap_{q \in Q} V_q$, and $f : U \times Q \to V$ is the *information function* defined such that $f(x, q) \in V_q$ for each attribute q and object $x \in U$. Q is often divided into a sub-set $C \neq \emptyset$ of *condition attributes* and a sub-set $D \neq \emptyset$ of *decision attributes* such that $C \cup D = Q$ and $C \cap D = \emptyset$. In this case, $\mathbf{S}$ is called a *decision table*.

In multicriteria decision-making, the *domain* of condition attributes are supposed to be ordered according to decreasing or increasing preference. Such attributes are called *criteria*. We assume that the preference is increasing with a value of $f(\cdot, q)$ for every $q \in C$. We also assume that the set of decision attributes $D = \{d\}$ is a singleton. The unique decision attribute d makes a partition of U into a finite number of preference-ordered decision classes $\mathbf{Cl} = \{Cl_1, \cdots, Cl_n\}$ such that each $x \in U$ belongs to one and only one class.

In DRSA the represented knowledge is a collection of *upward union* $Cl_t^{\geq}$ and *downward union* $Cl_t^{\leq}$ of classes defined as follows:

$$Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s, Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s.$$

The assertion "$x \in Cl_t^{\geq}$" means that "x belongs to at least class Cl_t" while assertion "$x \in Cl_t^{\leq}$" means that "x belongs to at most class Cl_t". The basic idea of DRSA is to replace *indiscernibility* relation used in conventional rough sets theory with *dominance* relation. The dominance relation Δ_P associated with P is defined for each pair of objects x and y as follows:

$$x \Delta_P y \Leftrightarrow f(x, q) \succeq f(y, q), \forall q \in P.$$

To each object $x \in U$, we associate two sets: (i) the P*-dominating set* $\Delta_P^+(x) = \{y \in U : y \Delta_P x\}$ containing objects that dominate x, and (ii) the P*-dominated set* $\Delta_P^-(x) = \{y \in U : x \Delta_P y\}$ containing the objects dominated by x. The P-lower and P-upper approximations of $Cl_t^{\geq}$ with respect to $P \subseteq C$ are defined as follows:

- $\underline{P}(Cl_t^{\geq}) = \{x \in U : \Delta_P^+(x) \subseteq Cl_t^{\geq}\}$,

- $\bar{P}(Cl_t^{\geq}) = \{x \in U : \Delta_P^-(x) \cap Cl_t^{\geq} \neq \emptyset\}$.

Analogously, the P-lower and P-upper approximations of $Cl_t^{\leq}$ with respect to $P \subseteq C$ are defined as follows:

- $\underline{P}(Cl_t^{\leq}) = \{x \in U : \Delta_P^-(x) \subseteq Cl_t^{\leq}\}$,
- $\bar{P}(Cl_t^{\leq}) = \{x \in U : \Delta_P^+(x) \cap Cl_t^{\leq} \neq \emptyset\}$.

The P-boundaries of $Cl_t^{\geq}$ and $Cl_t^{\leq}$ are defined as:

- $Bn_P(Cl_t^{\geq}) = \bar{P}(Cl_t^{\geq}) - \underline{P}(Cl_t^{\geq})$,
- $Bn_P(Cl_t^{\leq}) = \bar{P}(Cl_t^{\leq}) - \underline{P}(Cl_t^{\leq})$.

The *quality of classification* (or *approximation*) of a partition **Cl** by means of a set of criteria P is measured by the ratio γ_P, which expresses the ratio of all P-correctly classified objects to all objects in the system.

Let $H = \{1, \cdots, i, \cdots, h\}$ and let $\mathbf{S}_i = \langle U, C \cup \{E_i\}, V, f_i\rangle$, $\forall i \in H$ be n decision tables where E_i and f_i are respectively the decision attribute and the information function relative to the ith decision table. We assume that a preference order for U represented by a finite set of preference-ordered classes $\mathbf{Cl}_i = \{Cl_{t,i}, t \in T_i\}$, $T_i = \{0, \cdots, n_i\}$, such that $\bigcup_{t=1}^{n_i} Cl_{t,i} = U$, $Cl_{t,i} \cap Cl_{r,i} = \emptyset$, $\forall r, t \in Ti$, $r \neq t$, and if $x \in Cl_{r,i}$, $y \in Cl_{s,i}$ and $r > s$, then x is better than y for the ith decision table. The n_i is the number of decision classes for the ith decision table. The approximation of the ith decision table $\mathbf{S}_i$ is characterized, among others, by: (i) the P-lower approximation and P-boundary of $Cl_{t,i}^{\leq}$ and $Cl_{t,i}^{\geq}$, for each $t \in T_i$, and (ii) the quality of classification γ_P^i.

The first step of rough approximation of decision tables consists in the aggregation of these tables into a collective decision table with one collective decision attribute. The problem of decision tables aggregation can be stated as follows: Let $\mathbf{S}_i = \langle U, C \cup \{E_i\}, V, f_i\rangle$ ($\forall i \in H$). Then, construct a collective decision table $\mathbf{S} = \langle U, C \cup \{E\}, V, g\rangle$ where E is a decision attribute and g is an information function defined for each $x \in U$ as follows:

$$g(x, q) = \begin{cases} f(x, q), & \text{if } q \in C, \\ g(x, E), & \text{if } q = E. \end{cases} \tag{1}$$

The decision attribute E induces a partition of U into a set of decision classes $\mathbf{Cl} = \{Cl_1, \cdots, Cl_n\}$ such that each $x \in U$ belongs to one and only one class $Cl_t \in \mathbf{Cl}$. To define **S** it suffices to specify the values of $g(x, E)$ for all $x \in U$.

Decision tables aggregation algorithm

As stated above, the objective of the aggregation algorithm is to construct a decision table **S** by aggregating the decision tables $\mathbf{S}_1, \cdots, \mathbf{S}_h$. The idea of the aggregation algorithm is to use the upward and downward approximation of unions of classes in order to identify the possible assignments classes for each decision object. Two sets will be constructed: (i) set N_1 contains the possible assignments obtained based on the upward approximation of unions of classes; and (ii) set N_2 contains the possible assignments obtained based on the downward approximation of unions of classes. These sets will then be used to associate to each object $x \in U$ an assignment interval $I(x) = [l(x), u(x)]$ where $l(x)$ and $u(x)$ are respectively the lower and upper classes to which object x can be assigned. Finally, some simple rules are used to reduce the assignment interval $I(x)$ into a single element representing the value of the collective decision attribute E.

Before introducing the aggregation algorithm we need to introduce new concepts. More specifically, the definition of sets N_1 and N_2 requires the introduction of three concepts: concordance power, discordance power and the credibility indexes. Let first standardize the quality of classifications γ_P^k ($\forall k \in H$) as follows:

$$\gamma'_k = \frac{\gamma_P^k}{\sum_{r=1}^h \gamma_P^r} \tag{2}$$

We assume that $\diamond \in \{\geq, \leq\}$ and **Cl**=$\{Cl_1, \cdots, Cl_n\}$.

Concordance power For each $x \in U$ and $Cl_t \in \mathbf{Cl}$ we define the set: $L(x, Cl_t^{\diamond}) = \{i : i \in H \wedge x \in \underline{P}(Cl_{t,i}^{\diamond})\}$ where $\underline{P}(Cl_{t,i}^{\diamond})$ is the P-lower approximation of $Cl_t^{\diamond}$ in respect to the ith decision table. Then, the *concordance powers* for the assignment of x to $Cl_t^{\diamond}$ is then defined as follows.

Definition 1 *The concordance power for the assignment of x to $Cl_t^{\diamond}$ is computed as follows:*

$$S(x, Cl_t^{\diamond}) = \sum_{k=1}^{k=n} S_k(x, Cl_t^{\diamond}) \tag{3}$$

where:

$$S_k(x, Cl_t^{\diamond}) = \begin{cases} \gamma'_k, & \text{if } k \in L(x, Cl_t^{\diamond}), \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Discordance power For each $x \in U$ and $Cl_t \in \mathbf{Cl}$ we define the set $B(x, Cl_t^{\diamond}) = \{i : i \in H \wedge x \in Bn_P(Cl_{t,i}^{\diamond})\}$ where $Bn_P(Cl_{t,i}^{\diamond})$ is the boundary of $Cl_t^{\diamond}$ in respect to the ith decision table. Then, the *discordance powers* for the assignment of x to the boundary of $Cl_t^{\diamond}$ is defined as follows.

Definition 2 *The discordance power for the assignment of x to $Cl_t^{\diamond}$ is computed as follows:*

$$Z(x, Cl_t^{\diamond}) = \prod_{k=1}^{k=n} Z_k(x, Cl_t^{\diamond}) \tag{5}$$

where

$$Z_k(x, Cl_t^{\diamond}) = \begin{cases} \frac{1-\gamma'_k}{1-S(x, Cl_t^{\diamond})}, & \text{if } \gamma'_k > S(x, Cl_t^{\diamond}) \wedge \\ & \quad k \in B(x, Cl_t^{\diamond}), \\ 1, & \text{otherwise.} \end{cases} \tag{6}$$

We may distinguish two cases in the definition of the discordance power. The first case holds when $\gamma'_k \leq S(x, Cl_t^{\diamond})$, which leads to $Z_k(x, Cl_t^{\diamond}) = 1$. In this case, there is no veto effect for decision maker k and $Z_k(x, Cl_t^{\diamond})$ will have no effect on the definition of overall discordance power $Z(x, Cl_t^{\diamond})$ and on the value of the credibility indexes as explained in the next paragraph. The second case holds when $\gamma'_k > S(x, Cl_t^{\diamond})$, which leads to $0 < Z_k(x, Cl_t^{\diamond}) < 1$. Here, decision maker k do have a veto effect and $Z_k(x, Cl_t^{\diamond})$ will have an effect on the value of overall discordance power $Z(x, Cl_t^{\diamond})$ and on the value of the credibility indexes as explained later.

Credibility indexes Using the concordance and discordance powers, we may define the credibility index for assigning x to $Cl_t^\diamond$ as follows.

Definition 3 *Let $x \in U$ and $\diamond \in \{\geq, \leq\}$. The credibility indexes for the assignment of x to $Cl_t^\diamond$ is computed as follows:*

$$\sigma(x, Cl_t^\diamond) = S(x, Cl_t^\diamond) \cdot Z(x, Cl_t^\diamond) \quad (7)$$

This formula can be explained as follows. If there is no support for the assignment of x to $Cl_t^\diamond$, i.e., $S(x, Cl_t^\diamond) = 0$, then the credibility indexes will be $\sigma(x, Cl_t^\diamond) = 0$. In turn, if there is a full support, i.e., $S(x, Cl_t^\diamond) = 1$ (which imposes that $Z(x, Cl_t^\diamond) = 1$), then credibility indexes will be $\sigma(x, Cl_t^\diamond) = 1$. Finally, if there is a partial support, i.e., $0 < S(x, Cl_t^\diamond) < 1$ (which imposes that $0 < Z(x, Cl_t^\diamond) \leq 1$), then $0 < \sigma(x, Cl_t^\diamond) < 1$. In the last case, we may distinguish two subcases, according to the verification or not of the condition $\gamma_k' > S(x, Cl_t^\diamond)$. The first subcase holds when the condition $\gamma_k' > S(x, Cl_t^\diamond)$ is not verified. This leads to $Z(x, Cl_t^\diamond) = 1$ and then $\sigma(x, Cl_t^\diamond) = S(x, Cl_t^\diamond) < 1$. In this subcase, the credibility index is simply equal to the concordance power; hence the discordance power will have no effect on the value of the credibility indexes $\sigma(x, Cl_t^\diamond)$. The second subcase holds when condition $\gamma_k' > S(x, Cl_t^\diamond)$ is verified. This leads to $Z(x, Cl_t^\diamond) < 1$ and consequently $\sigma(x, Cl_t^\diamond) = S(x, Cl_t^\diamond) \cdot Z(x, Cl_t^\diamond) < 1$. In this subcase, the credibility index is obtained by decreasing the concordance power $S(x, Cl_t^\diamond)$ proportionally to the value of the discordance power $Z(x, Cl_t^\diamond)$.

Definition of assignment interval Let $\lambda \in [0.5, 1]$ be a *credibility threshold*. Then, based on the credibility indexes, we may define the sets N_1 and N_2 as follows.

Definition 4 *The credibility indexes, we may define the sets N_1 and N_2 as follows:*

- $N_1(x) = \{Cl_t : x \in U \wedge \sigma(x, Cl_t^{\geq}) \geq \lambda\}$,
- $N_2(x) = \{Cl_t : x \in U \wedge \sigma(x, Cl_t^{\leq}) \leq \lambda\}$.

Then, the idea for the definition of assignment intervals is to constraint possible assignment classes by the content of sets $N_1(x)$ and $N_2(x)$. Indeed, the set $N_1(x)$ is defined based on the upward union of classes $Cl_t^{\geq}$; it should be used to define the lower limit $l(x)$ of the assignment interval of x. In turn, the set $N_2(x)$ is defined based on downward union of classes $Cl_t^{\leq}$; it should be used to define the upper limit $u(x)$ of the assignment interval of x.

Definition 5 *Let $x \in U$. Then, we associate to each object x an assignment interval $I(x) = [l(x), u(x)]$ where:*

$$l(x) = \begin{cases} \textit{argmax}_{Cl_t} N_1(x), & \textit{if } N_1(x) \neq \emptyset, \\ Cl_0, & \textit{otherwise.} \end{cases} \quad (8)$$

$$u(x) = \begin{cases} \textit{argmin}_{Cl_t} N_2(x), & \textit{if } N_2(x) \neq \emptyset, \\ Cl_n, & \textit{otherwise.} \end{cases} \quad (9)$$

Reduction of the assignment interval Let $I(x) = [l(x), u(x)]$ be the assignment interval for object $x \in U$ defined as previously. Two cases hold for the reduction of $I(x)$. The first case holds when $l(x) = u(x)$. Here, object x is assigned to a single class and consequently we can set $g(x, E) = l(x)$ (or $g(x, E) = u(x)$). The second case holds when $l(x) < u(x)$. This corresponds to the situation where object x can be assigned to more than one class. To specify the value of $g(x, E)$ when the second case holds we may apply one of the following rules to reduce the collective assignment interval $I(x)$ to a single class: the minimum value, the maximum value, the median value, the floor of the median value and the ceil of the median value.

Aggregation algorithm The aggregation procedure is formalized in Algorithm 1. This algorithm works as follows. It loops on the set of decision objects and for each object: (i) computes the credibly indexes for upward unions of classes (the first inner *for* loop); (ii) computes the credibly indexes for downward unions of classes (the second inner *for* loop)); (iii) computes the assignment interval $I(x) = [l(x), u(x)]$; and (vi) computes the values of the collective decision attribute E.

Functions SigmaUpward and SigmaDownward permit to compute the credibility indexes and function IntervalReduction permits to compute the assignment interval.

Application

We consider a real-world data relative to the management of post-accident nuclear risk in the PRIME project (Chakhar and Saad 2012). The problem involves 18 decision objects and 7 attributes (radioecological vulnerability of agricultural area (A_1), radioecological vulnerability of forest area (A_2), radioecological vulnerability of urban area (A_3), real estate vulnerability (A_4), Tourism vulnerability (A_5), economic vulnerability of companies (A_6), and employment vulnerability (A_7)).

The main input is three decision tables summarized in Table 1. Each object is described in terms of seven condition attributes ($A_1, A_2, \cdots$, and A_7) and three decision attributes (E_1, E_2, and E_3). The values of condition attributes correspond to vulnerability levels. The values of decision attributes correspond to the global vulnerability levels as specified by three experts. All condition and decision attributes are evaluated on a six-level ordinal scale (from normal situation (0) to major and long-lasting negative impact (5)).

The software 4eMka2 (which implements the DRSA) is used to approximate the individual decision tables. The outputs of individual approximations are then complied in a single .txt file and provided as input to a prototype implementing the aggregation algorithm.

The credibility indexes values computed using Equation (7) are given in Table 2. The assignment intervals along with the application of interval reduction rules are given in Table 3.

Algorithm 1: Aggregation

Input : **I** // where $\mathbf{I} = \langle U, C, V, f \rangle$ is the common decision table.
$\mathbf{S}_1, \cdots, \mathbf{S}_h$ //where $S_i = \langle U, C \cap \{E_i\}, V, f_i \rangle$.
λ // where $\lambda \in [0.5, \cdots, 1]$ is the credibility threshold.
ir_rule // where $ir - rule$ is the interval reduction rule.
$default\_rule$ // default rule to use when "median" rule do not apply.
Output: **S** // where $\mathbf{S} = \langle U, C \cap E, V, g \rangle$ is the aggregated decision table.
$E \longleftarrow$ decision attribute;
$Q \longleftarrow C \cup \{E\}$;
$H \longleftarrow \{1, 2, \cdots, h\}$
for $(all\ x \in U)$ **do**
  //...computes the credibly indexes for upward unions of classes...
  $N_1\ (x) \longleftarrow \emptyset$;
  for $(all\ t \in \{1, 2, \cdots, n\})$ **do**
    $\sigma\left(x, Cl_t^{\geq}\right) \longleftarrow$ SigmaUpward($\mathbf{S}_1, \cdots, \mathbf{S}_h, t, x$);
    if $\left(\sigma(x, Cl_t^{\geq}) \geq \lambda\right)$ **then**
      $N_1 \longleftarrow N_1 \cup Cl_t$;
    end
  end

  //...computes the credibly indexes for downward unions of classes...
  $N_2\ (x) \longleftarrow \emptyset$;
  for $(all\ t \in \{0, 1, \cdots, n-1\})$ **do**
    $\sigma\left(x, Cl_t^{\leq}\right) \longleftarrow$ SigmaDownward($\mathbf{S}_1, \cdots, \mathbf{S}_h, t, x$);
    if $\left(\sigma(x, Cl_t^{\leq}) \geq \lambda\right)$ **then**
      $N_2 \longleftarrow N_2 \cup Cl_t$;
    end
  end

  //...computes the assignment interval $I(x) = [l(x), u(x)]$...
  $l(x) \longleftarrow Cl_0$;
  $u(x) \longleftarrow Cl_n$;
  if $(N_1 \neq \emptyset)$ **then**
    $l(x) \longleftarrow$ argmax$_{Cl_t} N_1(x)$;
  end
  if $(N_2 \neq \emptyset)$ **then**
    $u(x) \longleftarrow$ argmin$_{Cl_t} N_2(x)$;
  end

  //...computes the values of the collective decision attribute $g(x, E)$...
  for $(all\ q \in C)$ **do**
    $g\ (x, q) \longleftarrow f(x, q)$;
  end
  if ($l(x) = u(x)$) **then**
    $g(x, E) \longleftarrow l(x)$;
  end
  else
    $g(x, E) \longleftarrow$
    IntervalReduction($l(x), u(x), ir\_rule, default\_rule$);
  end
end
S $\longleftarrow < U, Q, V, g >$;
return **S**

Conclusion

We proposed an algorithm for the approximation of a set of decision tables. The algorithm is illustrated using real-world data. In the future, we intend to study the mathematical properties of the introduced concepts. We also intend to conceive and to develop a full-featured decision support system supporting the aggregation algorithm.

Object	A_1	A_2	A_3	A_4	A_5	A_6	A_7	E_1	E_2	E_3
x_1	4	5	5	5	4	1	1	4	4	5
x_2	4	5	5	5	4	2	2	4	4	5
x_3	4	5	5	5	4	2	1	4	4	5
x_4	4	5	5	5	4	3	1	5	4	5
x_5	3	2	2	4	4	2	0	3	2	3
x_6	1	1	1	2	4	1	0	0	0	1
x_7	2	2	1	2	4	1	0	3	2	2
x_8	1	2	1	2	2	1	0	0	0	1
x_9	3	2	2	4	4	2	0	3	2	2
x_{10}	3	3	3	4	4	1	0	3	2	3
x_{11}	3	3	3	4	4	1	0	3	2	3
x_{12}	3	3	2	4	4	1	0	3	2	3
x_{13}	3	2	2	4	4	1	0	2	2	3
x_{14}	2	2	2	4	4	1	0	2	1	2
x_{15}	2	2	1	4	3	1	0	2	1	2
x_{16}	2	2	1	4	4	1	0	2	1	2
x_{17}	1	1	1	2	4	1	0	3	3	4
x_{18}	1	1	0	1	4	1	0	3	3	3

Table 1: Information table with assignment examples

Object	$\sigma(x_i, Cl_t^{\geq})$						$\sigma(x_i, Cl_t^{\leq})$				
x_i	t	1	2	3	4	5	0	1	2	3	4
x_1		1	1	1	1	0.26	0	0	0	0	0.74
x_2		1	1	1	1	0.26	0	0	0	0	0.74
x_3		1	1	1	1	0.26	0	0	0	0	0.74
x_4		1	1	1	1	0.74	0	0	0	0	0.26
x_5		1	1	0.48	0	0	0	0	0	0.74	1
x_6		0.18	0	0	0	0	0	0	0	0.74	1
x_7		1	0.74	0	0	0	0	0	0	0.74	1
x_8		0.26	0	0	0	0	0.74	1	1	1	1
x_9		1	1	0.48	0	0	0	0	0	0.74	1
x_{10}		1	1	0.74	0	0	0	0	0	0.74	1
x_{11}		1	1	0.74	0	0	0	0	0	0.74	1
x_{12}		1	1	0.74	0	0	0	0	0	0.74	1
x_{13}		1	1	0	0	0	0	0	0	0.74	1
x_{14}		1	0.74	0	0	0	0	0	0	0.74	1
x_{15}		1	0.74	0	0	0	0	0.26	1	1	1
x_{16}		1	0.74	0	0	0	0	0	0	0.74	1
x_{17}		0.183	0	0	0	0	0	0	0	0.74	1
x_{18}		0.183	0	0	0	0	0	0	0	1	1

Table 2: Credibility indexes values

Object x_i	l(x)	u(x)	min	max	floor	ceil
x_1	4	4	4	4	4	4
x_2	4	4	4	4	4	4
x_3	4	4	4	4	4	4
x_4	5	5	5	5	5	5
x_5	2	3	2	3	2	3
x_6	0	4	0	4	2	3
x_7	2	4	2	4	3	3
x_8	0	0	0	0	0	0
x_9	2	3	2	3	2	3
x_{10}	3	3	3	3	3	3
x_{11}	3	3	3	3	3	3
x_{12}	3	3	3	3	3	3
x_{13}	2	3	2	3	2	3
x_{14}	2	3	2	3	2	3
x_{15}	2	2	2	2	2	2
x_{16}	2	3	2	3	2	3
x_{17}	0	3	0	3	1	2
x_{18}	0	3	0	3	1	2

Table 3: Collective decision attribute E values for $\lambda = 0.70$

References

Bi, W.-J., and Chen, X.-H. 2007. An extended dominance-based rough set approach to group decision. In *Proceedings of the International Conference on Wireless Communications, Networking and Mobile Computing*, 5753–5756.

Chakhar, S., and Saad, I. 2012. Dominance-based rough set approach for groups in multicriteria classification problems. *Decision Support Systems* 54(1):372–380.

Chen, Y.; Kilgour, D.; and Hipel, K. 2012. A decision rule aggregation approach to multiple criteria-multiple participant sorting. *Group Decision and Negotiation* 21:727–745.

Greco, S.; Matarazzo, B.; and Słowiński, R. 2001. Rough sets theory for multicriteria decision analysis. *European Journal of Operational Research* 129(1):1–47.

Greco, S.; Matarazzo, B.; and Słowiński, R. 2006. Dominance-based rough set approach to decision involving multiple decision makers. In *Proceedings of the 5th International Conference Rough Sets and Current Trends in Computing, Kobe, Japan, November 6-8*, LNAI 4259. 306–317.

Pawlak, Z. 1991. *Rough set. Theoretical aspects of reasoning about data.* Dordrecht: Kluwer Academic Publishers.