PLEASE: Palm Leaf Search for POMDPs with Large Observation Spaces

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Abstract

This paper provides a novel POMDP planning method, called \texttt{Palm LEAf SEarch} (PLEASE), which allows the selection of more than one outcome when their potential impacts are close to the highest one during its forward exploration. Compared with existing trial-based algorithms, PLEASE can save considerable time to propagate the bound improvements of beliefs in deep levels of the search tree to the root belief because of fewer backup operations. Experiments showed that PLEASE scales up SARSOP, one of the fastest algorithms, by orders of magnitude on some POMDP tasks with large observation spaces.

Introduction

Partially Observable Markov Decision Process (POMDP) provides a rich mathematical framework for agent reasoning and planning in uncertain environments. In the past decade, point-based algorithms have made impressive progress in handling large POMDPs. Several representative algorithms among them, such as HSVI2, GapMin and SARSOP (Kurniawati, Hsu, and Lee 2008), do a trial-based search in their sampling strategies, where the current bounds of the optimal value function $V^*$ are used to select a \textit{single path} from the initial belief $b_0$ to one leaf belief during each trial.

This paper provides a new method called \texttt{Palm LEAf SEarch} (PLEASE). We use a simple example to explain the motivation behind the method. Assume that, for a POMDP, $n$ child beliefs of a belief at level $d-1$, denoted $b_{d-1}$, need to perform point-based value backups to propagate their bound improvements to the initial belief $b_0$ to guarantee to find a near optimal policy at $b_0$. For each trial, trial-based algorithms need to forward search one of $b_{d-1}$’s $n$ child beliefs so as to propagate its improvement to $b_0$. Since each trial length is $d+1$, totally, there are $n \times (d+1)$ beliefs that need to perform backups. The PLEASE method allows the selection of \textit{more than one} outcome during its forward exploration phase when their potential impacts are close to the highest one. Instead of repeatedly performing backups on $b_i$, where $i = 0, \ldots, d-1$, $n$ times, PLEASE can perform them \textit{once}. As a result, there are only $n + d$ beliefs that need to perform backups in PLEASE. When $d$ or $n$ becomes larger, $\frac{d(n-1)}{n(d+1)-n+d}$ increases. This suggests that PLEASE is more attractive when tackling a POMDP with large observation size (probably large $n$) and that needs to search \textit{deeply} (large $d$) to find a near optimal solution.

Palm Leaf Search

Compared with SARSOP, the \texttt{EXPLORE} procedure in PLEASE (see Algorithm 1) adds a for loop in Lines 6–10. This allows Algorithm 1 to search towards more than one outcome in its forward exploration phase when their potential impacts are close to the highest one. How to use the heuristics of beliefs to select promising child beliefs to make PLEASE do best is not trivial. Here, we control the palm leaf search at beliefs by setting $\zeta$ online (Line 7 in Algorithm 1).

\begin{algorithm}
\caption{PLEASE (see Algorithm 1) adds a for loop in Lines 6–10.}
\begin{algorithmic}[1]
\STATE if $b_0$’s gap termination condition == true then
\RETURN;
\ELSE
\STATE $a^* = \max_{a \in A} Q^b(a)$;
\STATE $z^* = \max_{z \in Z} [Pr(z|b,a^*) \cdot \text{excess}(\tau(b,a^*,z),d_0+1,\epsilon)]$;
\FOR {$z \in Z$}
\STATE if $Pr(z|b,a^*) \cdot \text{excess}(\tau(b,a^*,z),d_0+1,\epsilon) \geq \zeta \cdot Pr(z^*|b,a^*) \cdot \text{excess}(\tau(b,a^*,z^*),d_0+1,\epsilon)$ then
\STATE \texttt{EXPLORE}(\tau(b,a^*,z),d_0+1,\epsilon);
\ENDIF
\ENDFOR
\STATE Perform a backup operation of bounds at belief $b$;
\end{algorithmic}
\end{algorithm}

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$C > 0$ reduces the number of backups to propagate the bound improvements of selected leaf nodes to the root node. Please see (Zhang et al. 2015) for a variant of PLEASE (called PLEASE-Z) with a different definition of $C$.

We let PLEASE control $\theta$ online so that it obtains an ideal value of $\theta$ over time to make the actual ratio of #PLEASE to #SARSOP close to $C + 1$. Specifically, we define $\theta$ by using the following rule:

$$\theta = \begin{cases} 
\min(\theta + \Delta, 1) & \text{if } \#\text{PLEASE} > \#\text{SARSOP} \geq C + 1, \\
\max(\theta - \Delta, 0) & \text{otherwise.}
\end{cases}$$

Here, $\theta$ is set to $\theta_0 = 1$ in the beginning of PLEASE, $\theta_1 = 0.8$, and $\Delta = 0.01$ in this paper. PLEASE adjusts the value of $\theta$ in the beginning of each forward exploration phase from the initial belief $b_0$ in the PLEASE method.

PLEASE defines $\zeta(b, \theta) = \frac{\text{dis}(b, p^*) + \sqrt{\theta}}{1 + \sqrt{\theta}}$ by using $\theta$ and $b$'s heuristic information to achieve the goal of giving more time to do palm leaf search in promising beliefs with deep levels. Here, $p^*$ represents the path generated by SARSOP’s action and outcome selection strategies and its belief’s gap termination condition, and the distance from $b$ to $p^*$, denoted $\text{dis}(b, p^*)$, as the number of beliefs that $b$ needs to go through to arrive in $p^*$. Such a formula guarantees that PLEASE does more aggressive palm leaf search around the current best path $p^*$ in each forward exploration phase starting at $b_0$.

Essentially, palm leaf search can be viewed as a kind of complete anytime best-first beam search in POMDPs. Similar techniques of tree-trials have been used in Monte-Carlo tree search, such as d-UCT by Yoshizoe et al. (2011). At each step, the complexity of each PLEASE exploration step is at least the time and space complexity of each SARSOP step. However, its conservative theoretical time bound of finding an $\epsilon$-optimal policy is verifiable to be not worse than the original trial-based algorithm.

### Experiments

We use Tag(55) and Two-Robot Tag, the two POMDP problems with large observation spaces, to study the effects of varying the tuning parameter $C$. Empirical results show that $C = 4$ and 10 are good candidates for the two problems, respectively. We use $C = m \log_2 |Z|$, inspired by the fact that $C$ should be larger when $|Z|$ increases, as a simple formula to set the input constant automatically. Here, $m = 3.22$ is fitted by using the least squares method, which uses $C = 4$, $|Z| = 56$ from the Tag(55) problem and $C = 10$, $|Z| = 625$ from the Two-Robot Tag problem as the training set.

Table 1 compares PLEASE with SARSOP on 8 benchmark problems in terms of the gap between bounds at $b_0$ ($V^U(b_0) - V^L(b_0)$), the lower bound $V^L(b_0)$, the upper bound $V^U(b_0)$, and the total running time (in seconds). On most of these problems this table shows that PLEASE is substantially faster than SARSOP by orders of magnitude.

There are three other observations obtained from more detailed data in our experiments. First, the performance of HSVI2 on our test problems is worse than SARSOP in most cases, and GapMin variants are not efficient in tackling large problems. Second, compared with SARSOP, PLEASE needed much fewer $\alpha$-vectors to get the same gaps on test problems. For example, when the gap is 3.43 on HomeCare, the number of $\alpha$-vectors in PLEASE is 7,947, while 30,633 in SARSOP. Third, when the gaps were the same, SARSOP and PLEASE’s expected total rewards appeared to be similar with each other. Specifically, expected total reward was $-0.73 \pm 0.12$ on Tag(55) when the gap was 5.21; the reward was $-11.58 \pm 0.12$ on Two-Robot Tag(24) when the gap was 7.43; and the expected reward was $17.03 \pm 0.14$ on HomeCare when the gap was 3.43.

### Future Topics

One topic is to enrich the theoretical analysis of the heuristic for observation selection in the current PLEASE method. Another interesting topic is how to use more heuristic information of beliefs (e.g., the depth information of beliefs) in defining the threshold function $\zeta(b, \theta)$.

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### References

