Knocking in the Game of Gin Rummy

Ryzedon C. Maravich, Taylor C. Neller, Todd W. Neller
Gettysburg College
{marary01, nellta01, tneller}@gettysburg.edu

Abstract
We perform an empirical study of Gin Rummy knocking strategies, drawing insight from a population of AI players that vary in both discarding and knocking strategies. For our best performing player, simple linear regression yielded a knocking strategy that both affirmed the features expert players give attention to in making knock decisions, and yet called into question the way such features are conventionally used.

Introduction
In this paper, we will perform an empirical study focusing on knocking strategies for the game of Gin Rummy. We begin with the rules and terminology of Gin Rummy and survey the conventional wisdom for knocking decisions in Gin Rummy literature. Next, we discuss the two dimensions along which we vary our AI players, discard strategy and knocking strategy, overviewing the features we considered in applying linear regression for predicting the relative quality of knock decisions. After describing our experimental design, we share and discuss our experimental results, giving special attention to what those results suggest about the quality of conventional wisdom. Finally, we conclude by noting limitations of our study, suggesting directions for future work, and summarizing our contributions.

Gin Rummy
Gin Rummy is one of the most popular 2-player card games played with a standard (a.k.a. French) 52-card deck. Ranks run from aces low to kings high. The object of the game is to be the first player to score 100 or more points accumulated through the scoring of individual hands.

The play of Gin Rummy, as with other games in the Rummy family, is to collect sets of cards called melds. There are two types of melds: “sets” and “runs”. A set is 3 or 4 cards of the same rank, e.g. 3C-3H-3S, or KC-KH-KS-KD. A run is 3 or more cards of the same suit in sequence, e.g. 5C-6C-7C, or 9H-TH-JH-QH-KH. Melds are disjoint, i.e. do not share cards.

Cards not in melds are referred to as deadwood. Cards have associated point values with aces being 1 point, face cards being 10 points, and other number cards having points according to their number. Deadwood points are the sum of card points from all deadwood cards. Players play so as to reduce their deadwood points.

For each hand of a game, the dealer deals 10 cards to each player. After the deal, the remaining 32 cards are placed face-down to form a draw pile, and the top card is turned face-up next to the draw pile to start the face-up discard pile. The top of the discard pile is called the upcard.

In a normal turn, a player draws a card, either the upcard or the top of the draw pile, and then discards a card. The player may not discard a drawn upcard but may discard a card drawn face-down. For the first turn, play starts with the non-dealer having the option to take the first turn by drawing the upcard. If the non-dealer declines this option, the dealer is given the option. If both decline, the non-dealer must take the first turn drawing from the draw pile.

In the event that the hand has not ended after a turn with only 2 cards remaining in the draw pile, nothing is scored, all cards are shuffled, and the hand is replayed with the same dealer.

After a player has discarded, if that player’s hand has 10 or fewer deadwood points, that player may knock, i.e. end the hand. (Often this is indicated by discarding face-down.) The hand is then scored as follows: The knocking player displays melds and any deadwood. Next, if any deadwood was displayed, the non-knocking player may lay off cards, adding to the knocking player’s melds so as to reduce the non-knocking player’s deadwood. Then, the non-knocking player displays melds and any remaining deadwood.

If the knocking player had no deadwood, they score a 25-point gin bonus plus any opponent deadwood points. If the knocking player had less deadwood than their opponent, they score the difference between the two deadwood totals. Otherwise, if the knocking player had greater than or equal to their opponent’s deadwood points, the opponent scores a 25-point undercut bonus plus the difference between the two deadwood totals.

A player scoring a total of 100 or more points wins the game.

Abstract

\[ \text{Knocking in the Game of Gin Rummy} \]

Ryzedon C. Maravich, Taylor C. Neller, Todd W. Neller

Gettysburg College

\{marary01, nellta01, tneller\}@gettysburg.edu

Abstract
We perform an empirical study of Gin Rummy knocking strategies, drawing insight from a population of AI players that vary in both discarding and knocking strategies. For our best performing player, simple linear regression yielded a knocking strategy that both affirmed the features expert players give attention to in making knock decisions, and yet called into question the way such features are conventionally used.

Introduction
In this paper, we will perform an empirical study focusing on knocking strategies for the game of Gin Rummy. We begin with the rules and terminology of Gin Rummy and survey the conventional wisdom for knocking decisions in Gin Rummy literature. Next, we discuss the two dimensions along which we vary our AI players, discard strategy and knocking strategy, overviewing the features we considered in applying linear regression for predicting the relative quality of knock decisions. After describing our experimental design, we share and discuss our experimental results, giving special attention to what those results suggest about the quality of conventional wisdom. Finally, we conclude by noting limitations of our study, suggesting directions for future work, and summarizing our contributions.

Gin Rummy
Gin Rummy is one of the most popular 2-player card games played with a standard (a.k.a. French) 52-card deck. Ranks run from aces low to kings high. The object of the game is to be the first player to score 100 or more points accumulated through the scoring of individual hands.

The play of Gin Rummy, as with other games in the Rummy family, is to collect sets of cards called melds. There are two types of melds: “sets” and “runs”. A set is 3 or 4 cards of the same rank, e.g. 3C-3H-3S, or KC-KH-KS-KD. A run is 3 or more cards of the same suit in sequence, e.g. 5C-6C-7C, or 9H-TH-JH-QH-KH. Melds are disjoint, i.e. do not share cards.

Cards not in melds are referred to as deadwood. Cards have associated point values with aces being 1 point, face cards being 10 points, and other number cards having points according to their number. Deadwood points are the sum of card points from all deadwood cards. Players play so as to reduce their deadwood points.

For each hand of a game, the dealer deals 10 cards to each player. After the deal, the remaining 32 cards are placed face-down to form a draw pile, and the top card is turned face-up next to the draw pile to start the face-up discard pile. The top of the discard pile is called the upcard.

In a normal turn, a player draws a card, either the upcard or the top of the draw pile, and then discards a card. The player may not discard a drawn upcard but may discard a card drawn face-down. For the first turn, play starts with the non-dealer having the option to take the first turn by drawing the upcard. If the non-dealer declines this option, the dealer is given the option. If both decline, the non-dealer must take the first turn drawing from the draw pile.

In the event that the hand has not ended after a turn with only 2 cards remaining in the draw pile, nothing is scored, all cards are shuffled, and the hand is replayed with the same dealer.

After a player has discarded, if that player’s hand has 10 or fewer deadwood points, that player may knock, i.e. end the hand. (Often this is indicated by discarding face-down.) The hand is then scored as follows: The knocking player displays melds and any deadwood. Next, if any deadwood was displayed, the non-knocking player may lay off cards, adding to the knocking player’s melds so as to reduce the non-knocking player’s deadwood. Then, the non-knocking player displays melds and any remaining deadwood.

If the knocking player had no deadwood, they score a 25-point gin bonus plus any opponent deadwood points. If the knocking player had less deadwood than their opponent, they score the difference between the two deadwood totals. Otherwise, if the knocking player had greater than or equal to their opponent’s deadwood points, the opponent scores a 25-point undercut bonus plus the difference between the two deadwood totals.

A player scoring a total of 100 or more points wins the game.
Conventional Wisdom

Generally, the earlier on in the game, the better off one is to knock. Shankar advises that with fewer than ten cards discarded, it is advisable to knock as soon as one can (Shankar 2015, p. 32). With so many cards available in the stock, the chances of acquiring the card(s) you would need for gin are minimal. Waiting for these card(s) gives the opponent more time to craft a better hand, minimizing their own deadwood and possibly ending up undercutting or ginning.

This same logic can be applied when there are greater than twenty discards, in which case holding out for gin is wiser. In this case, you have a greater chance of drawing the cards you need, and the opponent’s hand has most likely developed to a point where it can only be marginally improved by subsequent draws.

In the literature we surveyed, the most important factor for determining if one should knock is the number of gin hits, or as Shankar calls it, the gin rating, defined as the number of cards that could be drawn to immediately yield gin. For example, if a player had a hand of 2H, 3H, 4H, 2C, 2D, 2S, J5, QS, KS, 8D, they would have a gin rating of 3, as one of AH, 5H, or TS would need to be drawn to go gin. A hand that requires more than one card/turn to go gin has a gin rating of 0. Cohen advises that gin should be played out in most circumstances when there are either 7 or 9 melded cards, because in these situations the gin ratings tend to be higher (Cohen 1973, p. 115). Shankar advises at least a four-way hit before going for gin.

Crawford writes, “When in doubt—knock. Play for gin only if you have an excellent chance or if you strongly suspect that you will lose if you knock. . . . The only time to consider playing for gin is when you have seven or nine cards matched (except when you don’t dare knock because you may be undercut). . . . As a general rule, with four or more gin chances and nine cards matched, play for gin. With only two chances, knock. With three chances, use your judgment . . . ” (Crawford 1961, p. 125-127)

Steig writes of the late turns (“after the fifth”), “If an undercut is not certain, the best policy is usually to knock at the first opportunity. . . . Holding a one-pick hand with four or more live chances for Gin, try for it if you have reason to believe a knock short of Gin will be undercut.” (Steig 1971, p. 98-100)

Fry writes, “Despite the name of the game, going for Gin and gunning for the 25-point bonus is only a secondary or incidental objective. . . . don’t go for it unless your hand is such that you are forced to such a course of action, or unless it is late in the hand and you know your opponent is down very low and waiting to undercut you.” (Fry 1978, p. 6)

Cohen, Shankar, Crawford, Steig, and Fry agree to many nuances regarding going gin, especially if there is much known information about the opponent’s hand or playstyle which could be exploited. Some of these nuances include knowledge of aggressive or defensive play of the opponent, knowledge that the opponent tends to knock early or go for an undercut, and the number of turns that have been played. In any case, various standards used by experts when deciding whether to knock have some form of gin rating as the core principle around which the strategy is built, with the general consensus being a gin rating of at least 4.

Correctly deciding whether to knock or go gin is essential to skillful play. Well-informed decisions can decrease or increase the probabilities of being undercut or scoring gin, respectively. This can amount to a 50 point or more swing in one’s favor in a race for 100 points.

Algorithmic Approaches

To learn a more effective knocking policy in Gin Rummy, we began with the Gin Rummy EAAl Undergraduate Research Challenge code base2 and extended the behavior of the SimpleGinRummyPlayer along two dimensions: discard policy and knock policy. The SimpleGinRummyPlayer class implements a strategy that discards to maximally reduce deadwood, draws the upcard only when it creates a new meld or adds to an existing one, and knocks as soon as possible. We will refer to this as the Simple Player.

Due to the strategic deficiencies of this player, we created another player, which we will refer to as the Hits-Deadwood-Turn player (HDT player), whose discard decisions were based on a function consisting of deadwood points and hit count (e.g., the number of possible draws that could create a new meld or add to an existing one) being scaled by the total turns taken and respective constants. To do this, we created functions for determining the values of these variables on every turn of the game, starting on turn 0, and then used this data to evaluate our function. One turn is counted as a player’s combined draw and discard decision, or in the case of the beginning of the game, a player’s decision to decline the face up card.

Starting with the Simple Player, we used the existing variables and class framework to have a starting player from which we modified the discard decisions. By simulating many games and hand-tuning coefficients, we decided on a simple heuristic function scoring states after discarding based on the hit count, the deadwood, and the turn number. The heuristic score is calculated as follows:

\[
\text{score} = -\text{deadwood} - \frac{1.6 \times \text{deadwood} \times \text{turn}}{10} + \frac{3.6 \times \text{hitCount} \times (15 - \text{turn})}{15}
\]

Instead of choosing the discard that would leave the hand with the least deadwood, i.e. the Simple Player strategy, the HDT player discards so as to maximize the heuristic score. This player makes plays more akin to human plays with management of melds and weighting of priorities based on the turn.

Any player can be made from a combination of drawing, discarding, and knocking decisions. Each of our players share the same draw decision, which is to draw the upcard only when it creates or adds to an existing meld. The two different discard decisions that we have outlined are as follows:

\[\text{http://cs.gettysburg.edu/~tneller/games/ginrummy/eaaI/}\]
1. Simple: This player discards to reduce deadwood.

2. HDT: Discard decisions are based upon the HDT heuristic as described above.

The different knocking policies are as follows:

1. Go Gin: This player only knocks with no deadwood.

2. First Knock: This player knocks at the first possible opportunity.

3. Regression: This player knocks if this expression is positive:

   $$25.878 - 0.152 \cdot \text{deadwood}^2 - 1.148 \cdot \text{turn total} - 1.202 \cdot \text{gin rating}^2$$  \hspace{1cm} (2)

   Data collection for this regression is based on the HDT Go Gin player playing against itself for 10,000 games. Discussion of these regression features is given later in this section.

4. Regression (Simple Data): This player knocks if this expression is positive:

   $$22.115 - 0.151 \cdot \text{deadwood}^2 - 1.017 \cdot \text{turn total} - 0.995 \cdot \text{gin rating}^2$$  \hspace{1cm} (3)

   Data collection for this knocking regression is based on the Simple Go Gin player playing against itself for 10,000 games.

5. Rule-Based: Knocking decisions are based on the following hard-coded rule-based approach. The player will go for gin when the turn total is greater than 7, the player has at least 2 hit cards, and one of the following is true: the player has 3 or more hit cards, 1 unmelded card, or fewer than 3 unmelded cards with less than 6 deadwood. Otherwise this player will knock as soon as possible.

6. Wisdom: Knocking decisions are based on the following rules, collected from the conventional wisdom section. The player will go for gin when the turn total is greater than 9, the player has at least 4 gin hits, and the player has either 7 or 9 melded cards. Otherwise the player will knock as soon as possible.

The HDT and Simple Go Gin Players were used for data collection because this allowed for as long of a game as possible, so that we could collect a greater sample of the state space.

The players used for experimentation were created from combinations of aforementioned discarding and knocking strategies.

The core idea of our knocking regression rests on the method of data collection we used, in which we compared various game states with what would happen if the hand was played out without knocking, including only the turns where the player had 10 or fewer deadwood points. The key feature, which we call the knock difference, is the difference between the scores of the given state had the player knocked then and the outcome had the player gone for gin. By using the knock difference as a dependent variable in a regression, we proceeded to make a linear regression, which could be used to determine if the player should knock if the regression’s expected difference between first-knocking score and attempted-ginning score is positive.

In order to create a successful regression, we tried many logical combinations of features according to the ideas of preexisting conventional wisdom. These features included gin rating, turns taken, deadwood, hit count, the number of upcards the opponent has drawn, the number of sets, the number of runs, and the number of known lay off cards a player currently has. We started with gin rating and turn total as the only two features in our regression. From here we added new features to the regression one by one, noting both the performance of the resulting player, and the weights (size of constant) for each individual feature. The inclusion of deadwood increased the performance of our player. The inclusion of all other features we tested were detrimental to this player’s overall performance. Thus the features that we decided upon for the regression were deadwood count, turns taken, and gin rating.

Interestingly, deadwood and gin rating contributed to better player performance when they were squared. While we were looking at the graph for gin rating versus knock difference isolated from the rest of the regression (Figure 1), we noted that the correlation looked like it was slightly nonlinear, with the higher gin rating values indicating a much greater likelihood of gaining more points by going gin. While further investigation found that a linear regression seemed to suit the data better, the play of the player improved by squaring the terms, so we kept these changes. This was not observed to be true with turns taken; a linear regression both fit the data better and led to better player performance.

One concern we had was that a different regression other than a linear regression would work better at fitting the data, so we tried out a quadratic regression by running our linear regression on the features, the features squared, and the 3 combinations of the features. This strategy, however, yielded a player with poorer performance, so we concluded that selective use of squared terms yielded better play performance.

**Experimental Design**

To compare the performance of the different players, we played each of them against every other player for 10,000 games, collecting the results into Table 1. The table of players is ordered by average number of wins, ordered from highest to lowest.

Our main method for determining statistical significance is the Wilson confidence interval, processed with the number of wins out of 10,000 games between two players. For every pair of players in Table 1, the one with the higher win rate according to the collected data will, with 99% Wilson interval confidence, win more than 50% of the games. Thus each entry in Table 1 is statistically significant. This Wilson interval was also used in determining statistical significance during feature selection. As noted in the Algorithmic Approaches section, we tried many different logical combinations of features to make our regression, using the Wilson confidence interval as our standard for determining better play.
Table 1: Win rates from 10,000 games.

<table>
<thead>
<tr>
<th></th>
<th>HDT Reg</th>
<th>HDT RB</th>
<th>Sim Reg (Sim)</th>
<th>HDT GG</th>
<th>Sim Reg</th>
<th>Sim GG</th>
<th>HDT Wisdom</th>
<th>HDT FK</th>
<th>Sim FK</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDT Reg</td>
<td>-</td>
<td>53.7</td>
<td>57.9</td>
<td>59.6</td>
<td>61.4</td>
<td>64.8</td>
<td>57.5</td>
<td>59.3</td>
<td>70.2</td>
</tr>
<tr>
<td>HDT RB</td>
<td>46.3</td>
<td>-</td>
<td>52.8</td>
<td>54.9</td>
<td>56.9</td>
<td>58.2</td>
<td>59.2</td>
<td>62.6</td>
<td>72.3</td>
</tr>
<tr>
<td>Sim Reg (Sim)</td>
<td>42.1</td>
<td>47.3</td>
<td>-</td>
<td>54.2</td>
<td>53.0</td>
<td>57.8</td>
<td>50.3</td>
<td>52.5</td>
<td>64.0</td>
</tr>
<tr>
<td>HDT GG</td>
<td>40.4</td>
<td>45.1</td>
<td>45.8</td>
<td>-</td>
<td>52.3</td>
<td>52.2</td>
<td>59.7</td>
<td>63.5</td>
<td>73.5</td>
</tr>
<tr>
<td>Sim Reg</td>
<td>38.6</td>
<td>43.1</td>
<td>47.0</td>
<td>47.7</td>
<td>-</td>
<td>52.7</td>
<td>49.3</td>
<td>51.5</td>
<td>63.5</td>
</tr>
<tr>
<td>Sim GG</td>
<td>35.3</td>
<td>41.8</td>
<td>42.2</td>
<td>47.8</td>
<td>47.3</td>
<td>-</td>
<td>53.2</td>
<td>55.8</td>
<td>68.0</td>
</tr>
<tr>
<td>HDT Wisdom</td>
<td>42.5</td>
<td>40.8</td>
<td>49.7</td>
<td>40.3</td>
<td>50.7</td>
<td>46.8</td>
<td>-</td>
<td>52.1</td>
<td>62.4</td>
</tr>
<tr>
<td>HDT FK</td>
<td>40.7</td>
<td>37.4</td>
<td>47.6</td>
<td>36.5</td>
<td>48.5</td>
<td>44.2</td>
<td>47.9</td>
<td>-</td>
<td>60.6</td>
</tr>
<tr>
<td>Sim FK</td>
<td>29.8</td>
<td>27.7</td>
<td>36.0</td>
<td>26.5</td>
<td>36.7</td>
<td>32.0</td>
<td>37.6</td>
<td>39.4</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1 shows the win rate of each player against every other player for 10,000 games, and is ordered from best to worst player based on the number of winning matchups, with ties being broken by highest average win rate. The Simple Regression, HDT Wisdom, and Simple Go Gin players are all tied with three winning matchups because of their cyclical dominance relationships against each other: the Simple Regression player beats the Simple Go Gin player, the Simple Go Gin player beats the HDT Wisdom player, and the HDT Wisdom player beats the Simple Regression player. It makes sense that the Simple Regression player beats the Simple Go Gin player, because the regression selectively goes gin only when it perceives it to be beneficial, and it makes sense that the Simple Regression loses to the HDT Wisdom player, because the different knocking strategies are not enough to account for the performance difference caused by the HDT and Simple discard strategies. We conjecture that the HDT Wisdom player loses to the Simple Go Gin player because it is too selective when going for gin, as described later.

Results and Discussion

Table 1 shows the win rate of each player against every other player for 10,000 games, and is ordered from best to worst player based on the number of winning matchups, with ties being broken by highest average win rate. The Simple Regression, HDT Wisdom, and Simple Go Gin players are all tied with three winning matchups because of their cyclical dominance relationships against each other: the Simple Regression player beats the Simple Go Gin player, the Simple Go Gin player beats the HDT Wisdom player, and the HDT Wisdom player beats the Simple Regression player. It makes sense that the Simple Regression player beats the Simple Go Gin player, because the regression selectively goes gin only when it perceives it to be beneficial, and it makes sense that the Simple Regression loses to the HDT Wisdom player, because the different knocking strategies are not enough to account for the performance difference caused by the HDT and Simple discard strategies. We conjecture that the HDT Wisdom player loses to the Simple Go Gin player because it is too selective when going for gin, as described later.

Except for this situation, we observe transitivity in the performance ranking between players, because all other values are above 50% with statistical significance above the main diagonal. We also note that win rates against any player are not monotonic with player rank.

For example, against the two First Knocking players, one might expect that the highest tier player (the HDT Regression player) would have the highest win rate against them. However, in both cases, the HDT Go Gin and HDT Rule-Based players outperform the top player, with the HDT Go Gin taking the highest win rate, the HDT Rule-Based taking second highest, and the HDT Regression taking third. The conclusion that this suggests is that certain types of players have certain weaknesses in their strategy, and the best response to a First Knocking player is to always go for the undercut. We conjecture that this is the case because a First Knocking player is susceptible to undercuts; knocking early usually means less developed hands and therefore higher deadwood points. Chances of undercutting against the First Knocking are greater later in the game when the opponent has played for gin and did not play for gin are shown in Table 2. This data was collected to analyze the difference between the players in the situations where they knocked or didn’t knock.

Figure 1: Graph of Gin Rating vs Knock Difference with one percent Gaussian jitter. Means are graphed in orange.

We also collected feature data with the HDT Regression, HDT Rule-Based, HDT Wisdom, Simple Regression (Simple Data), and Simple Regression players against the HDT Rule-Based player for 10,000 games. These five players were chosen because they are the only ones that make a game-state-dependent decision about whether to knock or not, whereas the other players either knock as soon as possible or go for gin. The HDT Rule-Based player was chosen as the opponent to provide a standard, well-rounded player for feature data collection.

We recorded how many times each player knocked at the first given opportunity, and how many times they did not, an indication that they are playing for gin. Note that if the player has gin at the first knocking opportunity, this counts as an attempt to play for gin. This is given by the ratio “Knock % Without Gin” in Table 2. We then recorded each of the three features in our regression at the first opportunity the player could have knocked, regardless of whether they did or not. The averages for the features when they...
more time to develop their hand and minimize their own deadwood. Because the Go Gin and the Rule-Based players both play for gin more than the HDT Regression player, they are able to exploit this weakness of the First Knock player the most, and have a better win rate as a result. The HDT Regression player wins against the Rule-Based player because it is more selective in when it goes for gin, as shown by greater percentage of times knocked in Table 2. Thus, while the HDT Regression player has more winning matchups than the Rule-Based player, the Rule-Based player performs better against the First Knocking players.

We also assume that HDT Wisdom player’s poor performance can be attributed to its highly selective criteria for attempting gin. Table 2 shows that the HDT Wisdom player knocks without gin 95% of the time, which is significantly higher than the other regression or rule-based players. This is because the player has the opportunity to knock before the three conditions for going gin can be met. Therefore it plays very much like, but slightly better than, the First Knocking players, which explains its similar win rate to the HDT Regression player. Thus, the conventional wisdom does not seem to predict well when a person should go gin.

Overall, our results indicate that many of the general ideas expressed in the conventional wisdom section were correct. However, there are a few key takeaways to improve performance. Firstly, noting how much results changed in matchups against the First Knock player, we can see that one should note opponent knocking habits, and if they are knocking early consistently, it would probably be a good idea to simply never knock unless you have gin. Additionally, given the results in Table 2, the better policy seems to be to go gin selectively, usually when there is more than 1 gin hit. Lastly, as seen by the large discrepancy between the turn averages in Table 2, it can be inferred that the number of turns heavily affects knocking policy, showing that if one can knock early, i.e. before turn 10, one should probably do so.

**Comparison to Conventional Wisdom**

Overall, our results indicate that many of the general ideas expressed in the conventional wisdom section were correct. However, there are a few claims that seem to be either incorrect or lacking in explicit specification.

First of all, just crafting a player to the knocking specifications of the conventional wisdom section leads to a player that does not play well against our players. This player that we made would go for gin if the turns taken was greater than 9, the gin rating was greater than or equal to 4, and the number of melded cards was 7 or 9, given same draw/discard decisions. This player had a 40.8% win rate against our HDT Rule-Based player and a 42.5% win rate against the HDT Regression player. Thus, the conventional wisdom does not seem to predict well when a person should go gin.

One claim that was mentioned several times was that going for gin is much more promising if one has 7 or 9 melded cards as opposed to 6 or 8 melded cards, which tend to have lower gin ratings. By collecting data at the point where the

<table>
<thead>
<tr>
<th></th>
<th>Knock % Without Gin</th>
<th>Averages when not playing for gin</th>
<th>Averages when playing for gin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Deadwood</td>
<td>Turn</td>
</tr>
<tr>
<td>HDT Reg</td>
<td>.44</td>
<td>6.56</td>
<td>9.24</td>
</tr>
<tr>
<td>HDT RB</td>
<td>.34</td>
<td>7.27</td>
<td>9.91</td>
</tr>
<tr>
<td>HDT Wisdom</td>
<td>.95</td>
<td>6.71</td>
<td>12.77</td>
</tr>
<tr>
<td>Sim Reg (Sim Data)</td>
<td>.50</td>
<td>6.37</td>
<td>8.81</td>
</tr>
<tr>
<td>Sim Reg</td>
<td>.20</td>
<td>6.56</td>
<td>13.51</td>
</tr>
</tbody>
</table>

Table 2: Average of game state features for different players collected at first knocking opportunity for 10,000 games.
regression player could first knock, we found that our HDT Regression player knocks 54.0% of the time when it has 6 or 8 melded cards and 39.2% of the time when it has 7 or 9 melded cards. This makes sense and supports the conventional wisdom, which says that 7 or 9 melded cards is preferable.

However, it should be noted that the regression player still goes for gin 46.0% of the time with 6 or 8 melded, so it appears that the conventional wisdom over-discourages promising gin plays with 6 or 8 melded cards. It is more accurate to say that there is a higher average gin rating among hands with 7 or 9 melded cards, leading to a greater number of hands where it is desirable to go for gin with 7 or 9 melded cards. The average gin rating for 6 or 8 melded cards is .13 as opposed to an average rating of 2.07 for 7 or 9 melded cards. This shows that our player probably goes for gin with 6 or 8 cards because of a late turn and probability of an undercut. It seems that there are still good incentives for going gin much of the time with 6 or 8 melded cards.

One claim that seems to be correct is the turn on which one should knock. The conventional wisdom states that before the 10th turn, one should knock, and the data in Table 2 corroborates this: the two best players, the HDT Regression and HDT Rule-Based players, both seem to have an average knocking turn of around 9, which means that the turns on which they knock are likely under or around 10, strikingly close to the conventional advice.

One claim that seems to be incorrect is the gin rating required to try for gin. The HDT Regression player, which is the best player we have, has an average gin rating of around 1.57 when it goes for gin, which is far from the suggested gin rating of 4. This player goes for gin 56% of the time, much more often than conventional wisdom would advise. The players in Table 2 are not optimal, but the large discrepancy in gin rating when knocking between the HDT Wisdom player and the others raises the question of whether the experts are correct in their assumptions. Additionally, the stipulation that the HDT Wisdom player cannot go for gin without at least a gin rating of four can account for its poor performance.

Conventional wisdom has its focus on good features, but our empirical study recommends different use of such features. As with most expert strategies, there is probably room for improvement in some areas, one of which might be the gin rating at which one should go gin. While wisdom from experts is hard to come by because decisions in the game are very situational and nuanced, this play data suggests that a better strategy for at least a newer player is to go gin more often, maybe settling for a gin rating of 2 on turns later than 10. In any case, good play will consist in part of consideration of gin rating and turns taken, with deadwood being a variable not touched on by the books, and a variable probably involved in situational tie-breaker decisions. These books give a short analysis of knocking decisions that is not bad, but probably could be improved on significantly with further research.

Future Work

As shown with the inclusion of the Simple player with the HDT Regression data, the regression quality depends partly on the player that generates such play data. This means that better players yield better knocking policies, and as progressively better players converge to optimal play, so too will knocking policy.

Indeed, one limitation of this work is that it is conducted within a limited population of strategies. While it is clear that conventional wisdom does not work well in this population of players, it remains to be seen how conventional wisdom varies from optimal knocking play in the context of optimal Gin Rummy play of both players.

Another interesting research direction involves incorporating the other features we mentioned but did not use in our final regression. Features like the number of upcards drawn by the opponent, the number of sets, and the number of runs all seemed to have a positive impact on the quality of the player against the best regression, but the overall play against other types of AI suffered as a result. These features might be excellent for situational matchups, or they might be more applicable against better AI. Another feature or line of features that could be added regards hand estimation, e.g. how far the opponent’s hand has developed.

Even our best player, being based on a high-bias regression, can only make decisions of limited complexity, and thus can only reach a certain threshold of good play. There are, however, numerous other approaches to model-building and machine learning that could yield better play performance. Some of these might include gradient boosted machines, neural networks, or any other regression that could unravel more intricate relationships between different features, especially if more were to be included in the regression.

In this work, we develop a deterministic policy with a focus on knocking decisions. We would expect that optimal knocking policy would be a mixed strategy, i.e. knocking some fraction of the time in some cases so as to conceal information across many hands. Further, we expect that, as with other games, a suboptimal opponent may be exploited by one's knocking strategy. Knowing that the SimpleGinRummyPlayer knocks at the first opportunity, for example, informs us that we will less likely be undercut when knocking and that we can, in turn, have more opportunities to undercut since the deadwood distribution of the SimpleGinRummyPlayer will skew higher from its strategy.

Future work might not only include computation of optimal knocking strategy, but also the detection of suboptimality in opponent knocking and the adjustment of knocking strategy to exploit such suboptimality.

Conclusion

In this paper, we constructed a population of varied discard and knock strategies for the purpose of gaining insight into improved knocking strategy. We were especially interested in how these strategies compared to conventional wisdom in Gin Rummy literature.
The best player of our population of players, HDT Regression, made use of the most important features of conventional knocking strategy, yet made significantly different use of such features, suggesting that conventional wisdom may overly discourage attempts at a gin bonus in common situations where one has 6 or 8 melded cards or a gin rating less than 4.

Our HDT regression player plays as follows: It draws the upcard only when it creates or extends a meld, discards so as to maximize score (1), and knocks when score (2) is positive. We do not claim this deterministic player as optimal, and expect that optimal knocking policy would vary and present mixed strategies in some situations. Nonetheless, we believe that this empirical study suggests that the decision to knock should be more nuanced and less frequent than conventional wisdom dictates.

Acknowledgments
This work was supported, in part, by the Cross-Disciplinary Science Institute at Gettysburg College (X-SIG).

References