A Model for Concurrent Actions having Temporal Extent

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Abstract

In this paper we present a semantic model that is used to interpret a logic that represents concurrent actions having temporal extent. In an earlier paper [Pelavin and Allen, 1986] we described how this logic is used to formulate planning problems that involve concurrent actions and external events. In this paper we focus on the semantic structure. This structure provides a basis for describing the interaction between actions, both concurrent and sequential, and for composing simple actions to form complex ones. This model can also treat actions that are influenced by properties that hold and events that occur during the time that the action is to be executed. Each model includes a set of world-histories, which are complete worlds over time, and a function that relates world-histories that *differ solely on the account of* an action executed at a particular time. This treatment derives from the semantic theories of conditionals developed by Stalnaker [Stalnaker, 1968] and Lewis [Lewis, 1973].

I. Introduction

One of the most successful approaches to representing events and their effects in Artificial Intelligence has been situation calculus [McCarthy and Hayes,1969]. In this logic, an event is modeled by a function from situation, i.e. instantaneous snapshot of the world, to situation. This function captures the state changes produced by the event in different situations.

A deficiency of this representation is that simultaneous events cannot be directly modeled; one cannot describe the result produced by two events initiated in the same situation (see, however, Georgeff [Georgeff, 1986] who extends and modifies situation calculus so this can be done). Another deficiency is that situation calculus does not capture what is happening while an event is occurring. Thus, one cannot directly treat events that are affected by conditions that hold during execution, such as the event "sailing across the lake" which can occur only if the wind is blowing while the sailing is taking place.

Allen [Allen, 1984] and McDermott [McDermott, 1982] have put forth logics that represent simultaneous events and events with temporal extent. Allen develops a linear time model based on intervals, i.e. contiguous chunks of time. McDermott describes a branching time model where a set of instantaneous states are arranged into a tree that branches into the future. McDermott uses the term "interval" to refer to a convex set of states that lie along a branch in the tree of states.

In both logics, an event is equated with the set of intervals over which the event occurs. Properties, which capture static aspects of the world, are treated in a similar manner. Each property is equated with the set of intervals over which the property holds. Simultaneous events can be described by stating that two events occur over intervals that overlap in time. One can also describe the properties that hold and the events that occur while some event takes place.

Although these logics overcome some of the deficiencies of situation calculus, they are not adequate for reasoning about actions and forming plans. These logics lack a structure analogous to the result function in situation calculus that describes the result of executing different actions in different contexts. In situation calculus, the context is given by the situation in which an action is to be initiated. At each situation, one can describe whether an action can be successfully executed and describe whether an action negates some property or does not affect it. This structure also provides a simple basis for constructing complex actions, i.e. sequences of actions.

Without extension, similar statements cannot be made in Allen's and McDermott's logics. For example, one cannot describe that an action does not affect some property or event, such as stating that raising one's arm does not affect whether it is raining out. One cannot represent that some action can be executed only under certain conditions, such as stating that the agent can edit a document during interval i only if the text editor is operational during i.

Allen's and McDermott's logics do not provide a basis for relating the conditions under which a set of actions, concurrent or sequential, can be executed together to the conditions under which the actions, making up the set, can be executed individually. Whether two actions can be executed together depends on how they interact. For example, one may be able to execute two actions individually, but not concurrently, such as "moving one's hand up" and

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"moving one's hand down". It might be the case that two actions can be executed together only under certain conditions, such as two concurrent actions that share the same type of resource. Allen's and McDermott's logics can express "if actions a1 and a2 both occur during i, then there must be at least two resources available during i". These logics, however, cannot distinguish whether "there are at least two resources available during i" is a necessary condition that must hold in order to execute a1 and a2 together, or whether this condition is an effect produced by the joint execution of a1 and a2. A detailed discussion of these issues is given in [Pelavin, 1987].[1]

To remedy these problems, we develop a semantic model that contains a structure analogous to the result function in situation calculus. In our models, *world-histories* and *action instances* take the place of situations and actions. A world-history refers to a complete world over time, rather than an instantaneous snapshot. An action instance, refers to an action to be performed at a specified time. A world-history serves as the context in which the execution of an action instance is specified. This enables us to model the influence of conditions that may hold during the time that an action instance is to be executed, and, as we will see, provides a simple basis for modeling concurrent interactions and for defining the joint execution of a set of action instances.

To describe these models, we extend Allen's language, which is a first order language, with two modal operators. In this paper, we only describe the underlying semantic structure and do not discuss the syntax or interpretation of this modal language. Moreover, we focus on the portion of the model that pertains to modeling actions, after briefly describing the other components in the model structure. The reader interested in the language, axiomatics, or other details omitted in this paper can refer to [Pelavin, 1987] and [Pelavin and Allen, 1986].

II. Overview of the model structure

In each model, a set of world-histories and set of temporal intervals are identified. Each temporal interval picks out a common time across the set of world-histories. The intervals are arranged by the *MEETS* relation to form a global date line. The relation MEETS(i1,i2) is true if interval *i1* is immediately prior to interval *i2*. In [Allen and Hayes, 1985], it is shown that all temporal interval relations, such as "overlaps to the right" and "starts", can be defined in terms of MEETS.

The model identifies the set of properties and events that hold (occur) at various times in the different world-histories. Formally, events and properties are sets of ordered pairs, each formed by an interval and a world-history. If $\langle i,h \rangle \in ev$, then event *ev* occurs during interval *i* in world-history *h*. Similarly, if $\langle i,h \rangle \in pr$, then property *pr* holds during interval *i* in world-history *h*. To capture the relation "if property *pr* holds over an interval *i* then *pr* holds over all intervals contained in *i*", we restrict the models so that if *i1* is contained in *i2* and $\langle i2,h \rangle \in pr$, then $\langle i1,h \rangle \in$ pr.

World-histories are arranged into trees that branch into the future by the *R* accessibility relation which takes an interval and two world-histories as arguments. Intuitively, R(i,h1,h2) means that *h1* and *h2* share a common past through the end of interval *i* and are possible with respect to each other at *i*. This structure is identical to one found in [Haas, 1985] with the exception that Haas uses a time point to relate world-theories, rather than the end of an interval. Constraints are placed on R to insure that i) it is an equivalence relation for a fixed interval, ii) if R(i1,h1,h2), then *h1* and *h2* agree on all events and properties that end before or at the same time as *i1*, and iii) if R(i1,h1,h2), then R(i2,h1,h2) for all intervals *i2* that end at the same time as or before *i1*.

In situation calculus, the execution of an action is given with respect to a situation, and an action is modeled as a function from situation to situation. In our model, the execution of an action instance is given with respect to a world-history, and an action instance is modeled as a function from world-history to set of world-histories. The rest of the paper is devoted to describing this function and showing how a function associated with a set of action instances can be constructed from the functions associated with its members.

In this paper, we will only discuss a type of action instance called a ***basic action instance***. Basic actions [Goldman, 1970] refer to actions that are primitive in the sense that all non-basic actions are brought about by performing one or more basic actions under appropriate conditions. In [Pelavin, 1987], we describe how all other action instances (which we refer to as "plan instances") are defined in terms of basic action instances.

III. The F_{cl} function

The result of executing a basic action instance with respect to a world-history is given by the F_{cl} function. F_{cl} takes a basic action instance *bai* and a world-history *h* as arguments and yields a nonempty set of world-histories that "differ from *h* solely on the account of the occurrence of *bai*". Equivalently, we say that the world-histories belonging to F_{cl}(bai,h) are the "closest world-histories" to *h* where basic action instance *bai* occurs. The term "closest" is a vestige from Stalnaker's [Stalnaker, 1968] and Lewis' [Lewis, 1973] semantic theories of conditionals from which our treatment derives.

In the remainder of this section, we explain what we mean by "differing solely on the account of the occurrence of a basic action instance" and present the constraints that are imposed on F_{cl} in accordance with these intuitions. Very briefly, if *h2* belongs to F_{cl}(bai,h), then *h* and *h2* will coincide on all conditions that are not affected by the occurrence of *bai*. This includes conditions out of the agent's control, such as whether or not it is raining during some interval, and conditions that only refer to times that end before *bai*. One reason that F_{cl} yields a set of world-histories, rather than a single one, is to provide for nondeterministic basic actions. Another reason for treating F_{cl} as a set is explained later.

1 For example, in [Pelavin, 1987] we show why a branching time model cannot be used to interpret "action a1 can be executed during time i" if we want to treat actions, such as "sailing", that are influenced by conditions that hold during execution.

We use a term of the form "*ba@i*" to refer to a basic action instance whose time of occurrence is *i*. The treatment of F_{cl}(ba@i,h) is trivial when *ba@i* occurs in *h*. In this case, F_{cl}(ba@i,h) is equal to {h} reflecting the principle that a world-history is closer to itself than any other world-history. This is captured by the following constraint which is imposed on our models:

BA1)
For all basic action instances (ba@i),
and world-histories (h), if h $\in$ OC(ba@i),
then F_{cl}(ba@i,h) = {h}

where OC(ba@i) is the set of world-histories
in which ba@i occurs

F_{cl}(ba@i,h) is also set to {h} when *ba@i*'s *standard conditions* do not hold in *h*. The term "standard conditions" is taken from Goldman [Goldman, 1970] although we use it in more general way. A basic action's standard conditions are conditions that must hold in order to execute the action. For example, the standard conditions for "the agent moves its right arm up during time *i*" include the condition that the arm is not broken during time *i*. We also use standard conditions to refer to the conditions under which a move is legal when modeling a board game.

If *ba@i*'s standard conditions do not hold in *h*, then F_{cl}(ba@i,h), which equals {h}, contains a world-history in which *ba@i* does not occur. In effect, if *ba@i*'s standard conditions do not occur in *h*, we are not defining "the closest world-history to *h* where bai@i occurs". We treat the lack of standard conditions this way because we want to restrict F_{cl} so that if *h2* belongs to F_{cl}(ba@i,h) then *h2* and *h* agree on all conditions that are not affected, directly or indirectly, by *ba@i*. This restriction would be violated if bai@i's standard conditions did not hold in *h*, but F_{cl}(bai@i,h) contained a world-history *h2* where *ba@i* occurs. This stems from an assumption that a basic action cannot affect whether or not its own standard conditions hold.

F_{cl}(ba@i,h) yields a non-trivial result when *ba@i*'s standard conditions hold in *h*, but *ba@i* does not occur in *h*. In this case, all the members belonging to F_{cl}(ba@i,h) differ from *h* and *ba@i* occurs in all these world-histories. Consequently, we impose the following constraint:

BA2)
For all world-histories (h and h2)
and basic action instances (ba@i),
if F_{cl}(ba@i,h) $\neq$ {h} then
F_{cl}(ba@i,h) $\subseteq$ OC(ba@i)

Typically, when *h2* belongs to F_{cl}(ba@i,h) and *h2* is distinct from *h* (which we will assume in the rest of this section), the two world-histories will differ on more than the status of "*ba@i* occurs". We assume that the set of world-histories adhere to a set of laws that govern the relations between events, properties, and other objects in the world-histories. A world-history formed by just modifying *h* to make "*ba@i* occurs" true may violate some laws. Consequently, *h* and *h2* will also differ on some conditions that are related, directly or indirectly, to "*ba@i* occurs" by some set of laws.

As an example, suppose that property *pr2* does not hold during interval *i2* in *h*, but there is a law that entails that if *ba@i* occurs then *pr2* holds during *i2*. Consequently, *h* and *h2* must differ on the status of "*pr2* holds during *i2*" since *ba@i* occurs in *h2*. World-histories *h* and *h2* may also differ on conditions that are indirectly affected by *ba@i*. Suppose that there is a second law that entails that if *pr2* holds during *i2* then *pr3* holds during *i3*. If *pr3* does not hold during *i3* in *h*, then *h* and *h2* will also differ on this condition.

As a second example, consider a law that entails that *ba@i* and *ba2@i* cannot occur together. Thus, if *ba2@i* occurs in *h*, any world-history *h2* belonging to F_{cl}(ba@i,h) will differ from *h* because *ba2@i* does not occur in *h2*. This type of relation, as we will see, forms the basis for detecting interference between basic action instances and is used when composing basic action instances together.

We assume that the difference between *h* and *h2* are minimal in that changes are only made in going from *h* to *h2* to satisfy laws that would be violated if these changes were not made. They agree on all other conditions. This includes conditions out of the agent's control such as whether or not it is raining out. We also constrain our models so that *h* and *h2* agree on all conditions that refer to times that are prior to *ba@i*'s time of occurrence. This is captured by a constraint relating F_{cl} to the R relation which is given as follows:

BA-R1)
For all world-histories (h1 and h2),
basic action instances (ba@i), and intervals (i0),
if h2 $\in F_{cl}$(ba@i,h) and MEETS(i0,i), then R(i0,h,h2)

BA-R1 entails the relation that two world-histories differing on the occurrence of *ba@i* must coincide on all conditions that end before the beginning of interval *i*. This restriction presupposes that there are no laws specifying whether or not a basic action instance, whose standard conditions hold, occurs.

One reason why F_{cl}(ba@i,h) yields a set of world-histories, instead of a single one, is that there may be many ways to minimally modify *h* to account for *ba@i*'s occurrence. For example, suppose that only two of the three basic action instances, *ba1@i*, *ba2@i*, and *ba3@i*, can be executed together. Also assume that both *ba2@i* and *ba3@i* occur in *h*. In this case, F_{cl}(ba1@i,h) will contain (at least) two world-histories: one where both *ba1@i* and *ba2@i* occur, but not *ba3@i*, and another where *ba1@i* and *ba3@i* occur, but not *ba2@i*.

It is important to emphasize that the F_{cl} function is part of the semantic model and thus there is no need to precisely specify this function when reasoning in our logic. We describe the world using a set of sentences in our language (which is described in [Pelavin and Allen, 1986] and [Pelavin, 1987]) Typically, a set of sentences only partially describe a model; there may be many models that satisfy a set of sentences. The F_{cl} function provides a simple underlying structure to interpret sentences that describe what a basic action instance affects and does not affect with respect to a context that may include conditions that hold while the basic action instance is to be executed. As we will see, it also provides a simple basis for modeling basic action instance interactions and for treating the joint execution of a set of basic action instances.

IV. Composing action instances

The result of executing a set of basic action instances together is computed from the individual members in the set. In other words, F_{cl} applied to a set of basic action instances *bai-set* is defined in terms of F_{cl} applied individually to each member in *bai-set*. In this section, we will let F_{cl} take a set of basic actions instances as an argument, rather than a single one; F_{cl} applied to the singleton set {bai} is to be treated as we described F_{cl} applied to *bai* in the last section.

Any set of basic action instances can be composed together regardless of their temporal relation. Moreover, the definition of F_{cl} applied to *bai-set* does not need to be conditionalized on the temporal relations between the members of *bai-set*. So for example, the composition of two concurrent basic action instances is defined in the same way as the composition of two basic action instances that do not overlap in time.

The following notation is introduced to succinctly present the definition of F_{cl} applied to a basic action instance set and to present two related constraints.

The constructor function "*" combines two functions from H to 2^H to form a function from H to 2^H, where H denotes a set of world-histories:

$$fx*fy(h) =_{def} \bigcup_{hx \in fx(h)} fy(hx)$$

The set of *composition functions of a basic action instance set* is recursively defined by:

i) A singleton basic action instance set {bai} has one composition function: $\lambda h.F_{cl}(\{bai\},h)$

ii) The composition functions of a basic action instance set *bai-set* with more than one element: {bai*cmp | bai ∈ bai-set and cmp is a composition function of (bai-set — bai)}

If *cmp* is a composition function of *bai-set*, then cmp(h) yields the set of world-histories that would be reached by successively modifying *h* by the basic action instances belonging to *bai-set* in some order.

ALL-OC relates world-histories and composition functions:

For any composition function of bai-set (cmp), ALL-OC(h,cmp) $=_{def}$ cmp(h) ⊆ OC(bai-set)

If *cmp* is a composition function of *bai-set*, then ALL-OC(h,cmp) is true iff the result of modifying *h* successively by all the members in *bai-set*, in the order implicit in *cmp*, yields a set of world-histories where all the members in *bai-set* occur.

Finally, the definition of F_{cl} and constraints BA-CMP1 and BA-CMP2 are given by:

$$F_{cl}(\text{bai-set},h) =_{def} \begin{cases} \text{cmp}(h) & \text{If there exists a composition function of bai-set (cmp) such that ALL-OC(h,cmp)} \\ \{h\} & \text{Otherwise} \end{cases}$$

BA-CMP1)
For all world-histories (h) and basic action instance sets (bai-set), if there exists two composition functions of bai-set (cmp1 and cmp2) such that ALL-OC(h,cmp1) and ALL-OC(h,cmp2), then cmp1(h) = cmp2(h)

BA-CMP2)
For all world-histories (h) and composition functions (cmp1 and cmp2),if ALL-OC(h,cmp2) and ALL-OC(h,cmp1*cmp2) then ALL-OC(h,cmp2*cmp1)

In the following discussion, we examine BA-CMP1, BA-CMP2, and the definition of F_{cl}(bai-set) for the case where *bai-set* consists of two basic action instances, *bai1* and *bai2*, that yield unique closest world-histories when F_{cl} is applied to either of them at any world-history. In [Pelavin, 1987], a detailed explanation is provided for the other cases, such as when *bai-set* contains three or more members.

The set {bai1,bai2} has two composition functions, which we will denote by bai1*bai2 and bai2*bai1. bai1*bai2(h) yields a singleton set containing the world-history obtained by modifying *h*, first by *bai1*, then by *bai2*. bai2*bai1(h) yields a singleton set containing the world-history obtained by modifying *h*, first by *bai2*, then by *bai1*. It is important to keep in mind that *bai1* and *bai2* have fixed times associated with them and consequently may have any temporal relation. Thus, bai1*bai2(h) does not necessarily describe the results of executing *bai1* before *bai2*, since *bai2* may be prior to or concurrent with *bai1*.

Let us first consider the case where *bai1*'s and *bai2*'s standard conditions hold at all world-histories. We say that *bai1* and *bai2* interfere at world-history *h* if they cannot be executed together in the context given by *h*. If they interfere, we set F_{cl}({bai1,bai2},h) to {h}, treating {bai1,bai2} as if its standard conditions do not hold at *h*. As an example, "move right hand up during i" and "move right hand down during i" are basic action instances that interfere at all world-histories (when modeling a typical world). Conversely, "move right hand up during i" and "move left hand down during i" do not interfere at any world-history.

We may also model basic action instances that conditionally interfere, ones that interfere at some world-histories but not at others. For example, if two concurrent basic action instances share the same type of resource, they interfere only at world-histories where there is not enough of this resource available during their time of execution. It is important to note that interference is defined relative to world-histories. Consequently, whether two or more basic actions interfere can depend on conditions that hold <u>during</u> execution. Some other treatments of interference in the AI literature, such as Georgeff [Georgeff, 1986], provide for conditional interference, but only in the case when interference depends on conditions that hold just prior to execution.

We can detect whether *bai1* and *bai2* interfere at a world-history *h* by examining F_{cl} applied to *bai1* and *bai2* individually. Since we are assuming that *bai1*'s (and *bai2*'s) standard conditions hold everywhere, F_{cl}({bai1},h) yields a world-history in which *bai1* occurs. Call this world-history *hx*. If *bai1* and *bai2* interfere at *h*, and consequently at *hx*, F_{cl}({bai2},hx) yields a world-history where *bai2* occurs (since its standard conditions hold at *hx*), but not *bai1*. If they do not interfere, both *bai1* and *bai2* occur in $F_{cl}(\{bai2\},hx)$ in which case we set $F_{cl}(\{bai1,bai2\},h)$ to $F_{cl}(\{bai2\},hx)$.[2] Since $F_{cl}(\{bai2\},hx)$ is the result of modifying *h* first by *bai1*, then by *bai2*, it is equivalent to bai1*bai2(h).

We can also detect if *bai1* and *bai2* interfere by modifying *h* first by *bai2*, then by *bai1*. bai2*bai1(h) yields this world-history. If *bai1* and *bai2* interfere at *h*, then *bai1*, but not *bai2*, occurs in bai2*bai1(h). If they do not interfere, both *bai1* and *bai2* occur in bai2*bai1(h). Moreover, if they do not interfere, we assume that modifying *h*, first by *bai1*, then by *bai2* yields the same world-history obtained by modifying *h*, first by *bai2*, then by *bai1*.

The definition of F_{cl} and constraints BA-CMP1 and BA-CMP2 capture the treatment described above. If *bai1* and *bai2* interfere with each other at *h*, then *bai1* and *bai2* do not both occur together in either bai1*bai2(h) or bai2*bai1(h). Consequently, $F_{cl}(\{bai1,bai2\},h)$ is defined as $\{h\}$. If *bai1* and *bai2* do not interfere, then they occur together in both bai1*bai2(h) and bai2*bai1(h). In this case $F_{cl}(\{bai1,bai2\},h)$ is set to bai1*bai2(h) which equals bai2*bai1(h) by constraint BA-CMP1. For the case where *bai1*'s and *bai2*'s standard conditions hold everywhere, constraint BA-CMP2 insures that bai1*bai2(h) and bai1*bai2(h) are compatible; they would be incompatible, if both *bai1* and *bai2* occurred together in one of them, signifying that *bai1* and *bai2* did not interfere at *h*, but did not occur together in the other, signifying they did interfere at *h*.

The analysis described above also applies in less restrictive cases where *bai1*'s and *bai2*'s standard conditions may not hold at all world-histories. This analysis is applicable as long as *bai1*'s standard conditions hold at both *h* and $F_{cl}(\{bai2\},h)$, and *bai2*'s standard conditions hold at both *h* and $F_{cl}(\{bai1\},h)$.

Let us now consider the case where both *bai1*'s and *bai2*'s standard conditions hold at *h*, but the occurrence of one of the basic action instances, say *bai1*, ruins the others standard conditions. This situation is treated as interference; $F_{cl}(\{bai1,bai2\},h)$ is set to $\{h\}$. If *bai1* ruins *bai2*'s standard conditions with respect to *h* then *bai2*'s standard conditions do not hold in $F_{cl}(\{bai1\},h)$. Consequently, *bai1*, but not *bai2*, occurs in bai1*bai2(h). By constraint BA-CMP2, both *bai1* and *bai2* will not occur together in bai2*bai1(h) either. Thus, by the definition of F_{cl}, we see that $F_{cl}(\{bai1,bai2\},h)$ is set to $\{h\}$.

The next case to consider is where *bai1*'s, but not *bai2*'s, standard conditions hold at *h*. In this situation, $F_{cl}(\{bai1,bai2\},h)$ is set to $\{h\}$ unless the following two conditions hold: i) the occurrence of *bai1* with respect to *h* brings about *bai2*'s standard conditions, and ii) they do not interfere with each other at *h*. If both i) and ii) hold, then both *bai1* and *bai2* occur together in bai1*bai2(h). Consequently, by the definition of F_{cl}, $F_{cl}(\{bai1,bai2\},h)$ is set to bai1*bai2(h). This case differs from the situation where both *bai1*'s and *bai2*'s standard conditions hold in *h*; if *bai1* brings about *bai2*'s standard conditions, then *bai1* and *bai2* do not necessarily occur together in bai2*bai1(h) even though they do not interfere. Appropriately, constraints BA-CMP1 and BA-CMP2 are not applicable in this case.

The last case to consider is where both *bai1*'s and *bai2*'s standard conditions do not hold in *h*. In this situation, $F_{cl}(\{bai1,bai2\},h)$ is simply set to $\{h\}$.

V. Conclusion

We have presented a model that provides for concurrent actions having temporal extent. We have integrated Allen's model [Allen,1984], which can treat simultaneous events having temporal extent, with a structure analogous to the result function in situation calculus. This structure captures the result of executing an action at a specified time with respect to a context given by a world-history, i.e. a complete world over time. This enables us to model actions and action interactions that are affected by conditions that hold during execution. This structure also provides a simple framework for composing simple actions, both concurrent and sequential, to form complex ones.

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2 World-history *hx* and the world-history in $F_{cl}(\{bai1\},hx)$ are not necessarily distinct from *h*. For example, if both *bai1* and *bai2* occur in *h*, then $F_{cl}(\{bai1\},h) = F_{cl}(\{bai2\},h) = \{h\}$ by constraint BA1; consequently $F_{cl}(\{bai1,bai2\},h)$ equals $\{h\}$.